On Special Problems of Robustness in Control Systems

L. KEVICZKY, CS. BÁNYÁSZ
Institute of Computer Science and Control†

Abstract: The paper discusses some interesting mainly philosophical paradigms of robustness in the modelling and control areas, which are still partly unsolved and/or only partially studied..

Keywords: regulator, design, performance, parameterization, robustness

1. Introduction

Some scientists believe that everything has been solved in control, consequently nothing remained to study and/or research. The purpose of this paper is to recall some interesting philosophical paradigms in the areas of modelling and control to prove the contrary.

Only a few questions are discussed here, but there are many. Our aim is to encourage scientists to find further unsolved problems, blazes and interesting paradigms partly based on the modelling and control literature, partly on other disciplines.

If we can invite only a few further authors to continue our discussions then this effort is worth while.

In the sequel the YOULA parameterization [1], [2], [4], [5] will be used to discuss regulator and control system design. We found that this is very good basis for

1.1. The YOULA parameterization

The *YOULA*- (Y or Q) *-parameterization* is a classical method for linear time invariant control system to characterize all realizable stabilizing regulators (*ARS*) by

$$C = \frac{Q}{1 - QP} \tag{1}$$

for open-loop stable plant $P \in \mathcal{S}$, where $\mathcal{S}$ is the closed set of all stable proper real-rational systems, having all poles within the closed unit disc. The "parameter"

$$Q = \frac{C}{1 + C\,P} \quad ; \quad Q \in \mathcal{S} \tag{2}$$

ranges over all proper ($Q(\omega = \infty)$ is finite), stable transfer functions [1], [5]. Observe that Q is the transfer function from the r to u in the closed-loop (see Fig. 1), where y_n is the output disturbance (or noise) signal in a *SISO* (*S*ingle *I*nput *S*ingle *O*utput) system.

The transfer characteristics of the closed-loop can be easily computed

$$y = Q\,P\,r - (1 - Q\,P)\,y_n = y_t + y_d \tag{3}$$

where y_t is the tracking (servo) and y_d is the regulating (or disturbance rejection) independent behaviors of the closed-loop response, respectively.

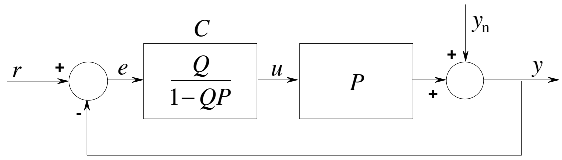

Fig. 1. Closed-loop with an *ARS* regulator

Because the *ARS* regulator represented in Fig. 3 was formulated for an one-degree of freedom (*1DF*) control system, it is not surprising that the tracking part y_t of the transfer characteristics between y and r can not be set independently of the regulating behavior y_d, i.e. independently of Q.

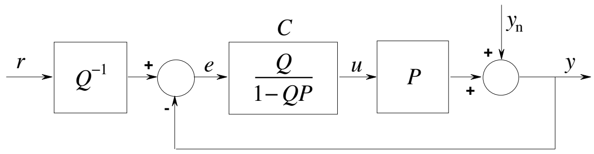

Fig. 2. The modified control system with an *ARS* regulator opening the closed-loop

The *Y-parameterization* "almost" opens the closed-loop. Here "almost" means that $y_t = Q\,P\,r$ is obtained instead of a real open-loop case with $y_t = P\,r$. So we need a Q^{-1} prefilter shown in Fig. 2, when the *ARS* regulator really "virtually" opens the closed-loop as

$$y = P\,r - (1 - Q\,P)\,y_n = y_t + y_d \tag{4}$$

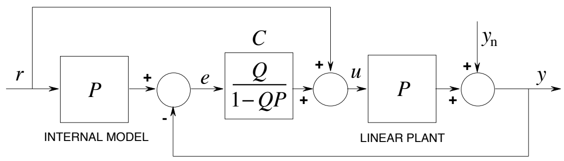

Fig. 3. The *K-B-parameterized 2DF* system with an *ARS* regulator

An important and new observation of the authors was that the scheme in Fig. 2 is equivalent to the special control system given in Fig. 4 and its parameterization has been named as *Keviczky-Bányász-*(*KB*) *parameterization* [1], [2]. Since in the case of the special structure presented in Fig. 3 we have $y_t = Pr$, i.e., (3) holds, it is easy to introduce a new general form of any *2DF* control systems providing

$$y = Q_r P y_r - (1 - QP) y_n = y_t + y_d \tag{5}$$

if a serial compensator Q_r is applied additionally as the Fig. 5 shows.

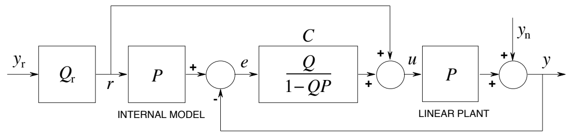

Fig. 4. The general form of the *K-B-parameterized 2DF* control system

Here and in the sequel the general notation y_r will be used for the reference signal for general *2DF* systems. Equation (5) shows that the tracking properties $y_t = Q_r P y_r$ can independently be designed from the regulating behavior $y_d = (1 - QP) y_n$ by Q_r.

The last scheme was later named as a *generic two-degree of freedom* (*G2DF*) system [1], [2]. The *K-B parameterization* for closed-loop control is not so widely known as the *Youla-Kucera* (*Y-K*) *parameterization* [4] however, it is much closer to a control engineering view and its most important advantage in *2DF* systems is that it virtually opens the closed-loop. However, this parameterization can only be applied for open-loop stable processes.

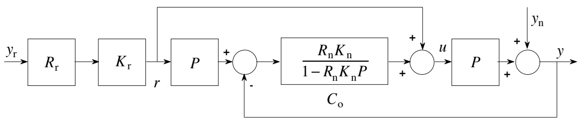

Fig. 5. The *generic 2DF* (*G2DF*) control system

A *G2DF* control system is shown in Fig. 5, where y_r, u, y and y_n are the reference, process input, output and disturbance signals, respectively. The optimal discrete-time *ARS* regulator of the *G2DF* scheme [1], [2] is given by

$$C_o = \frac{R_n K_n}{1 - R_n K_n P} = \frac{Q_o}{1 - Q_o P} = \frac{R_n G_n P_+^{-1}}{1 - R_n G_n P_- z^{-d}} \tag{6}$$

where

$$Q_o = Q_n = R_n K_n = R_n G_n P_+^{-1} \tag{7}$$

is the associated *Y-parameter* [1], furthermore

$$Q_r = R_r K_r = R_r G_r P_+^{-1} \; ; \; K_n = G_n P_+^{-1} \; ; \; K_r = G_r P_+^{-1} \tag{8}$$

assuming that the continuous-time process is factorable as

$$P = P_+ \overline{P}_- = P_+ P_- e^{-sT_d} \tag{9}$$

and a discrete-time process is factorable as

$$G = G_+ \overline{G}_- = G_+ G_- z^{-d} \tag{10}$$

where P_+, G_+ means the inverse stable (*IS*) and P_-, G_- the inverse unstable (*IU*) factors, respectively. Here T_d is the continuous time delay and z^{-d} corresponds to the discrete time delay, which is the integer multiple of the sampling time T_s.

It was shown [1], [2] that the optimization of the *G2DF* scheme can be performed in $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norm spaces by the proper selection of the serial K_r and embedded K_n filters (compensators). These optimizations are reduced to the optimal computation of the G_r and G_n embedded filters. If G_r and G_n are optimally selected, then C_o denotes the optimal *ARS* regulator in (6). It is interesting to see how the transfer characteristics of this system look like:

$$\begin{aligned} y &= R_r K_r P y_r - (1 - R_n K_n P) y_n = \\ &= R_r G_r P_- e^{-sT_d} y_r - \left(1 - R_n G_n P_- e^{-sT_d}\right) y_n = y_t + y_d \end{aligned} \tag{11}$$

or

$$\begin{aligned} y &= R_r K_r G y_r - (1 - R_n K_n G) y_n = \\ &= R_r G_r G_- z^{-d} y_r - \left(1 - R_n G_n G_- z^{-d}\right) y_n = y_t + y_d \end{aligned} \tag{12}$$

Here R_r and R_n are stable and proper transfer functions, that are partly capable to place desired poles in the servo and the regulatory transfer functions, furthermore they are usually referred as reference signal and output disturbance predictors. They can even be called as reference models, so reasonably $R_r(\omega = 0) = 1$ and $R_n(\omega = 0) = 1$ are selected. In this case the obtained regulator is always an integrating one.

2. Prejudice Free Control

The knowledge of a process is never exact, independent of the method how its model is determined, whether measurement-based identification (ID) or physico-chemical theoretical considerations are used. The uncertainty of the plant can be expressed by the absolute model error

$$\Delta P = P - \hat{P} \tag{13}$$

and the relative model error

$$\ell = \frac{\Delta P}{\hat{P}} = \frac{P - \hat{P}}{\hat{P}} \tag{14}$$

where $\hat{P}$ is the available nominal model used for regulator design and P is the real plant.

The parameters of the plant may change in terms of their nominal values in a given range. The closed-loop control system needs to be stable under the given uncertainty ranges of the parameters.

Suppose that the open loop is stable. The regulator designed for the nominal plant ensures the stability of the nominal closed-loop control system. Let us analyze whether the system remains stable with the parameter uncertainties of the open loop. Stability is maintained if the NYQUIST diagram of the modified open loop does not encircle the $-1+0j$ point.

If there is an uncertainty ΔP (or parameter change) in the transfer function of the plant, then if we apply the same regulator this uncertainty appears in the absolute error $\Delta L = C\Delta P$ of the loop transfer function, whereas its relative model error is

$$\ell_{\mathrm{L}} = \frac{\Delta L}{\hat{L}} = \frac{L-\hat{L}}{\hat{L}} = \frac{CP-C\hat{P}}{C\hat{P}} = \frac{P-\hat{P}}{\hat{P}} = \ell \qquad (15)$$

Here $\hat{L}$ denotes the nominal and L denotes the real loop transfer function.

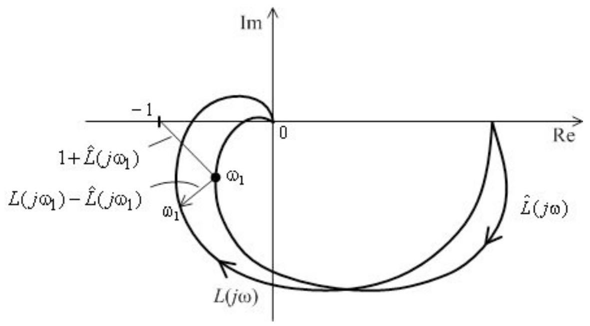

Fig. 6. Change in the NYQUIST diagram of an uncertain system

Robust stability means that the closed-loop control system should not display unstable behavior even in the "worst case" parameter changes. The bound for ΔL can be formulated based in Fig. 6 by taking the simple geometrical considerations into account: the NYQUIST diagram will not encircle the $-1+0j$ point, if the following relationship is satisfied for all frequencies:

$$\left|\Delta L(j\omega)\right| = \left|\ell(j\omega)\right| \left|\hat{L}(j\omega)\right| < \left|1+\hat{L}(j\omega)\right| \qquad \forall\omega \qquad (16)$$

With further straightforward manipulations the necessary and sufficient condition for robust stability is obtained as

$$\left|\ell(j\omega)\right| < \left|\frac{1+\hat{L}(j\omega)}{\hat{L}(j\omega)}\right| = \frac{1}{\left|\hat{T}(j\omega)\right|} \qquad \forall\omega \qquad (17)$$

or

$$\left|\hat{T}(j\omega)\right| \left|\ell(j\omega)\right| < 1 \qquad \forall\omega \qquad (18)$$

where $\hat{T} = \hat{L}/(1+\hat{L})$ is the nominal complementary sensitivity function.

This form is also called the dialectic relationship of robust stability. In the design the first factor $\left|\hat{T}(j\omega)\right|$ is calculated for the supposed (known) nominal parameters of the plant, and thus it depends on the designer. The second factor $|\ell|$ does not (or only partly) depends on the designer, as it contains the uncertainties in the knowledge of the plant or its unexpected parameter changes. In those frequency ranges where the uncertainty is large, unfortunately only small transfer gain can be designed for the closed-loop. Where $\left|\hat{T}(j\omega)\right|$ is high, very accurate information is needed to reach a small error. The higher the absolute value of the complementary sensitivity function, the smaller the permissible parameter uncertainty.

$$|C| \le \eta \left|\hat{P}^{-1}\right| = \frac{1}{\left|1-|\ell|\right|} \left|\hat{P}^{-1}\right| \qquad (19)$$

where the $\eta(|\ell|)$ function is plotted in Fig. 7. The interpretation of this function is very interesting. For small $|\ell|$ modelling error a model based controller design is suggested, which usually based on the inverse of the nominal model $\hat{P}^{-1}$. For very large errors no regulator design is advised. However, in the midrange domain, where the error is around 100 %, the regulator design practically does not depend on our knowledge of the process. This area can be called "*prejudice free*" domain

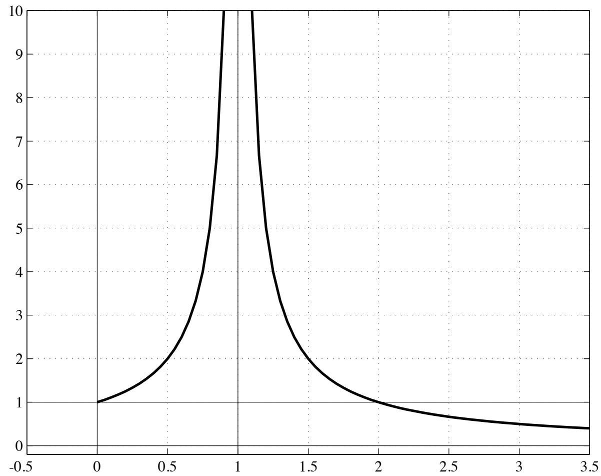

Fig. 7. The η in the function of ℓ

The robust stability condition (4), (5) and (6) can be rearrange in the form of

$$|C| \le \eta \left|\hat{P}^{-1}\right| = \frac{1}{\left|1-|\ell|\right|} \left|\hat{P}^{-1}\right| \qquad (20)$$

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Prejudice free control for Youla parameterized systems

The condition of robust stability for the *YP* control loops can be further simplified so the expression (18) becomes

$$\begin{aligned} y &= R_r K_r G y_r - \left(1 - R_n K_n G\right) y_n = \\ &= R_r G_r G_- z^{-d} \left|\hat{Q}\hat{P}\ell\right| = \left|R_n G_n \hat{P}_+^{-1} \hat{P}\ell\right| = \\ &= \left|R_n G_n \hat{P}_- e^{-s\hat{T}_d} \ell\right| = \left|R_n G_n \hat{P}_- \ell\right| = \left|R_n G_n \hat{P}_-\right|\left|\ell\right| < 1 \\ & y_r - \left(1 - R_n G_n G_- z^{-d}\right) y_n = y_t + y_d \end{aligned} \qquad \forall\omega \quad (21)$$

where $\hat{T}_d$ is the dead time of the model and $\left|e^{-s\hat{T}_d}\right| = 1$. The inequality (17), limiting the relative error, is now

$$\left|\ell(j\omega)\right| < \frac{1}{\left|R_n\right|\left|G_n \hat{P}_-\right|} \qquad \forall\omega \qquad (22)$$

If the process is *IS*, i.e., $\hat{P}_- = 1$, then $G_n = 1$ can be chosen and the condition of robust stability can be further simplified as

$$\left|\ell(j\omega)\right| < \frac{1}{\left|R_n\right|} \qquad \forall\omega \qquad (23)$$

i.e., it does not depend on the model $\hat{P}$ but only on the reference model or the design goal.

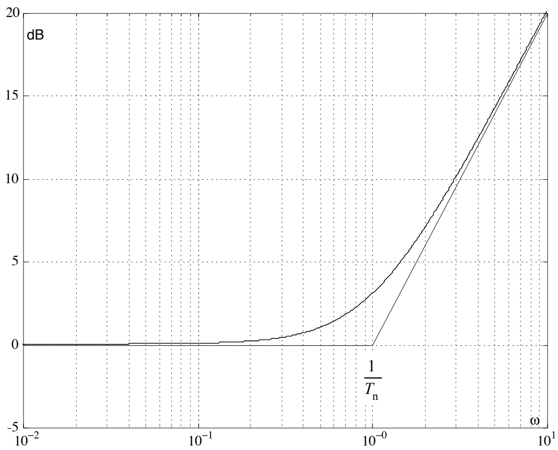

Fig. 8. Condition constraining the relative model error in the case of the first-order reference model

The reference model is an important parameter of the general YOULA design, by means of which the condition of robust stability (23) can be guaranteed. Let

$$R_n = \frac{1}{1+sT_n} \qquad (24)$$

then the constraining condition of the right side of (23) can be seen in Fig. 8. Given the latter condition and choosing first-order reference model R_n, we see that robust stability can be ensured even in the case of 100 % relative model error. Furthermore for the high frequency domain a real prejudice free case is obtained,

If the process is *IU,* even the factor $\left|G_n \hat{P}_-\right|$ appears in (22), can significantly modify (22). Fig. 9 shows the case when two unstable zeros seriously decrease the prejudice free character of the stability. The worst case is when this factor has a large value in the region of the cut-off frequency.

KALMAN was who tried to investigate the possibility of a *prejudice free* identification/modelling methodology [6]. He could not find any general applicable results, however many interesting, almost philosophical statements were developed.

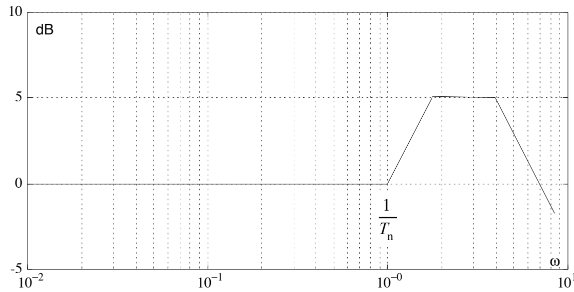

Fig. 9. Conditions constraining the relative model error n the case of two unstable zeros

3. The Heisenberg Uncertainty of Control

The condition of robust stability (18) already contains a product inequality. Here $\left|\hat{T}(j\omega)\right|$ (although it is usually called a design factor) can be considered as the quality factor of the control. The other factor, however, can be considered as the relative correctness of the applied model. In the light of practical experience control engineers favor applying a mostly heuristic expression

(quality of the control) $\times$ (robustness of the control) $\leq$ limit

This product inequality can be simply demonstrated by the integral criteria of classical control engineering. Let I_2 be a square integral criterion (<u>I</u>*ntegral* <u>S</u>*quare of* <u>E</u>*rror: ISE*) whose optimum is I_2^* when the regulator is properly set, and the NYQUIST stability limit (i.e., robustness measure) is ρ_m. The well-known empirical, heuristics formula is

$$\frac{I_2^*}{I_2} \rho_m \leq \text{limit} \qquad (25)$$

The inequality is illustrated in Fig. 15. The fact that the quality of the identification (which is the inverse of the model correctness) can have a certain relationship with the robustness of the control, is not very trivial. This can be observed only in a special case, namely in the identification technique based on *KB* parameterization [1] [2] when $\varepsilon_{\mathrm{ID}}=-\tilde{e}$. Introduce a new relationship for the characterization of the quality of the control

$$\delta=\delta\left(\omega,\hat{C}\right)=\frac{\left|-\tilde{e}\left(j\omega\right)\right|}{\left|y_{\mathrm{n}}\left(j\omega\right)\right|}=\left|\frac{1}{1+\hat{C}P}\right|=\frac{1}{\left|1+\tilde{L}\right|} \tag{26}$$

Notice that δ is the absolute value of the sensitivity function. Obviously, $\delta\rho=1$ for all frequencies (here $\rho=\left|1+\tilde{L}\right|$). Of course, the same equalities are valid for the minimum and maximum values, i.e.,

$$\begin{aligned}\rho_{\mathrm{m}}\left(\hat{C}\right)&=\min_{\omega}\left[\rho\left(\omega,\hat{C}\right)\right]=\min_{\omega}\left(\rho\right)\\ \delta_{\mathrm{M}}\left(\hat{C}\right)&=\max_{\omega}\left[\delta\left(\omega,\hat{C}\right)\right]=\max_{\omega}\left(\delta\right)\end{aligned}\quad ;\ \text{i.e.,:}\ \delta_{\mathrm{M}}\,\rho_{\mathrm{m}}=1 \tag{27}$$

Denote the worst value of these measures by

$$\begin{aligned}\hat{\rho}_{\mathrm{m}}&=\max_{\hat{C}}\left\{\min_{\omega}\left[\rho\left(\omega,\hat{C}\right)\right]\right\}=\max_{\hat{C}}\left[\min_{\omega}\left(\rho\right)\right]\\ \breve{\delta}_{\mathrm{M}}&=\min_{\hat{C}}\left\{\max_{\omega}\left[\delta\left(\omega,\hat{C}\right)\right]\right\}=\min_{\hat{C}}\left[\max_{\omega}\left(\delta\right)\right]\\ &\breve{\delta}_{\mathrm{M}}\,\hat{\rho}_{\mathrm{m}}=1\end{aligned} \tag{28}$$

The above three basic relationships can be summarized in the inequalities below

$$\delta\rho=1\quad;\quad\delta_{\mathrm{M}}\,\rho_{\mathrm{m}}=1\quad;\quad\breve{\delta}_{\mathrm{M}}\,\hat{\rho}_{\mathrm{m}}=1 \tag{29}$$

where the following simple calculations prove the existence of (34) and (35)

$$\begin{aligned}\rho_{\mathrm{m}}\left(\hat{C}\right)&=\min_{\omega}\left|1+\hat{C}P\right|=\frac{1}{\max_{\omega}\left|\frac{1}{1+\hat{C}P}\right|}=\\ &=\frac{1}{\left\|\frac{1}{1+\hat{C}P}\right\|_{\infty}}=\frac{1}{\delta_{\mathrm{M}}\left(\hat{C}\right)}\end{aligned} \tag{30}$$

$$\begin{aligned}\breve{\delta}_{\mathrm{M}}&=\min_{\hat{C}}\left\{\max_{\omega}\left|\frac{1}{1+\hat{C}P}\right|\right\}=\frac{1}{\max_{\hat{C}}\left\{\max_{\omega}\left|\frac{1}{1+\hat{C}P}\right|\right\}}=\\ &=\frac{1}{\max_{\hat{C}}\left\{\min_{\omega}\left|1+\hat{C}P\right|\right\}}=\frac{1}{\hat{\rho}_{\mathrm{m}}}\end{aligned} \tag{31}$$

Given (27), (28) and (29) further basic, almost trivial, inequalities can also be simply formulated

$$\begin{aligned}&\breve{\delta}_{\mathrm{M}}\le\delta_{\mathrm{M}}\left(\hat{C}\right)\quad;\quad\rho_{\mathrm{m}}\left(\hat{C}\right)\le\hat{\rho}_{\mathrm{m}}\\ &\frac{1}{\hat{\rho}_{\mathrm{m}}}=\breve{\delta}_{\mathrm{M}}\le\delta_{\mathrm{M}}\left(\hat{C}\right)=\frac{1}{\rho_{\mathrm{m}}\left(\hat{C}\right)}\\ &\frac{1}{\delta_{\mathrm{M}}\left(\hat{C}\right)}=\rho_{\mathrm{m}}\left(\hat{C}\right)\le\hat{\rho}_{\mathrm{m}}=\frac{1}{\breve{\delta}_{\mathrm{M}}}\end{aligned} \tag{32}$$

The above results are not surprising. The fact, that they are valid even for the modelling error in the case of *KB*-parameterized identification methods makes them special. So it can be clearly seen that when the modelling error decreases, the robustness of the control increases. Namely, if the minimum of the modelling error $\breve{\delta}_{\mathrm{M}}$ is decreased, then the maximum of the minimum robustness measure $\hat{\rho}_{\mathrm{m}}$ is increased, since $\breve{\delta}_{\mathrm{M}}\,\hat{\rho}_{\mathrm{m}}=1$.

Similar relationships can be obtained if the $\mathcal{H}_2$ norm of the “joint” modelling and control error is used instead of the absolute values. Introduce the following relative fidelity measure

$$\sigma=\frac{\left\|\varepsilon_{\mathrm{ID}}\right\|_2}{\left\|y_{\mathrm{n}}\right\|_2}=\frac{\left\|\tilde{e}\right\|_2}{\left\|y_{\mathrm{n}}\right\|_2}\quad;\ \left\|y_{\mathrm{n}}\right\|_2\ne0 \tag{33}$$

The upper limit for this measure can be formulated as

$$\sigma=\frac{\left\|\varepsilon_{\mathrm{ID}}\right\|_2}{\left\|y_{\mathrm{n}}\right\|_2}=\frac{\left\|\tilde{e}\right\|_2}{\left\|y_{\mathrm{n}}\right\|_2}\le\left\|\frac{1}{1+\hat{C}P}\right\|=\delta_{\mathrm{M}}\left(\hat{C}\right) \tag{34}$$

so it is very easy to find similar equations for σ. Let

$$\sigma_{\mathrm{M}}\left(\hat{C}\right)=\max_{\ell}\left[\sigma\left(\ell,\hat{C}\right)\right]\quad;\quad\breve{\sigma}_{\mathrm{M}}=\min_{\hat{C}}\left\{\max_{\ell}\left[\sigma\left(\ell,\hat{C}\right)\right]\right\}$$

Using these definitions and the former equations we obtain the following interesting relationship

$$\breve{\sigma}_{\mathrm{M}}\le\sigma_{\mathrm{M}}\left(\hat{C}\right)\le\delta_{\mathrm{M}}\left(\hat{C}\right)=\frac{1}{\rho_{\mathrm{m}}\left(\hat{C}\right)} \tag{35}$$

for the relative quadratic identification error.

Use the first-order reference model

$$R_{\mathrm{n}}=\frac{b_{\mathrm{n}1}z^{-1}}{1+a_{\mathrm{n}1}z^{-1}} \tag{36}$$

for the design of the noise rejection in the *IS* process.

Here the maximum of the robustness measure is $\hat{\rho}_{\mathrm{m}}^{\mathrm{o}}=\hat{\rho}_{\mathrm{m,IS}}^{\mathrm{o}}=0.9$ according to

$$\hat{\rho}_{\mathrm{m,IS}}^{\mathrm{o}}=\hat{\rho}_{\mathrm{m,IS}}\left(\ell=0\right)=\frac{1}{\left\|1-R_{\mathrm{n}}\right\|_{\infty}}=\min_{\omega}\left|\frac{1}{1-R_{\mathrm{n}}}\right| \tag{37}$$

and

$$\hat{\rho}_{\mathrm{m,IS}}^{\mathrm{o}}=\frac{\left|a_{\mathrm{n}1}-1\right|}{2} \tag{38}$$

This gives a robust measure of $\hat{\rho}_{\mathrm{m,IS}}^{\mathrm{o}}=0.9$ under the parameters $b_{\mathrm{n}1}=0.2$ and $a_{\mathrm{n}1}=-0.8$ chosen for reference models with unity gain (same as in (24)).

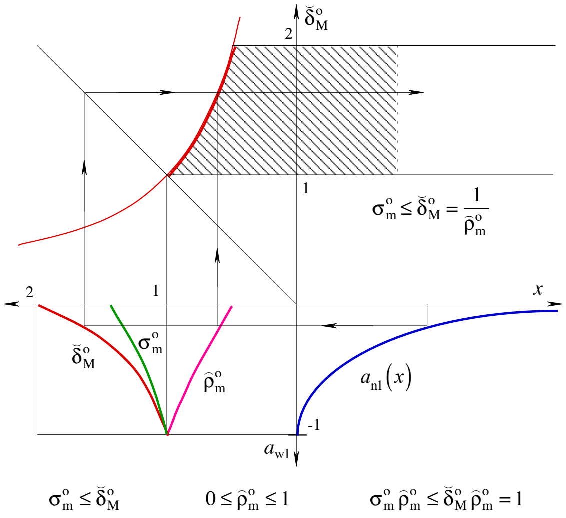

Figure 10. Illustration of uncertainty relationships (41)

The values of the typical variables (see above) are

$$\breve{\delta}_M^o = \frac{1}{\hat{\rho}_m^o} = \frac{2}{|a_{n1}-1|} = 1.1111 \quad \Rightarrow \quad \breve{\delta}_M^o \hat{\rho}_m^o = 1 \tag{39}$$

$$\sigma_M^o = \frac{1}{\sqrt{\hat{\rho}_m^o}} = \sqrt{\frac{2}{|a_{n1}-1|}} = 1.054 \quad \Rightarrow$$

$$\sigma_M^o \hat{\rho}_m^o = 0.9486 \le \breve{\delta}_M^o \hat{\rho}_m^o = 1 \tag{40}$$

Considering the data of (39) and applying again the relative sampling time $x = T_s / T_n$, the different measures in (35) are illustrated in Fig. 10. Here T_n is the time constant of the continuous time (CT) first-order reference model.

Introduce the following coefficient for the excitation caused by the reference signal

$$\xi = \frac{\|y_r\|_2}{\|y_n\|_2} \quad ; \quad \|y_n\|_2 \ne 0 \tag{40}$$

which represents a signal/noise ratio. Investigate the product $\sigma\rho$ (which is called the uncertainty product) in an iterative procedure where the relative error ℓ of the model is improved gradually. For simplicity, let us assume an IS process. It can be simply derived that

$$\sigma\rho_m \le \sigma\hat{\rho}_m^o \le \frac{1+\xi\|R_n\ell\|_\infty}{\min|1+R_n\ell|} \le \frac{1+\xi\|R_n\|_\infty\|\ell\|_\infty}{1-\xi\|R_n\|_\infty\|\ell\|_\infty}\Bigg|_{\ell\to 0} = \tag{42}$$

$$= \sigma_o \hat{\rho}_m^o = 1$$

i.e.,

$$\sigma\rho_m \le \sigma\hat{\rho}_m^o \underset{\ell\to 0}{\le} \sigma_o \hat{\rho}_m^o = 1 \quad \text{or}$$

$$\sigma \le \sigma_o = \frac{1}{\hat{\rho}_m^o} = \breve{\delta}_M^o \le \delta_M = \frac{1}{\rho} \tag{43}$$

where $\sigma_o = \sigma(\ell = 0)$. Similarly to the notations $\sigma_M(\hat{C})$ and $\breve{\sigma}_M$ applied above, the notations $\sigma_m(\ell) = \min_\ell\left[\sigma(\ell,\hat{C})\right]$ and $\sigma_m^o = \sigma_m(\ell = 0)$ can also be introduced. It is not an easy task, however, to derive the relationship between σ_m^o and σ_o or $\breve{\sigma}_M$ and $\sigma_M(\hat{C})$. The simplest case to investigate (43) is when $\ell = 0$, since then

$$\sigma_m^o \le \sigma_M^o(\hat{C}) \le \delta_M^o(\hat{C}) = \frac{1}{\rho_m^o(\hat{C})} \tag{44}$$

This equation gives a new uncertainty relationship, according to which

$$\frac{\|\tilde{e}\|_2}{\|y_n\|_2} \min_{\hat{C}} |1 + \hat{C}P| \underset{\ell\to 0}{\le} 1 \tag{45}$$

The product of the modelling accuracy and the robustness measure of the control must not be greater than one, when the optimality condition $\ell = 0$ is reached. The obtained uncertainty relation can be written in another form, since

$$\sup\left\{\frac{\|\tilde{e}\|_2}{\|y_n\|_2}\right\} \min_{\hat{C}} |1 + \hat{C}P|_{\ell=0} = 1 \tag{46}$$

The earlier results of control engineering referred only for the statement that the quality of the control cannot be improved, only at the expense of the robustness, so this result, which connects the quality of the identification and the robustness of the control, can be considered, by all mean, novel.

This phenomenon can arguably be considered as the HEISENBERG type uncertainty relationship of control engineering, according to which

$$\frac{1}{\Delta z}\frac{1}{\Delta p} \le 1 \tag{47}$$

Here Δz and Δp are the alterations of the canonical coordinate and the impulse variables, respectively, and thus their inverse corresponds to the generalized accuracy and "rigidity" which are known as performance and robustness in control engineering.

The consequence of the new uncertainty relation is very simple: *KB*-parameterized identification is the only method where the improvement of the modelling error also increases the robustness of the control. With other methods, and other identification topology, modelling and control errors are interrelated in a very complex way, and in many cases this relation cannot be given in an explicit form. This is the main reason why it is difficult to elaborate a method which guarantees, or at least forces, similar behavior by the two errors, though some results can be found in the literature [3].

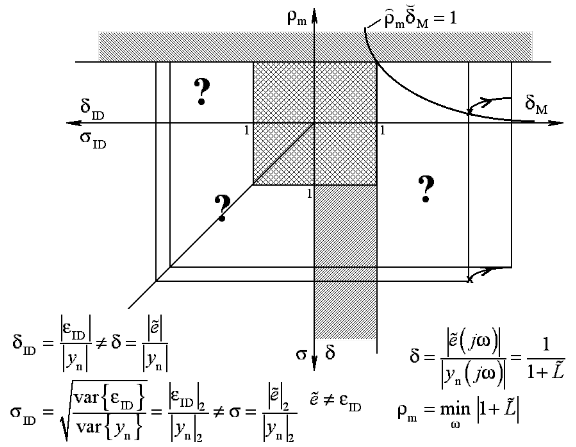

Fig. 11. Relationship between the control and identification error in the general case

There is a myth in the literature concerning the antagonistic conflict between control and identification. A "good" regulator minimizes the internal signal changes in the closed loop and therefore most of the identification methods, which use these inner signals provide worse modelling error, if the regulator is better. The exciting signal of *KB*-parameterized identification is an outer signal and therefore the phenomenon does not exist. The relevant feature of this relationship is shown in Figs. 11 and 12 for a general identification method and a *KB*-parameterized technique.

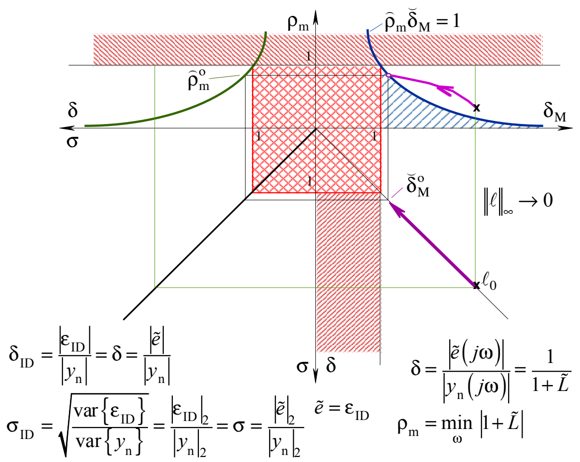

Fig. 12. Relationship between the control and identification error in the case of the *KB*-parameterized identification method

In Fig. 11, there is no clear relation between δ_{ID} and δ, or σ_{ID} and σ, and therefore there it is not guaranteed that minimizing δ_M increases ρ_m. In Fig. 12 $\delta_{ID} = \delta$ and $\sigma_{ID} = \sigma$, and thus the minimization of δ_M directly maximizes ρ_m. Thus if during the iterative identification the condition $\|\ell_k\|_\infty \underset{k\to\infty}{=} 0$ is guaranteed then, at the same time, the convergences $\breve{\delta}_M^k \underset{k\to\infty}{=} \breve{\delta}_M^o$ and $\hat{\rho}_m^k \underset{k\to\infty}{=} \hat{\rho}_m^o$ are ensured.

4. Conclusions

The purpose of this paper is to highlight some interesting, may be philosophical, paradigm of modelling and control.

Such problems are discussed here, which are worth further study and investigation.

"I believe that the progress of science should be rather measured by the raised and not by the solved problems !" as Eddington stated !!!

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