Uncertainty Cost Functions in Cubic Wind Speed-Power Relationships for Controllable Energy Systems: Monte Carlo, Numerical, and Analytical Validation Approaches

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Abstract: - The energy dispatch presents high variability due to the high increase due to the stochasticity of renewable resources. The deviations from unforeseen events increase the marginal cost of power generation, thus improving cost estimation It turns out to be essential. The estimation of uncertainty costs by the energy use wind presents an impact significant due to its stochasticity. This study aims to validate the uncertainty cost function for wind energy systems using Monte Carlo simulations, numerical integration, and analytical methods, where the power output is related to wind speed by ($P = k \cdot V^3$). Wind speed is modeled using the Weibull distribution, and uncertainty costs are computed for overestimation and underestimation scenarios. Numerical integration, and closed-form analytical expressions are formulated based on the lower incomplete gamma function. Monte Carlo simulations are used to generate wind speed scenarios, while numerical integration and analytical formulations are employed to derive expected costs. The mean uncertainty cost obtained from Monte Carlo simulations matches the results from numerical integration and analytical methods, validating the proposed approach. The study demonstrates the reliability of the uncertainty cost function for wind energy systems, providing a robust framework for managing uncertainty in renewable energy integration. This framework allows system operators to accurately quantify uncertainty costs, thereby improving dispatch decisions. Relative errors between methods ranged from 0.008% to 0.042%. Other methods evaluated include Kalman filtering and neural network wind forecasting, which significantly reduced costs for two low-power scheduled dispatch cases: 1 MW and 10 MW, to 50.694% and 62.285%.

Key-Words: - Uncertainty Cost Functions, Cubic Wind Speed-Power Relationships, Controllable Energy Systems, Monte Carlo, Wind Energy, Stochastic Modeling.

1 Introduction

Economic dispatch of wind-integrated power systems suffers from high uncertainty costs due to the cubic wind–power law and wind variability. This paper addresses the lack of a fully validated uncertainty-cost function for Weibull-distributed wind speeds. The incorporation of wind energy into electrical power systems inevitably introduces a degree of uncertainty, primarily stemming from the inherent intermittency of wind resources [1,14]. This intermittency directly affects the predictability and reliability of power generation, posing significant challenges for economic dispatch and overall energy management strategies. Electrical power systems that integrate renewable energy sources such as solar or wind energy, must contend with the inherent uncertainty regarding the availability of injected or demanded power. This uncertainty subsequently leads to uncertainty costs, which must be carefully considered within stochastic economic dispatch models to ensure accurate energy resource allocation and system management [1,14]. The stochastic nature of wind turbine speed introduces uncertainties that can significantly affect the economic dispatch of electric power [2,15]. These uncertainties necessitate the development and implementation of robust methodologies for quantifying and mitigating the associated costs, thereby ensuring the stability and economic viability of power grids with substantial renewable energy penetration.

Accurate modelling and quantification of uncertainty costs are essential for the effective management of energy resources and the reliable operation of power grids [1, 14]. By accurately assessing the economic impact of wind power variability, system operators can make informed decisions regarding resource allocation, grid infrastructure investments, and the implementation of advanced control strategies. The estimation of these costs is crucial for proper management of energy resources and accurate allocation of the amount of energy available for the system [1, 14]. Furthermore, the availability of reliable uncertainty cost functions facilitates the development of robust economic dispatch models that can effectively balance the trade-offs between cost minimization and system reliability, thereby ensuring the long-term sustainability of wind-integrated power systems [2-7].

Uncertainty cost formulations have been analyzed to contrast the real power generation failures from overestimation due to penalties due to failure to supply agreed upon and underestimation due to losses due to not taking advantage of opportunities [7, 18]. In literature, some reports are made of studies for the power dispatch wind with techniques inadequate, which bring network effects such as increase in the frequency and magnitude of fluctuations [8, 20]. Therefore, the need to use reliable forecasting techniques, which allow carry out a sophisticated management of these resources [15-19].

This article proposes a technique for energy estimation of wind power, which provides an approach to uncertainty cost estimation associated with the formulation of distribution functions that allow the characterization of air speed [20-21]. The formulation incorporates properties statistics of the office air formulation economical with air speed. This formulation allows improved market competitiveness economically in those that require efficiency computational, since the stochasticity of the power supply is represented by Monte Carlo simulations, allowing an answer competitiveness with scenarios with uncertainty wind power. The presented article has the following contributions.

1. Derive closed-form uncertainty cost expressions for $P = k V^3$ under Weibull wind.
2. Validate these expressions against numerical integration and Monte Carlo simulation.
3. Quantify computational performance for different power ratings.
4. Analysis of the Kalman filtering method and neural network wind forecasting to estimate uncertainty costs due to overestimation and underestimation.

The article is organized with the following Sections. Section 2 presents a review of the state of the art in cost estimation under uncertainty in wind energy systems. Section 3 describes the methodology proposed for this research. Section 4 presents the results obtained from the simulation and its corresponding validation. Finally, Section 5 presents the conclusions reached and proposes future research.

2 State of the art

Seminal work incorporating both over- and underestimation penalties for wind using Weibull wind speed into economic dispatch. Cost terms explicitly split, numerical scenario-based optimization used, setting conceptual framework for risk-aware wind integration in power systems [5, 14-15]. Shi (2010) Extended distance with explicit reserve and environmental cost modeling, tied to uncertainty in wind output [4]. Adopted similar probability-based approaches, using evolutionary programming for optimal power flow [2,16-19]. Shi (2012) systematized optimal power flow solutions with wind, refined cost function approach incorporating opportunity costs of shortfall or surplus, and further emphasized Monte Carlo scenario generation driven by Weibull Wind probability distribution functions [21-22].

Rapid adoption of metaheuristic algorithms (Cuckoo Search, Harmony Search, Aquila Optimizer) to cope with increased problem complexity due to wind uncertainty [1, 3, 5, 6, 7]. Monte Carlo scenario generation with Weibull wind modeled as standard; focus on solver performance and integration of cost penalty functions into optimal power flow. Cost models largely follow [5], with minor variations in presenting or scaling under and overestimation penalties. Extensive benchmarking on IEEE test systems.

Yan (2017) Introduced statistical learning for improved wind forecasting, defined quadratic and probability-of-shortfall-based cost metrics, used probabilistic wind power output but did not reach closed-form gamma- function results [8]. Arevalo (2019) shifted towards analytical closed-form formulas for expected uncertainty costs (though mainly Rayleigh for wind), and crucially validated them with Monte Carlo simulations, directly connecting analytics to simulation [9]. Reyes (2020) Extended [9], derived marginal and minimum uncertainty cost functions for multiple renewables [10]. Cross- comparison of analytical approaches and Monte Carlo simulation; introduced concept of uncertainty cost function and marginal uncertainty cost for wind.

In literature, not single paper fully validates an uncertainty cost function for wind power, assuming $P = k \cdot V^3$ and Weibull- distributed wind speeds, using all three methods: Monte Carlo simulation, numerical integration, and closed-form analytical expressions based on the gamma function. However, partially relevant studies do exist. optimal power flow and economic dispatch papers employ Monte Carlo methods with Weibull distributions and the cubic power law [1–7], while only [9-14] provide partial analytical cost derivations, primarily for The Rayleigh case, supported by Monte Carlo validation.

3 Proposed Methodology

Describes the Weibull distribution parameters (scale c and shape k) and their role in modeling wind speed variability. Wind speed (V) is modeled using the Weibull distribution, characterized by scale parameter (c = 10) and shape parameter (k = 2). The probability density function of the Weibull distribution is given by [9, 10]:

$$f(v) = \frac{k}{c} \cdot \left(\frac{v}{c}\right)^{k-1} \cdot \exp\left(-\left(\frac{v}{c}\right)^{k}\right) \qquad (1)$$

v is the wind speed, $k$is the shape parameter, and $c$is the scale parameter. Explain the relationship $P = k \cdot V^3$and its significance in wind energy systems, the constant used in the study is k = 1/100.

$$P = k \cdot V^3 \qquad (2)$$

According to these values, the programmed power and the number of Monte Carlo simulations are established. To generate 100,000 stochastic scenarios, we have: generate wind speeds, compute power outputs, and calculate overestimation and underestimation costs. For each scenario i, the wind speed $v_i$is generated from the Weibull distribution, and the power output (W_i) is computed as:

$$W(v) = \frac{v^3}{100} \qquad (3)$$

Present the numerical integration approach for computing expected overestimation and underestimation costs. Explain the integration limits and the use of the Weibull probability density function.

$$C_O = \{W_s - W_i \text{ if } W_i\ W_s \geq\ W_i, 0 \text{ otherwise},$$
$$C_U = \{\ W_i -\ W_s if\ W_i >\ W_s, 0 \text{ otherwise}. \qquad (4)$$

3.1 Neural Network Forecasting of Wind Speed

To forecast wind speed, it is employed a feedforward neural network trained on historical wind data using a sliding window of fixed length. Given a time, series of wind speeds $V = \{v_1, v_2, \dots, v_T\}$, the network learns the mapping.

$$\hat{v}_{t+1} = f(v_t, v_{t-1}, \dots, v_{t-n+1}) \qquad (5)$$

where $\hat{v}_{t+1}$ is the forecasted wind speed at time $t-n+1$, and n is the window size. The network function f is defined as:

$$f(X) = \phi^{(2)}\left(\boldsymbol{W}^{(2)}\phi^{(1)}\left(\boldsymbol{W}^{(1)}\boldsymbol{x} + \boldsymbol{b}^{(1)}\right) + \boldsymbol{b}^{(2)}\right)$$
(6)

where $\mathbf{W}^{(1)}$, $\mathbf{W}^{(2)}$ are weight matrices for the input and output layers, $\mathbf{b}^{(1)}$, $\mathbf{b}^{(2)}$ are bias vectors, and $\phi^{(1)}$, $\phi^{(2)}$ are activation functions.

Kalman Filtering for Wind Speed Estimation

To improve prediction accuracy, it is applied a Kalman filter to correct the neural network output based on sensor measurements. The system is modelled as a linear dynamic process:

$$x_t = Ax_{t-1} + w_t, \qquad w_t \sim N(0, Q),$$
$$z_t = Hx_{t-1} + v_t,\ v_t \sim N(0, R), \qquad (7)$$

where x_t is the true wind speed at time t, z_t is the noisy measurement, and A, H are system matrices (usually $A = H = 1$).

At each time step, the filter proceeds through prediction and update phases. The following is the prediction step.

$$\hat{x}_{t|t-1} = A\hat{x}_{t-1|t-1}, \qquad (8)$$

$$P_{t|t-1} = AP_{t-1|t-1}A^{\top} + Q, \qquad (9)$$

The following is the update step.

$$K_t = \frac{P_{t|t-1}H^{\top}}{HP_{t|t-1}H^{\top} + R}, \qquad (10)$$

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t\left(z_t - H\hat{x}_{t|t-1}\right), \qquad (11)$$

$$P_{t|t} = (I - K_tH)P_{t|t-1}. \qquad (12)$$

The corrected estimate $\hat{x}_{t|t}$ is then used in the Monte Carlo simulation to compute the expected penalty cost.

3.2 Numerical integration

Present the numerical integration approach for computing expected overestimation and underestimation costs. Explain the integration limits and the use of the Weibull probability density function.

Expected Costs via Numerical Integration, is calculated the expected overestimation cost and expected underestimation cost.

$$C_O^{expected} = \int_0^\infty C_{O(v)} \cdot f(v)dv \quad (13)$$

$$C_U^{expected} = \int_0^\infty C_{U(v)} \cdot f(v)dv \quad (14)$$

The total expected cost can be calculated as

$$C_{total}^{expected} = C_O^{expected} + C_U^{expected} \quad (15)$$

where

$$C_O(v) = \left(W_s - W(v)\right) \cdot I(W(v) < W_s) \quad (16)$$

$$C_U(v) = (W(v) - W_s) \cdot I(W(v) > W_s) \quad (17)$$

3.2.1 Underestimation costs

The steps to determine cost underestimation are described below, $I(.)$which is the indicator of the function.

$$C_U^{\text{expected}} = \int_{v_s}^\infty (W(v) - W_s) \cdot f(v) \; dv \quad (18)$$

The integral is split into two parts,

$$C_U^{\text{expected}} = \int_{v_s}^\infty W(v).f(v)\, dv - \cdots$$

$$\ldots W_s \int_{v_s}^\infty f(v)\, dv \quad (19)$$

Both terms are resolved for cost underestimation.

$$I_1 = \int_{vs}^\infty W(v) \cdot f(v)dv, \quad (20)$$

$$I_2 = \int_{vs}^\infty f(v)dv \quad (21)$$

Replacing both terms, we obtain the following expression.

$$C_U^{\text{expected}} = I_1 - W_s \cdot I_2 \quad (22)$$

Substitute $W(v) = \frac{v^3}{100}$ and $f(v)$

$$I_1 = \frac{c^3}{100} \cdot \Gamma\left(\frac{k+3}{k}, u_s\right) \quad (23)$$

where $u_s = \left(\frac{v_s}{c}\right)^k$, the integral I_2 is

$$I_2 = \exp(-u_s) \quad (24)$$

The expected underestimation cost is

$$C_U^{\text{expected}} = \frac{c^3}{100} \cdot \Gamma\left(\frac{k+3}{k}, u_s\right) - W_s \cdot \ldots$$

$$\ldots \exp(-u_s) \quad (25)$$

3.2.2 Overestimation costs

The steps for determining cost overestimation are described below. The overestimation cost $C_O(v)$ es given by:

$$C_O^{\text{expected}} = \int_0^\infty \left(W_s - W(v)\right) \cdot f(v)\, dv \quad (26)$$

where $f(v)$is the Weibull, the integral is split into two parts with the probability distribution functions.

$$C_O^{\text{expected}} = \int_0^\infty W_s \cdot f(v) \cdot I(W(v) < W_s)dv \ldots$$

$$\ldots - \int_0^\infty W(v) \cdot f(v) \cdot I(W(v) < W_s)dv \quad (27)$$

The two terms are presented from the two expressions shown

$$I_1 = \int_0^\infty W_s \cdot f(v) \cdot I(W(v) < W_s)dv, \quad (28)$$

$$I_2 = \int_0^\infty W(v) \cdot f(v) \cdot I(W(v) < W_s)dv \quad (29)$$

Therefore, the expression for overestimation costs is obtained from the two terms.

$$C_O^{\text{expected}} = I_1 - I_2 \quad (30)$$

The integral I_1simplifies to:

$$I_1 = W_s \cdot \int_0^{vs} f(v)dv \quad (31)$$

where the expression can be represented by the function

$$F(v_s) = 1 - \exp\left(-\left(\frac{v_s}{c}\right)^k\right) \quad (32)$$

The integral I_1simplifies to:

$$I_1 = W_s \cdot \int_0^{vs} f(v)dv \tag{33}$$

The integral I_2 es:

$$I_2 = \int_0^{vs} W(v) \cdot f(v)dv \tag{34}$$

Substitute $W(v) = \frac{v^3}{100}$ and f(v):

$$I_2 = \frac{k}{100 \cdot c^k} \int_0^{v_s} v^{k+2} \cdot \exp\left(-\left(\frac{v}{c}\right)^k\right) dv \tag{35}$$

Let $u = \left(\frac{v}{c}\right)^k$, then $du = \frac{k}{c} \cdot \left(\frac{v}{c}\right)^{k-1} dv$, and $v = c \cdot u^{\frac{1}{k}}$

$$I_2 = \frac{c^3}{100} \int_0^{u_s} u^{\frac{k+3}{k-1}} \cdot e^{-u} du \tag{36}$$

where $u_s = \left(\frac{v_s}{c}\right)^k$.The integral is expressed in terms of the lower incomplete gamma function:

$$I_2 = \frac{c^3}{100} \cdot \gamma\left(\frac{k+3}{k}, u_s\right) \tag{37}$$

where $\gamma(a, x)$is the lower incomplete gamma function. The expected overestimation cost es:

$$C_O^{\text{expected}} = I_1 - I_2 \tag{38}$$

substitute $I_1$and I_2:

$$C_O^{expected} = W_s \cdot \left(1 - \exp(-u_s)\right) - \cdots$$

$$\ldots \left(\frac{c^3}{100}\right) \cdot \gamma\left(\frac{k+3}{k}, u_s\right) \tag{39}$$

using the relationship between the gamma function and the incomplete gamma function:

$$\gamma(a, x) = \Gamma(a) \cdot gammainc(x, a, 'lower') \tag{40}$$

where gammainc$(x, a, 'lower')$is the regularized lower incomplete gamma function. Thus:

$$I_2 = \frac{c^3}{100} \cdot \Gamma\left(\frac{k+3}{k}\right) \cdot$$

$$gammainc\left(u_s, \frac{k+3}{k}, 'lower'\right) \tag{41}$$

substitute $I_1$and I_2:

$$C_O^{expected} = W_s \cdot \left(1 - \exp(-u_s)\right) \ldots$$

$$\ldots - \left(\frac{c^3}{100}\right) \cdot \gamma\left(\frac{k+3}{k}, u_s\right) \tag{42}$$

Using the relationship between the gamma function and the incomplete gamma function:

$$\gamma(a, x) = \Gamma(a) \cdot gammainc(x, a, ' lower') \tag{43}$$

where gammainc $(x, a, 'lower')$ is the regularized lower incomplete gamma function. Thus:

$$I_2 = \frac{c^3}{100} \cdot \Gamma\left(\frac{k + 3}{k}\right) \cdot \ldots$$

$$\ldots\ gammainc\left(u_s, \frac{k+3}{k}, 'lower'\right) \tag{44}$$

The final analytical expression for the expected overestimation cost is:

$$C_O^{expected} = W_s \cdot \left(1 - exp(-u_s)\right) - \cdots$$

$$\frac{c^3}{100} \cdot \Gamma\left(\frac{k + 3}{k}\right) \cdot \ldots$$

$$\ldots gammainc\left(u_s, \frac{k+3}{k}, 'lower'\right) \tag{45}$$

4 Problem Solution

The analysis is performed for four case studies, each of which presents the following values. The studies are evaluated for $W_{s,1}$=1 MW, $W_{s,2}$=10 MW, $W_{s,3}$=35 MW, and $W_{s,4}$=50 MW. The results are presented below. Table 1 shows that simulation times range from 4.26 to 6.18 s, and total expected costs range from \$12.49 to \$39.84/MW.

Table 1. Simulation time and expected value.

Description	$W_{s,1}$	$W_{s,2}$	$W_{s,3}$	$W_{s,4}$
Elapsed time (s)	6.18	4.41	4.26	4.55
Total expected cost (\$/MW)	12.49	11.9	27.1	39.84

Table 2 shows that using the numerical integration method the total expected estimate is made cost ranging from \$11.9 to \$39.88/MW, costs are also separated into two types, underestimation costs and overestimation costs.

Table 2. Simulation of total expected cost.

Description	$W_{s,1}$	$W_{s,2}$	$W_{s,3}$	$W_{s,4}$
Total expected overestimation cost (\$/MW)	0.12	4.32	24.4	38.29

Total expected underestimation cost ($/MW)	12.41	7.61	2.69	1.59
+Total expected cost ($/MW)	12.53	11.93	27.09	39.88

The values found by Monte Carlo simulations show the expected costs according to the four cases analysed, the costs range between 11,925 and 39,877 according to the total expected cost.

Table 3. Analytical Costs Development.

Description	$W_{s,1}$	$W_{s,2}$	$W_{s,3}$	$W_{s,4}$
Total expected cost ($/MW)	12.533	11.925	27.094	39.877

Costs with neural network forecasting and Kalman filter show a variation between 4.4975 and 42.79 for the power case studies.

Table 4. Kalman filtering and neural network wind forecasting.

Description	$W_{s,1}$	$W_{s,2}$	$W_{s,3}$	$W_{s,4}$
Total expected cost ($/MW)	6.18	4.498	27.699	42.79

The relative error is calculated as the difference in absolute value between the numerical integration and the analytical cost calculations, divided by the percentage of the analytical calculations. These values range from 0.008% to 0.042%, demonstrating that the method used offers outstanding approximations compared to the analytical method. The method combining the Kalman filter and the neural network for wind forecasting presents higher error rates than Monte Carlo simulations. However, when evaluating the uncertainty costs due to overestimation and underestimation, these costs are reduced by 50.694% and 62.285% for scheduled powers of $W_{s,1}$ = 1 MW and $W_{s,2}$ = 10 MW, respectively. The cost reduction is beneficial for power dispatch. In contrast, for scheduled powers of $W_{s,3}$=35 MW and $W_{s,4}$ = 50 MW, uncertainty costs increase.

Table 5. Estimation of relative error

Description	$W_{s,1}$	$W_{s,2}$	$W_{s,3}$	$W_{s,4}$
Relative error with analytical costs (%)	0.024%	0.042%	0.015%	0.008%
Relative error with Kalman filtering (%)	50.694%	62.285%	2.232%	7.305%

Figure 1 presents histograms describing the distribution of wind speed, generated power, and costs associated with estimation errors for a given $W_{s,1} = 1$MW capacity. The first histogram shows that most wind speeds are between 5 and 15 m/s, while the second indicates that power generation is usually low, concentrated below 50 MW. This relationship reflects the direct dependence between wind speed and power generation. The last two histograms illustrate the economic costs due to estimation errors. Underestimating generation can lead to significant costs, with most scenarios below $50, although some reach up to $400. In contrast, overestimating generates much lower costs, generally less than $1. This highlights that the economic impact of underestimating wind generation is considerably more critical than that of overestimating it.

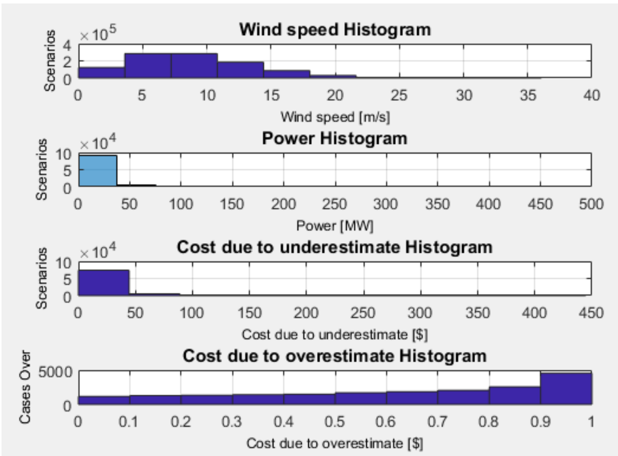

Figure 1. Histograms of costs, power and wind speed.

Figure 2, Figure 3, Figure 4 and Figure 5 show the frequency of uncertainty costs used for $W_{s,1}$=1 MW, $W_{s,2}$=10 MW, $W_{s,3}$=35 MW, and $W_{s,4}$=50 MW. In all cases, penalty costs are evaluated based on multiple scenarios. For all the capacities analysed, the vast majority of scenarios present low penalty costs, typically less than $50. This indicates that, regardless of the system power, prediction errors generally do not generate large costs. However, as the nominal power increases, especially in the 50 MW case, a slight dispersion towards higher costs is observed, although these cases remain rare. This suggests that, although larger systems have the potential to incur higher uncertainty costs, the general behaviour is still dominated by low penalties.

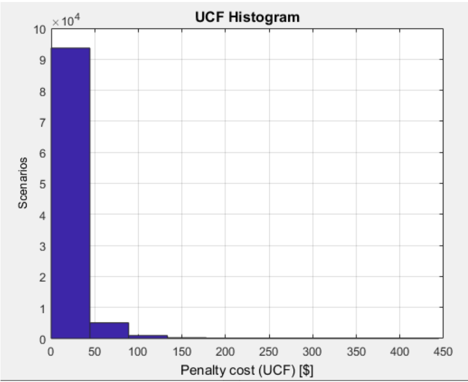

Figure 2. Uncertainty cost function for different powers at 1 MW wind speed.

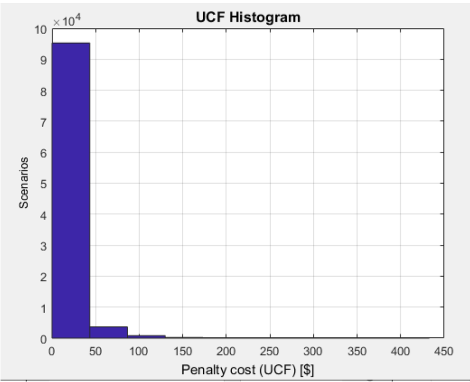

Figure 3. Uncertainty cost function for different powers at 10 MW wind speed.

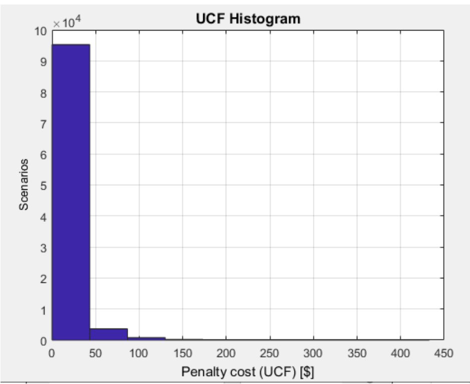

Figure 4. Uncertainty cost function for different powers at 35 MW wind speed.

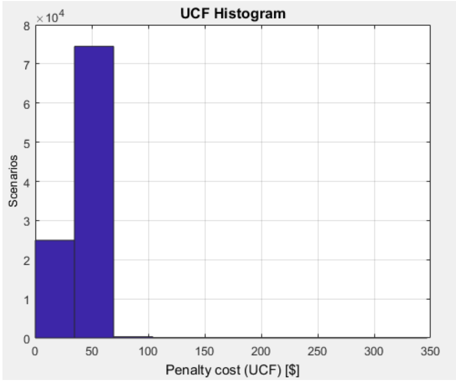

Figure 5. Uncertainty cost function for different powers at 50 MW wind speed.

4 Conclusions

The study is validated for the use of Monte Carlo simulations, numerical integration, and analytical methods to calculate uncertainty costs in wind power systems. The power-to-wind speed relationship, expressed as $P = k \cdot V^3$, is suitable for modeling power output, and the Weibull distribution is effective in representing wind speed variability. The results obtained from Monte Carlo simulations are in good agreement with the results derived from numerical integration and analytical methods, confirming the accuracy of the proposed approach. The case studies conducted for different wind generation capacity levels of 1 MW, 10 MW, 35 MW, and 50 MW show that the simulation times are reasonably fast, ranging between 4.26 s and 6.18 s. This suggests that the proposed model is efficient in terms of computational time, which is crucial for its practical application in assessing uncertainty costs in wind power systems. The calculation of the relative error between numerical integration and analytical cost development showed exceptional results, ranging from 0.008% to 0.042%. This suggests that the method used is highly accurate and a reliable tool for estimating uncertainty costs in wind energy systems, with minimal deviation from the analytical results. The method combining the Kalman filter and neural network for wind forecasting shows higher error rates compared to Monte Carlo simulations. However, when assessing the uncertainty costs due to overestimation and underestimation, these costs are reduced by 50.694% and 62.285% for scheduled power values of $W_{s,1}$=1 MW and $W_{s,2} = 10$ MW, respectively. This reduction in costs proves beneficial for power dispatch. In contrast, for scheduled power values of $W_{s,3} = 35$ MW and $W_{s,4}$=50MW, uncertainty costs increase. This study provides a solid foundation for the implementation of uncertainty models in wind energy systems, facilitating informed cost decisions and investment planning in this type of renewable energy. Validating the methods and verifying their accuracy increases confidence in the large-scale application of these models.

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