Adaptive Second Order Sliding Mode Controller for 2-Dof Serial Flexible Link

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Abstract: - Flexible link manipulator system dynamics and control has become a key research area by many researchers around the globe due to many benefits it affords like low weight, low power consumption leading to low universal cost. However, because of the inherent link flexibility they go through vibrations and take time to return the favored position as soon as the actuator force is eliminated. The difficulty is designing feedback control system for the manipulator because the system is non-minimal phase, under-actuated and non-collected due to physical separation of sensor and actuator. The important awareness of this thesis is to improve upon the present sliding mode control approach with the high goal of chattering reduction. The benefit of adaptive tuning mechanism is estimating uncertainty adaptively is proposed in this thesis. As a result, prior expertise about the upper bound of system uncertainty is not an essential requirement in the proposed adaptive 2nd order sliding mode controller. The entire system response was simulated in MATLAB/ Simulink environment. The effectiveness of adaptive second order sliding mode control (ASOSMC) controller changed into compared with conventional, classical or 1st order SMC, 2nd order SMC, model predictive control and adaptive 2nd order SMC by using the six-performance index like IAE, ISE, ITSE, ITAS, MSE and MAE. The results endow that the adaptive 2nd order SMC controller outperforms the above listed controllers**.**

Key-words: **-**ASOSMC, super twisting algorithm, chattering mitigation, flexible links, parameter estimation, uncertainty

1. Introduction

Flexible link manipulators are fabricated from light-weight substance and flexible links. Those manipulators can be operated with the aid of using DC motor as an actuator. Flexible robots utilized widely in extraordinary as they could convey big payload and consumes less power in cooperation to rigid robots. Furthermore, because of their light-weight, they can flow trajectories faster and their price of construction is much less. However, because of less weight to volume ratio, they are suffered from vibrations and un-modeled dynamics as a result their control mechanism will becomes more tough.

In fact, all physical structures are tormented by uncertainties occurring because of modeling mistakes, external disturbance and parametric variation. Controlling dynamics system is hard in the presence of uncertainties because the capacity of the controller decrease and the flexible robotic link may additionally even be pushed to instability. As a result, lively research is continuing still today to increase controller's performance even with uncertainties. Robust manipulate strategies along with composite learning manipulate [1], two performance enhance control [2, 3], adaptive higher-order sliding manipulates [4, 5, 6], Second-order sliding mode approaches for the control of a class of underactuated systems [7, 8], hybrid sliding mode/ h-infinity control approach for uncertain flexible manipulators [9], higher order sliding mode [10], and backstepping manipulate [11, 12, 13] have enhanced to cope with uncertainties. Those control strategies are able to accomplished the specific control objectives despite of modeled blunders and uncertainties on controlled system interfere directly [14, 15]. Beginning within 1980s and continuing till these days, the strategies of sliding mode control has received wide research area due to its inherent insensitivity to parametric variations [16]. The sliding mode manipulate is a specific sort of variable structure control system (VSCS) and it can be used the discontinuous control signal [17]. Recently many a hit realistic software of sliding mode control have mounted the significance of sliding mode principle. Nowadays sliding mode control significantly utilized in a selection of utility areas like flexible robotic links and process control.

Sliding mode design involves the steps, (1) sliding surface towards the preferred closed-loop performance and (2) its control law. The surface must be designed optimally based on the given constraints and specified criteria [18]. The initial section while the trajectory flowed by the state is in directed of sliding surface is side to be reaching phase. At some stage in the achieving phase, the control law is sensitive to all sort of disturbances. However, the control law guarantees finite reaching in its design surface with existence of uncertainties and disturbances. For removing non-robust reaching section, adaptive 2nd order sliding mode changed in to proposed wherein evidently allowed SMC to be blended with different techniques [19]. The principal blessings for sliding mode control are:

I. At some point of sliding mode, the system is in touchy to matched uncertainties and disturbances
II. When the system at sliding manifold, it looks like minimum-order system concerning unique plant.

But, even with claimed robustness, real time implementation of the sliding mode controller is bottlenecked by principal downside called chattering that's the excessive frequency bang-bang form of control action [20]. Chattering is induced because of fast dynamical model which might be normally omitted in ideal mode [21]. In appropriate sliding mode, the control manipulates to exchange in boundless frequency. But, in real applicable plants especially in flexible robots, in addition to the presence of nonlinearities and inertia of sensor and actuator, the switching happens with very high but defined frequency handiest [22]. Because of finite attaining of control signal the state might be transfer approximately to the sliding surface rather than without delay on it [23] is switching can arise at every high frequency. The algorithm of Higher-order sliding mode manipulate may classified in to four; suboptimal, twisting, quasi-continuous, and super-twisting are proposed in the literature. But from the list super-twisting is simpler as it calls for the information of states only and no longer its derivative [24]. The effectiveness of the proposed controller is checked in simulation using MATLAB.

The remain part of the paper is prepared as follows: In Section 2 of this paper, we provide brief description with mathematical model of the 2-DOF serial flexible link robot. In section three design STA for both SOSM and ASOSMC. The Simulation of results are discussed in the fourth Section. Finally, in section five Conclusion and future works.

2. System Description and Identification

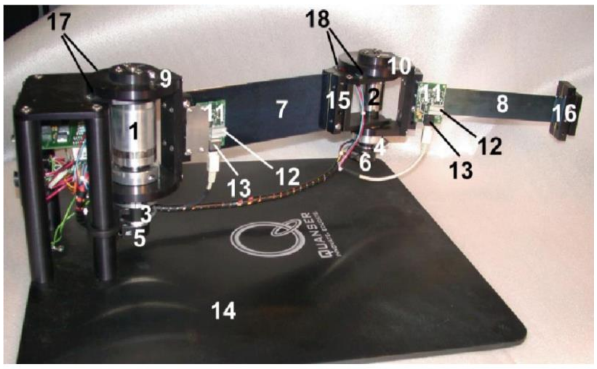


Figure 1: 2 DOF flexible link manipulator [24]

2.1 system parameters

J_{ij}: Moment of Inertia in $Kg.m^2$.

B_{ij}: Coefficient of viscous friction in N.m.s/rad.

k_{si}: Flexible Link's Stiffness Due to Torsional Action in N.m/rad
K_{ti}:Motor Torque constant in N.m/A

I_i: motor Armature current in A.

θ_{11}:Fflexible link One Angular position (rad)

θ_{12}: Flexible Link One End-Effector Angular Position (rad)

$\dot{\theta}_{11}$: First Flexible Link One End-Effector Angular Velocity (rad/s)

$\dot{\theta}_{12}$: First Flexible Link Relative End-Effector Angular Velocity (rad/s)

θ_{21}:Second (Elbow) Driving Shaft Angular Position Relative to Link1(rad)

θ_{22}: Second Flexible Link End-Effector Angular Position Relative to Link 1 (rad)

$\dot{\theta}_{21}$: Second (Elbow) Driving Shaft Angular Velocity Relative to Link 1 (rad/s)

$\dot{\theta}_{22}$: Second Flexible Link End-Effector Angular Velocity Relative to Link 1 (rad/s)

The two serial flexible links are actuated via mounted dc motor installed with strain gauges on clamped end to measure dimension of tip deflection. As shown in Fig 1 above the DC motor rotates in time of actuation to actuate the flexible link joint and the link follow the desired position [2].

2.2 Mathematical Model

Before driving equation of motion, we need to consider some assumptions to simplify the equation of motion.

i. $(\theta, \dot{\theta})$ *and* $(\alpha, \dot{\alpha})$ are time varying signals of system.
ii. The deflection of links is due most the bending mode and sufficiently small enough compared to the link length.
iii. Links neither expand nor contract rather bend.
iv. Each link moves in the horizontal plane only and not affected by gravity.

Assumption one indicates that the signal θ,$\dot{\theta}$, α and $\dot{\alpha}$ are available signals by taking sensor information like rotary encoder and strain gauges. Assumption two means the higher order mode are not excited by the deflection and it is reasonable for practical manipulator arm.

2.3 Dynamic Representation

Firstly, it is better to define generalized coordinates of the system [24].

$$q = [\theta_1 \quad \theta_2]^T = [\theta_{11} \ \ \theta_{21}]^T \qquad 1$$

Based on principle of virtual work, "the system is balanced if and only if the net virtual work along the generalized force is zero" [3].

$$\delta W = Q^T \delta q = 0 \qquad 2$$

where Q is the vector of generalized forces. The Euler Lagrange equations can be defined as,

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = Q_1 = \tau - B_{eq}\dot{\theta}$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = Q_2 = -B_{eq}\dot{\alpha} \qquad 3$$

where $[\theta_1 \quad \theta_2] = [\theta_{11} \ \ \theta_{21}]$ and$[\alpha_1 \quad \alpha_2] = [\theta_{12} \quad \theta_{22}]$,$L$ is the Lagrange function or Lagrangian which is the difference of kinetic and potential energies and is defined in [15, 24].

$$L = E_k(q, \dot{q}) - E_p(q) \qquad 4$$

The equation of motion based on equation (4) above can be written as follows.

$$\ddot{\theta}_{11}(J_{11} + J_{12}) + J_{12}\ddot{\theta}_{12} = \tau_1 - B_{11}\dot{\theta}_{11}$$
$$J_{12}(\ddot{\theta}_{11} + \ddot{\theta}_{12}) + k_{s1}\theta_{12} = -B_{12}\dot{\theta}_{12} \qquad 5$$

$$\ddot{\theta}_{21}(J_{21} + J_{22}) + J_{22}\ddot{\theta}_{22} = \tau_2 - B_{21}\dot{\theta}_{21}$$
$$J_{22}(\ddot{\theta}_{21} + \ddot{\theta}_{22}) + k_{s2}\theta_{22} = -B_{22}\dot{\theta}_{22} \qquad 6$$

From equations (5) and (6) we will find the value of $(\ddot{\theta}_{11})$, ($\ddot{\theta}_{21}$), ($\ddot{\theta}_{12}$) and $(\ddot{\theta}_{22})$.

$$\ddot{\theta}_{11} = \frac{k_{s1}}{J_{11}}\theta_{12} + K_{t1}\frac{I_m}{J_{11}} - \frac{B_{11}}{J_{11}}\dot{\theta}_{11} + \frac{B_{12}}{J_{11}}\dot{\theta}_{12} = -b_1\dot{\theta}_{11} + b_2\theta_{12} + b_3 I_m \qquad 7$$

$$\ddot{\theta}_{12} = -k_{s1}\left(\frac{J_{11}+J_{12}}{J_{11}J_{12}}\right)\theta_{12} + \frac{B_{11}}{J_{11}}\dot{\theta}_{11} - K_{t1}\frac{I_m}{J_{11}} - B_{12}\left(\frac{J_{11}+J_{12}}{J_{11}J_{12}}\right)\dot{\theta}_{12} = b_1\dot{\theta}_{11} - b_4\theta_{12} - b_3 I_m \quad 8$$

$$\ddot{\theta}_{21} = \frac{k_{s2}}{J_{21}}\theta_{22} + K_{t2}\frac{I_m}{J_{21}} - \frac{B_{21}}{J_{21}}\dot{\theta}_{21} + \frac{B_{22}}{J_{21}}\dot{\theta}_{22} = -b_7\dot{\theta}_{21} + b_5\theta_{22} + b_9\dot{\theta}_{22} + b_6 I_m \quad 9$$

$$\ddot{\theta}_{22} = -k_{s2}\left(\frac{J_{21}+J_{22}}{J_{21}J_{22}}\right)\theta_{22} + \frac{B_{21}}{J_{21}}\dot{\theta}_{21} - K_{t2}\frac{I_m}{J_{21}} - B_{22}\left(\frac{J_{21}+J_{22}}{J_{21}J_{22}}\right)\dot{\theta}_{22} = b_7\dot{\theta}_{21} - b_8\theta_{22} - b_{10}\dot{\theta}_{22} - b_6 I_m \quad 10$$

Within the model of system, the values that determine parameters of the predefined system that we've taken consideration are taken from the real-time system that is available in QUANSER laboratory tests. The way of determination for those values are found on the device issuer, i.e., QUANSER by various experiments [25]. And Now, from QUNASER laboratory test the value of viscous friction (B_{12}) is very small, assuming to be zero [24] the following section of this thesis, those parameters are represented as bi, where i=1–10.

3. Controller Design

Here the control goal is to permit a flexible link robot to monitor the rereferred role with the aid of parametric uncertainties and without the aid of parametric uncertainties. To overcome this requirement, a control law is adopted using SMC and adaptive SMC [26]. To analyze stability based on Lyapunov theory in order to regulate and track system [27]. SMC, SOSMC are applied while the parameters are recognized and ASMC, ASOSMC are applied while the parameter are not recognized, uncertain and time varying [14].

3.1 Control Design for Regulation Problem

Defining, $x1 = \theta_{11}$, $x2 = \theta_{12}$, $x3 = \dot{\theta}_{11}$, $x4 = \dot{\theta}_{12}$, $u = I_m$ and using equations (7 and 8), we can represent our system dynamics as [28] and [25] for sub system one:

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= -b_1x_3 + b_2x_2 + b_3I_m \\ \dot{x}_4 &= b_1x_3 - b_4x_2 - b_3I_m \end{aligned} \quad 11$$

Similarly, for sub system two we have $x1 = \theta_{21}$, $x2 = \theta_{22}$, $x3 = \dot{\theta}_{21}$, $x4 = \dot{\theta}_{22}$, $u = I_m$

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= -b_7x_3 + b_5x_2 + b_9x_4 + b_6I_m \\ \dot{x}_4 &= b_7x_3 - b_8x_2 - b_{10}x_4 - b_6I_m \end{aligned} \quad 12$$

To regulate the system and track desired trajectory of the dynamic equation in equation (11 and 12) the design procedure for the overall signal is achieved in two parts, design of suitable switching surfaces that assumes the stability of system dynamics and then design preferable control law which guarantees the available of sliding mode s(t) = 0*:*

3.1.1 SMC Regulation

To design the switching surface, we will use the sliding manifold as

$$\begin{aligned} s_1 &= c_1x_1 + x_3 \\ s_2 &= c_2x_2 + x_4 \\ s_3 &= c_3x_1 + x_3 \\ s_4 &= c_4x_2 + x_4 \end{aligned} \quad 13$$

$s_{11} = a_1s_2 + s_1$ and $s_{12} = a_2s_4 + s_3$ are equivalent surfaces of sub system one and two respectively and its surface derivative to get the derivative of sliding surface. Where a_1, a_2 and c_1, c_2, c_3, c_4 are positive design parameters and when it satisfies,

$$s_{11} = s_{12} = \dot{s}_{11} = \dot{s}_{12} = 0 \quad 14$$

The control input is given by u_1, u_2 for the above sub system one and two in equation (11 and 12) respectively:

$$u_1 = \left(\frac{-1}{p_1}\right)\left(x_2(b_2 - a_1b_4) + x_3(c_1 - b_1 + a_1b_1) + x_4a_1c_2 + k_1\mathrm{sign}(s_{11})\right) \quad 15$$

$$u_2 = \left(\frac{-1}{p_2}\right)(x_2(b_5 - a_2 b_8) + x_3(c_3 - b_7 + a_2 b_7) + x_4(b_7 - a_2 b_{10} + a_2 c_4) + k_2 \mathrm{sign}(s_{12})) \quad 16$$

Where $p_1 = (b_3 - a_1 b_3)$; $p_2 = (b_6 - a_2 b_6)$, $k_i > 0$

3.1.2 SOSMC Regulation

Control law that regulates the system proposed in (15 and 16) is discontinuous and take action to smooth control function and to dispose chattering by approximating the discontinues function k_i (sign (s_{ij})) by using some continuous function. To replace the discontinuous function in (15 and 16) use:

$$\dot{s} = -k_1|s|^{1/2}\mathrm{sign}(s) + z_i \qquad \dot{z}_i = -k_2 \mathrm{sign}(s) \quad 17$$

$$\dot{s} = -k_1|s|^{\frac{1}{2}}\mathrm{sign}(s) - k_2\int_0^{\tau} \mathrm{sign}(s)\, d\tau$$

The control function is

$$u_{1o} = \left(\frac{-1}{p_1}\right)\left(x_2(b_2 - a_1 b_4) + x_3(c_1 - b_1 + a_1 b_1) + x_4 a_1 c_2 + (k_{11}|s_{11}|^{\frac{1}{2}}\mathrm{sign}(s_{11}) + k_{12}\int_0^{\tau} \mathrm{sign}(s_{11}\right. \quad 18$$

$$u_{2o} = \left(\frac{-1}{p_2}\right)\left(x_2(b_5 - a_2 b_8) + x_3(c_3 - b_7 + a_2 b_7) + x_4(b_7 - a_2 b_{10} + a_2 c_4) + (k_{21}|s_{12}|^{1/2}\mathrm{sign}(s_{12}) + k_{22}\int_0^{\tau} \mathrm{sign}(s_{12})\, d\right. \quad 19$$

3.1.3 ASMC and ASOSMC Regulation

Adaptive control is used right now to adjust unknown and time varying controller parameters and to lessen chattering on dynamics of control input.

$$u_{a1} = \left(-\frac{1}{p_1}\right)\left((\hat{Q}x_2 + \hat{R}x_3 + a_1 c_2 x_4\right) + k_1 \mathrm{sign}(s_{11})) \quad 20$$

$$u_{a2} = \left(-\frac{1}{p_2}\right)\left((\hat{w}x_2 + \hat{Y}x_3 + \hat{Z}x_4\right) + k_2 \mathrm{sign}(s_{12})) \quad 21$$

$$u_{ao1} = \left(-\frac{1}{p_1}\right)\left(\hat{Q}x_2 + \hat{R}x_3 + a_1 c_2 x_4 + k_{11}|s_{11}|^{1/2} sign(s_{11}) + k_{12}\int_0^{\tau} sign(s_{11})\, d\right. \quad 22$$

$$u_{ao2} = \left(-\frac{1}{p_2}\right)\left(\hat{w}x_2 + \hat{Y}x_3 + \hat{Z}x_4 - \ddot{x}_{d2} + k_{21}|s_{12}|^{1/2} sign(s_{12}) + k_{21}\int_0^{\tau} sign(s_{12})\, d\right. \quad 23$$

The adaptive law for both the equations in (20,21,22 and 23) are

$$\begin{gathered}\dot{\hat{b}}_1 = \gamma_1(s_{11}x_3a_1)\\ \dot{\hat{b}}_2 = \gamma_2(s_{11}x_2)\\ \dot{\hat{b}}_4 = -\gamma_3(s_{11}x_2a_1)\\ \dot{\hat{b}}_5 = \gamma_4(s_{12}x_2)\\ \dot{\hat{b}}_7 = -\gamma_5(s_{12}x_3(1-a_2))\\ \dot{\hat{b}}_8 = -\gamma_6(s_{12}x_2a_2)\\ \dot{\hat{b}}_9 = \gamma_7(s_{12}x_4)\\ \dot{\hat{b}}_{10} = -\gamma_8(s_{12}x_4a_2)\end{gathered} \quad 24$$

Where $p_1 = (b_3 - a_1b_3)$; $p_2 = (b_6 - a_2b_6)$; $\hat{Q} = (\hat{b}_2 - a_1\hat{b}_4)$; $\hat{R} = (-\hat{b}_1 + c_1 + \hat{b}_1a_1)$; $\widehat{W} = (\hat{b}_5 + a_2\hat{b}_8)$; $\hat{Y} = (-\hat{b}_7 + a_2\hat{b}_7 + c_2)$; $\hat{Z} = (\hat{b}_9 - a_2\hat{b}_{10} + a_2c_4)$; $k_{i\,,}k_{ij} > 0$

3.2 Design of Tracking Problem

Considering external disturbance and uncertainty dt the above equation (11 and 12) and assume the desired trajectory is x_d. Then the error between actual and desired trajectory of flexible manipulator link given as follows.

$$\begin{gathered}e_1 = x_1 - x_{d1}\\ e_2 = x_2 - x_{d2}\end{gathered} \quad 25$$

We may use the derivative of desired trajectory and calculate its time derivative of the error as:

$$\begin{gathered}\dot{e}_1 = \dot{x}_1 - \dot{x}_{d1} = x_3 - \dot{x}_{d1}\\ \dot{e}_2 = \dot{x}_2 - \dot{x}_{d2} = x_4\end{gathered} \quad 26$$

The defined sliding surfaces are s_1 and s_2 for sub system one:

$$\begin{gathered}s_1 = \dot{e}_1 + c_1e_1\\ s_2 = \dot{e}_2 + c_2e_2\end{gathered} \quad 27$$

Further sliding surfaces of sub system two as s_3 and s_4 are defined as:

$$\begin{gathered}s_3 = \dot{e}_{12} + c_3e_{12}\\ s_4 = \dot{e}_{22} + c_4e_{22}\end{gathered} \quad 28$$

Where c_1, c_2, c_3 *and* c_4, are positive design constants to satisfy the equations bellow in in (27 and 28);

$$\begin{gathered}s_1 = s_2 = s_3 = s_4 = 0\\ \dot{s}_1 = \dot{s}_2 = \dot{s}_3 = \dot{s}_4 = 0\end{gathered} \quad 29$$

At the end we can design the equivalent sliding surface for the system that is mentioned on the above in (11 and 12) as:

$$\begin{gathered}s_{11} = a_{11}s_1 + a_{12}s_2\\ s_{12} = a_{21}s_3 + a_{22}s_4\end{gathered} \quad 30$$

Where a_{11} , a_{12}, a_{21} and a_{22} positive design constants s_{11} and s_{12} are sliding surface of sub system one and two respectively.

3.2.1 Sliding Mode Control Law

Design SMC control law to drive the trajectory towards the sliding surface $s = 0$;

$$\begin{aligned}\dot{s_{11}} = {} & x_2(a_{11}b_2 - a_{12}b_4) + x_3(-b_1a_{11} + b_1a_{12} + a_{11}c_1) + x_4(a_{12}c_2) + (I_m + dt)(a_1b_3 - a_2b_3) - \ddot{x}_d(a_{11}) - \dot{x}_d(a_{11}c_1)\\ \dot{s_{12}} = {} & x_2(a_{21}b_5 - a_{22}b_8) + x_3(-b_7a_{21} + b_7a_{22} + a_{21}c_3) + x_4(b_9a_{21} - b_{10}a_{22} + a_{22}c_4) + (I_m + dt)(a_{21}b_6 - a_{22}b_6) - \ddot{x}_d(a_3) - \dot{x}_d(a_3c_3)\end{aligned} \quad 31$$

Using Utkin's principle of proper control wherein the regular reaching law as

$$\dot{s_{11}} = \dot{s_{12}} = -k_i\text{sign}(s) \quad 32$$

Now from (31 and 32) we have;

$$\begin{aligned}u_1 &= \left(-\frac{1}{p_1}\right)\left(Qx_2 + Rx_3 + Tx_4 - \ddot{x}_d(a_{11}) - \dot{x}_d(a_{11}c_1) - k_1\text{sign}(s_{11})\right)\\ u_2 &= (-\frac{1}{p_2})\left(wx_2 + Yx_3 + Zx_4 - \ddot{x}_d(a_{21}) - \dot{x}_d(a_{21}c_3) - k_2\text{sign}(s_{12})\right)\end{aligned} \quad 33$$

Where, k_i is positive constant and $k_i \geq 0, \text{i} = 1{,}2, u_1, u_2 = I_m$ classical sliding mode control law

$p_1 = (a_{11}b_3 - a_{12}b_3)$; $p_2 = (a_{21}b_6 - a_{22}b_6)$; $Q = (a_{11}b_2 - a_{12}b_4)$; $R = (-b_1a_{11} + b_1a_{12} + a_{11}c_1)$; $T = (a_{12}c_2)$; $W = (a_{21}b_5 - a_{22}b_8)$; $Y = (-b_7a_{21} + b_7a_{22} + a_{21}c_3)$ and Z= $(b_9a_{21} - b_{10}a_{22} + a_{22}c_4)$, k_i >0; k_i can be designed suitably.

THEOREM 1: Control law u_1 and u_2 forces the trajectories of manipulator equation (11 and 12) converge to the direction of sliding surface *s*(*t*) = 0 and its dynamic follow the predefined or favored trajectory (x_{d1} and x_{d2}):

PROOF: Let's Consider the principle of Lyapunov as positive, symmetric and definite function that contains sliding surface.

$$V(s_{11}) = \frac{1}{2}(s_{11}{}^2)$$
$$V(s_{12}) = \frac{1}{2}(s_{12}{}^2) \quad 34$$

Taking time derivative of (34)

$$\dot{V}(s_{11}) = -k_1|s_{11}| + s_{11}|dt1| \le 0 \quad 35$$
$$\dot{V}(s_{12}) = -k_2|s_{12}| + s_{12}|dt2| \le 0$$

Where $|dti|$ is bounded, since $k_i > 0$ and $k_i = |dti|$ the Lyapunov function is proved and the switching surface $s(t)$ = 0 holds the Lyapunov stability and always converges to the designed sliding surface. So, the manipulator dynamics flows the desired trajectory and also the error dynamics are asymptotically stable on sliding manifold.

3.2.2 Second Order SMC Control

Supper twisted algorithm is more famous because it handles the states identification and not its derivative [28]. In this paper, a robust STA type 2nd Order Sliding Mode (SOSM) controller is designed for 2-DOF serial flexible link manipulator system. So, supper twisted algorithm is designed as (17):

$$u_{co1} = \left(-\frac{1}{P}\right)\Big((Qx_2 + Rx_3 + Tx_4 - \ddot{x}_d(a_{11}) - \dot{x}_d(a_{11}c_1)\Big) + (k_1|s_{11}|^{\frac{1}{2}}\text{sign}(s_{11}) + k_{12}\int_0^\tau \text{sign}(s_{11})\,d\tau)) \quad 36$$

$$u_{co2} = \left(-\frac{1}{P}\right)(wx_2 + Yx_3 + Zx_4 - \ddot{x}_d(a_{21}) - \dot{x}_d(a_{21}c_3) + (k_2|s_{12}|^{1/2}\text{sign}(s_{12}) + k_{22}\int_0^\tau \text{sign}(s_{12})\,d\tau$$

Hence, Control u_{co1} and u_{co2} forces the trajectories of manipulator equation (11 and 12) converge to the direction of sliding surface $s(t)$ = 0 and its dynamic follow the predefined or favored trajectory (x_{d1} and x_{d2}):

3.2.3 Adaptive Sliding Mode Control

Adaptive controller is designed right now to estimate the value of unknown and time varying controller parameters and to attenuate chattering on dynamics of ASMC control input.

$$u_{a1} = \left(-\frac{1}{p_1}\right)\Big((\hat{Q}x_2 + \hat{R}x_3 + Tx_4 - \ddot{x}_{d1}(a_{11}) - \dot{x}_{d1}(a_{11}c_1)\Big) + k_1\text{sign}(s_{11}))$$
$$u_{a2} = \left(-\frac{1}{p_2}\right)\Big((\hat{w}x_2 + \hat{Y}x_3 + \hat{Z}x_4 - \ddot{x}_{d2}(a_{21}) - \dot{x}_{d2}(a_{21}c_3)\Big) + k_2\text{sign}(s_{12})) \quad 37$$

Where, $\hat{Q} = (a_{11}\hat{b}_2 - a_{12}\hat{b}_4)$; $\hat{R} = (-\hat{b}_1 a_{11} + a_{11}c_1 + \hat{b}_1 a_{12})$; $\widehat{W} = (a_{21}\hat{b}_5 + a_{21}\hat{b}_8)$; $\hat{Y} = (-a_{21}\hat{b}_7 + a_{22}\hat{b}_7 + a_{21}c_3)$; $\hat{Z} = (a_{21}\hat{b}_9 - a_{22}\hat{b}_{10} + a_{22}c_4)$ and The adaptive law can be designed as

$$\dot{\hat{b}}_1 = \gamma_1(s_{11}x_3(a_{12} - a_{11}))$$
$$\dot{\hat{b}}_2 = \gamma_2(s_{11}x_2a_{11})$$
$$\dot{\hat{b}}_4 = -\gamma_3(s_{11}x_2a_{12})$$
$$\dot{\hat{b}}_5 = \gamma_4(s_{12}x_2(a_{21}))$$
$$\dot{\hat{b}}_7 = \gamma_5\big(s_{12}x_3(a_{22} - a_{21})\big) \quad 38$$
$$\dot{\hat{b}}_8 = -\gamma_6(s_{12}x_2a_{22})$$
$$\dot{\hat{b}}_9 = \gamma_7(s_{12}x_4a_{21})$$
$$\dot{\hat{b}}_{10} = -\gamma_8(s_{12}x_4a_{22})$$

Where $\gamma_i(i = 1,2,3,4,5,6,7 and\ 8) > 0$ (positive constant); $\widehat{b_i}$ is estimation value of b_i(i =1,2,4,5,7,8,9 and 10)

THEOREM 2 [14]: The system dynamics nature with unknown parametric values are stable as globally as well as asymptotically. The following control law with I_m replaced by u_a in dynamics (11 and 12) and realize this system without model information we can write systems in (11 and 12) using the proposed methods by Murphy and Ebrahim [29] as follows:

PROOF: In order to design controller without need model information, we choose [29] b_1, b_2, b_4, b_5, b_7, b_8, b_9 and b_{10} as unknown model parameters then the Lyapunov function is defined as:

$$\mathrm{V}_1 = \frac{1}{2}(\mathrm{s}_{11})^2 + \frac{1}{2\gamma_1}(\mathrm{b}_1 - \hat{\mathrm{b}}_1)^2 + \frac{1}{2\gamma_2}(\mathrm{b}_2 - \hat{\mathrm{b}}_2)^2 + \frac{1}{2\gamma_3}(\mathrm{b}_4 - \hat{\mathrm{b}}_4)^2$$

$$\mathrm{V}_2 = \frac{1}{2}(\mathrm{s}_{12})^2 + \frac{1}{2\gamma_4}(\mathrm{b}_5 - \hat{\mathrm{b}}_5)^2 + \frac{1}{2\gamma_5}(\mathrm{b}_7 - \hat{\mathrm{b}}_7)^2 + \frac{1}{2\gamma_6}(\mathrm{b}_8 - \hat{\mathrm{b}}_8)^2 + \frac{1}{2\gamma_7}(\mathrm{b}_9 - \hat{\mathrm{b}}_9)^2 + \frac{1}{2\gamma_8}(\mathrm{b}_{10} - \hat{\mathrm{b}}_{10})^2 \quad 39$$

$$\dot{\mathrm{V}}_1 = -\mathrm{k}_1|\mathrm{s}_{11}| + (\mathrm{b}_1 - \hat{\mathrm{b}}_1)\left(\mathrm{s}_{11}\mathrm{x}_2\mathrm{a}_{11} - \left(\frac{\dot{\hat{\mathrm{b}}}_1}{\gamma_1}\right)\right) + (\mathrm{b}_2 - \hat{\mathrm{b}}_2)\left(\mathrm{s}_{11}\mathrm{x}_3(\mathrm{a}_{12} - \mathrm{a}_{11}) - \left(\frac{\dot{\hat{\mathrm{b}}}_2}{\gamma_2}\right)\right) + (\mathrm{b}_4 - \hat{\mathrm{b}}_4)\left(\mathrm{s}_{11}\mathrm{x}_2(\mathrm{a}_{12}) + \left(\frac{\dot{\hat{\mathrm{b}}}_4}{\gamma_3}\right)\right) \quad 40$$

$$\dot{\mathrm{V}}_2 = -\mathrm{k}_2|\mathrm{s}_{12}| + (\mathrm{b}_5 - \hat{\mathrm{b}}_5)\left(\mathrm{s}_{12}\mathrm{x}_2\mathrm{a}_{21} - \left(\frac{\dot{\hat{\mathrm{b}}}_5}{\gamma_4}\right)\right) + (\mathrm{b}_7 - \hat{\mathrm{b}}_7)\left(\mathrm{s}_{12}\mathrm{x}_3(\mathrm{a}_{22} - \mathrm{a}_{12}) - \left(\frac{\dot{\hat{\mathrm{b}}}_7}{\gamma_5}\right)\right) - (\mathrm{b}_8 - \hat{\mathrm{b}}_8)\left(\mathrm{s}_{12}\mathrm{x}_2(\mathrm{a}_{22}) + \left(\frac{\dot{\hat{\mathrm{b}}}_8}{\gamma_6}\right)\right) + (\mathrm{b}_9 - \hat{\mathrm{b}}_9)\left(\mathrm{s}_{12}\mathrm{x}_4(\mathrm{a}_{21}) - \left(\frac{\dot{\hat{\mathrm{b}}}_9}{\gamma_7}\right)\right) - (\mathrm{b}_{10} - \hat{\mathrm{b}}_{10})\left(\mathrm{s}_{12}\mathrm{x}_4(\mathrm{a}_{22}) + \left(\frac{\dot{\hat{\mathrm{b}}}_{10}}{\gamma_8}\right)\right)$$

Since $k_i > 0, k_i > |dti|$ and the Lyapunov function is proved and the switching surface $s(t) = 0$ holds the Lyapunov stability and always converges to the designed sliding surface. So, the manipulator dynamics flows the desired trajectory and also the error dynamics are asymptotically stable on sliding manifold.

3.2.4 Adaptive Second Order Sliding Mode Control

In adaptive SMC and 1st order SMC chattering is not eliminated due to discontinuities in control law function that is mentioned in (33) and (37) equation of motion respectively although chattering is reduced in some extent when they use adaptive mechanism in SMC. At the end we need to propose Adaptive 2nd order sliding mode control to eliminate the chattering and we take the data to compare results to our proposed controller with new control law as:

$$u_{ao1} = \left(-\frac{1}{p_1}\right)\left(\hat{Q}x_2 + \hat{R}x_3 + \hat{T}x_4 - \ddot{x}_{d1}(a_{11}) - \dot{x}_{d1}(a_{11}c_1) + k_{11}|s_{11}|^{\frac{1}{2}}\mathrm{sign}(s_{11}) + k_{12}\int_0^{\tau} \mathrm{sign}(s_{11})\,d\tau\right) \quad 41$$

$$u_{a02} = \left(-\frac{1}{p_2}\right)\left(\hat{w}x_2 + \hat{Y}x_3 + \hat{Z}x_4 - \ddot{x}_{d2}(a_{21}) - \dot{x}_{d2}(a_{21}c_3) + k_{21}|s_{12}|^{1/2}\mathrm{sign}(s_{12}) + k_{22}\int_0^{\tau} \mathrm{sign}(s_{12})\,d\tau\right.$$

Here, the desired trajectory is tracked to stabilized all the states globally and asymptotically as we designed in (36) and parameter update laws in (37). because the integrative term handle or avoid the system discontinuity term as well.

3.2.5 Model Predictive Control Design

MPC is one of most powerful controllers for engineers. It is applicable for MIMO systems, input output interactions, constraint capability and used in many industries like process and robotic application. The continuous time mode of our plant can be converted in to discreet one corresponding m number of inputs and q number of outputs.

$$x_m(k+1) = A_m X_m(k) + B_m u(k) \quad 42$$
$$y_m(k) = c_m x_m(k)$$

$$x_m(k) = A_m X_m(k-1) + B_m u(k-1) \quad 43$$

Now we can write the output y_m in terms of state variables maintained in equation as

$$\Delta y_m(k+1) = C_m \Delta x_m(k+1) = C_m A_m \Delta x_m(k) + C_m B_m \Delta u_m(k) \quad 44$$

The above equations can be rewrite as in state space form as

$$\begin{vmatrix} \Delta x_m(k+1) \\ \Delta y_m(k+1) \end{vmatrix} = \begin{vmatrix} A_m & 0_m^T \\ C_m A_m & I_{q\times q} \end{vmatrix} \Delta x_m(k) + \begin{vmatrix} B_m \\ C_m B_m \end{vmatrix} \Delta u_m(k) \quad 45$$

$$y_m(k) = [0_m \quad I_{q\times q}] \begin{vmatrix} \Delta x_m \\ y_m \end{vmatrix}$$

Now we have the output equation from the predefined equation above

$$y = Fx(k) + \varphi \Delta u$$

Where the above terms are

$$F = \begin{vmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^N P \end{vmatrix}; \quad \varphi = \begin{vmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ CAB^2 & CAB & CB & 0\cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^N p^{-1}B & CA^N p^{-2}B & CA^N p^{-3}B & \cdots & CA^N p^{-N_c}B \end{vmatrix}$$

The control law that is increased in each time interval given by

$$\Delta U = (\varphi^T\varphi + R)^{-1}(\varphi^T R_s - \varphi^T F x(k)) \quad 46$$

And the signal given for set point operation is

$$r(k) = \left| \; r_1(k) \quad r_2(k)\cdots \quad r_q(k) \right|^T$$

4. Results and Discussions

We use ODE45 simulator to simulate system dynamics in equation (11 and 12), (13), (15), (16), and (18-24) by using a constant step size h = 10^{-3} in MATLAB simulation environment. The parametric values are b_1 = 62.95, b_2 = 628.88, b_3 = 140.47, b_4 = 863.3324, b_5 = 2271.1, b_6 = 288.12, b_7 = 496.76, b_8 = 3336.19, b_9 = 28.51, b_{10} = 41.64. The initial conditions for regulation problem and tracking problem are taken to figure out the system response of 2-DOF serial flexible link equation (11 and 12), sliding surface (13) controller (15), (16),(18-24) respectively as x(0) = $(0.15,0.1,0,0,0.2,0.15,0,0)^T$ and $(0,0,0,0,0,0,0,0)^T$, and the value of those constants used for simulation are c_1, c_3=3 and c_4, c_2=2, a_1, a_2=0.1 and a_4, a_2= 0.5,$k_1, k_2 = 100, k_{11}, k_{21} = 100, k_{12} = 95, k_{22} = 150$ for regulation and $c_1, c_3 = 3$ and $c_4, c_2 = 2, a_{11}, a_{21} = 0.5$ and $a_{12}, a_{22} = 0.3,\ k_1, k_2 = 100, k_{11} = 200, k_{21} = 170, k_{12} = 170, k_{22} = 150$ for tracking. The initial conditions for Adaptive control are taken to plot the dynamics of manipulator, as x(0) = $(0.15,0.1,0,0,0.2,0.15,0,0,0,0,0,0,0,0,0,0)^T$ and $(0,0,0,0,0,0,0,0,0,0)^T$ respectively and the value of different constants are c_1, c_3=3 and c_4, c_2=2, a_1, a_2=0.1 and a_4, a_2= 0.5,$k_1, k_2 = 100, k_{11}, k_{21} = 100, k_{12} = 95, k_{22} = 150$ for regulation and $c_1, c_3 = 3$ and $c_4, c_2 = 2, a_{11}, a_{21} = 0.5$ and $a_{12}, a_{22} = 0.3,\ k_1, k_2 = 100, k_{11} = 200, k_{21} = 170,$

$k_{12} = 170, k_{22} = 150$ for tracking and $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_7 = 10, \gamma_5, \gamma_6, \gamma_8 = 100.$

Prediction horizon=10, control horizon= 2

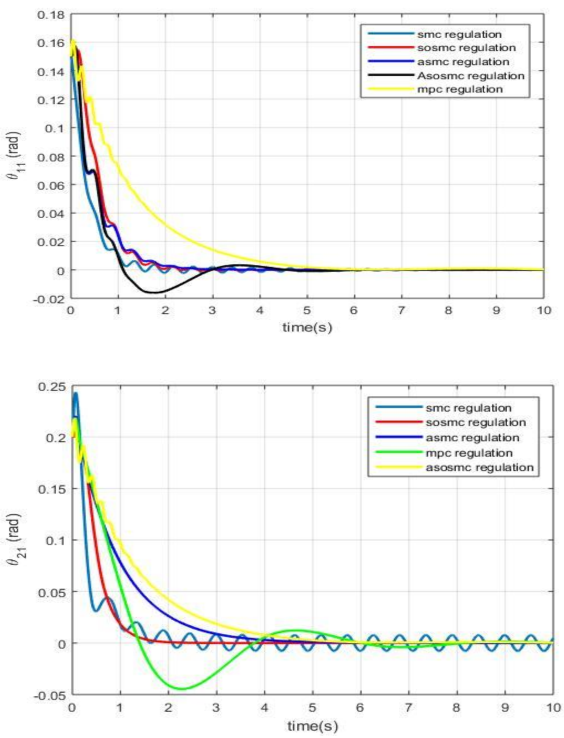


Fig 2:performance of controllers for position regulation

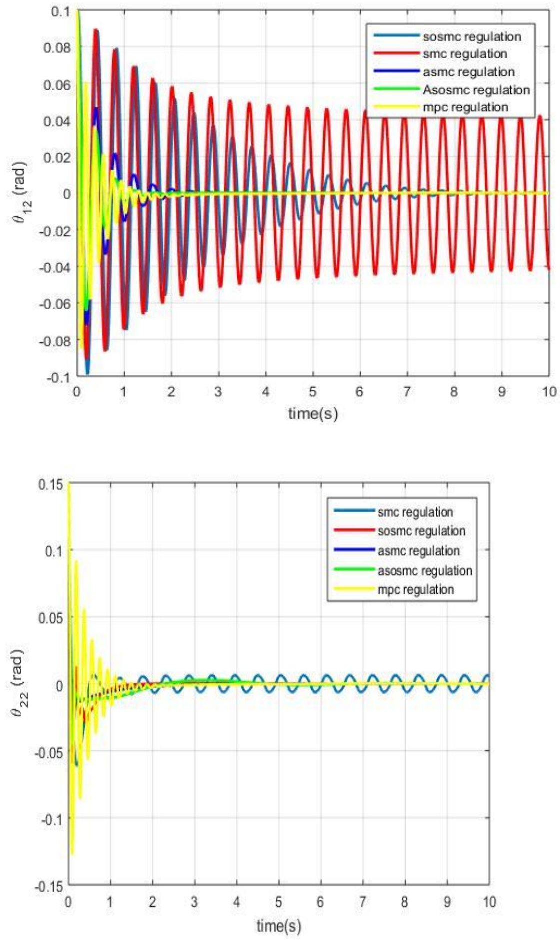
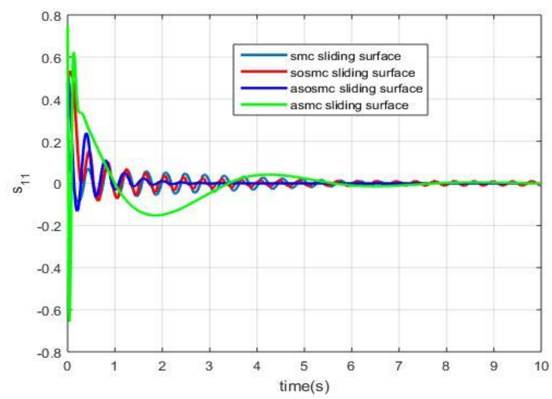


Fig 3:performance of controllers for deflection regulation



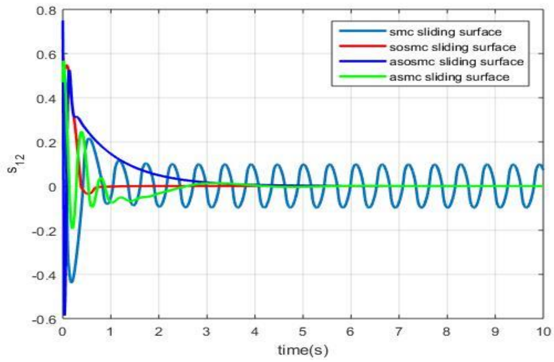


Fig 4: Sliding surfaces for regulation problem

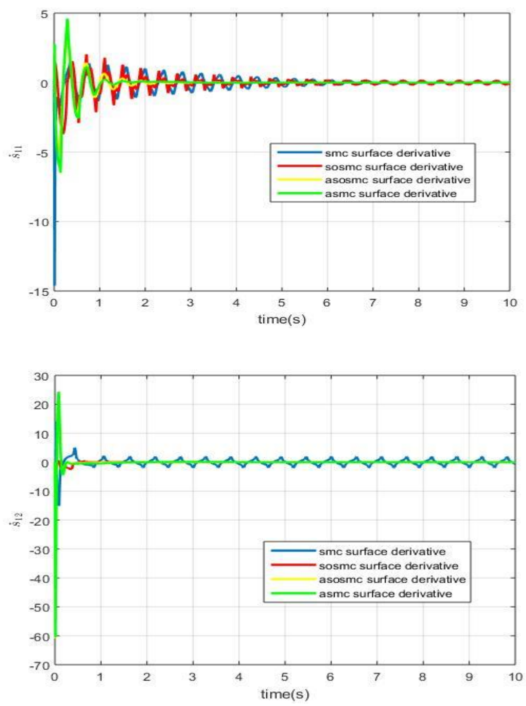


Fig 5: Sliding surface derivatives for regulation problem

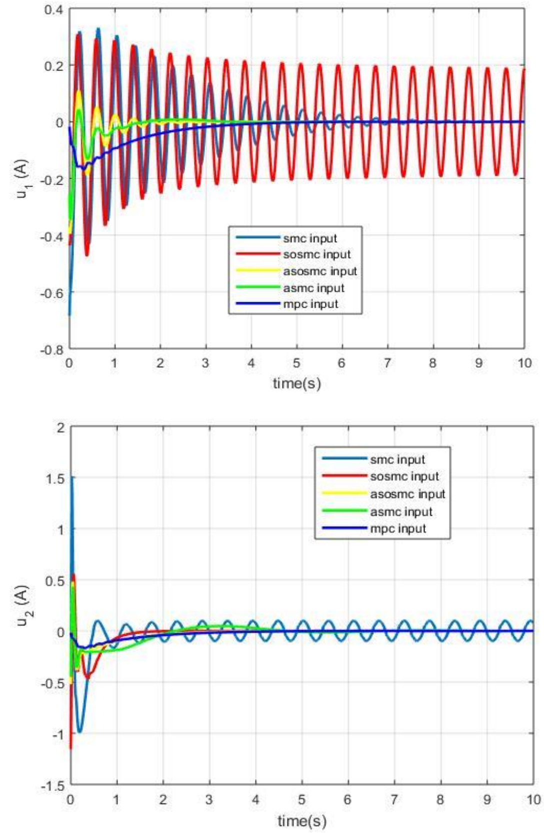


Fig 6: Control inputs for regulation problems

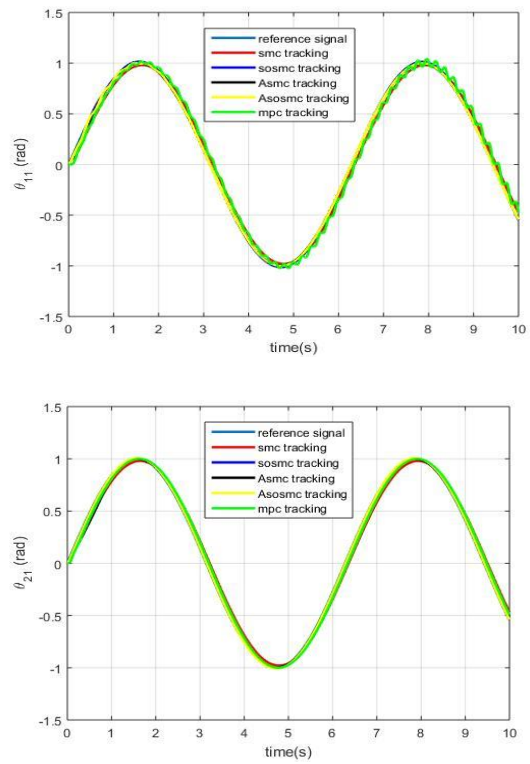


Fig 7: performance of controllers for position tracking

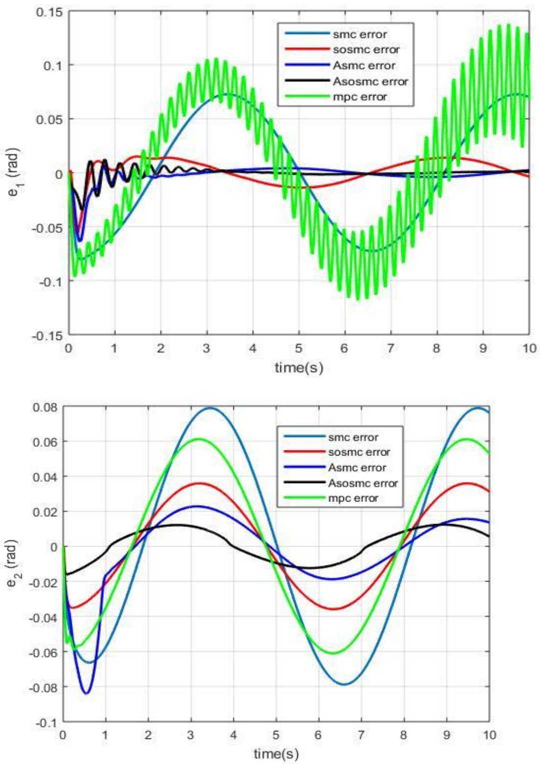


Fig 8: Position tracking error of controllers

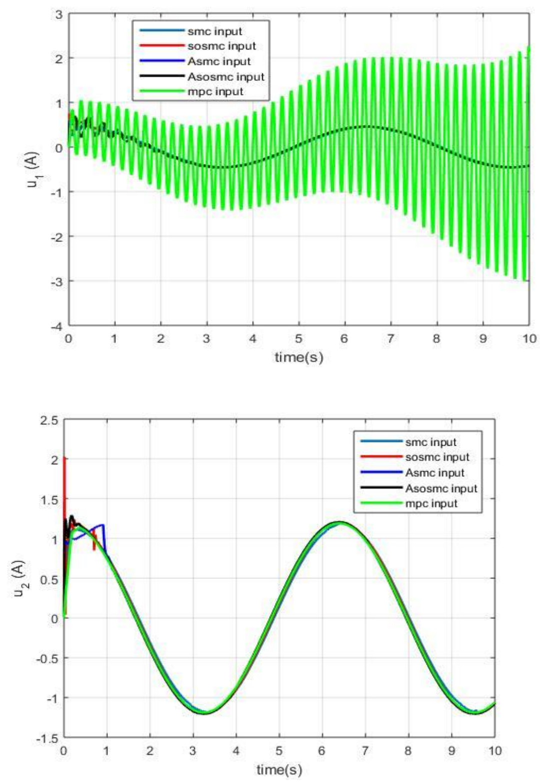


Fig 9: Position control signal for controllers in tracking

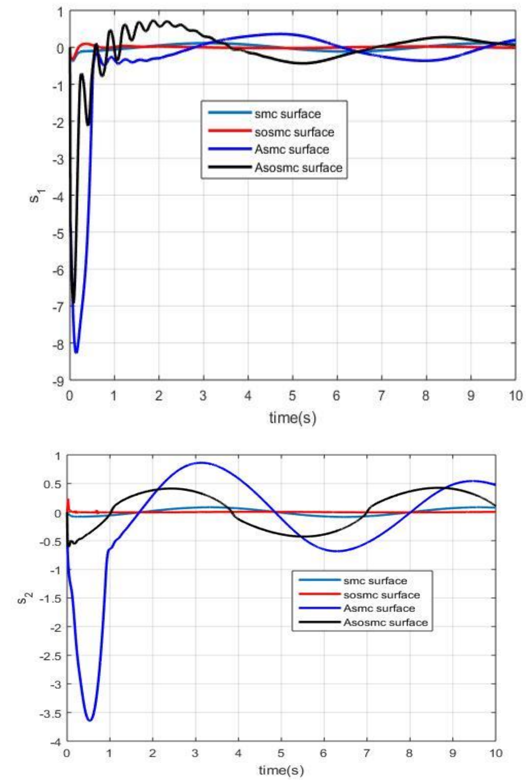
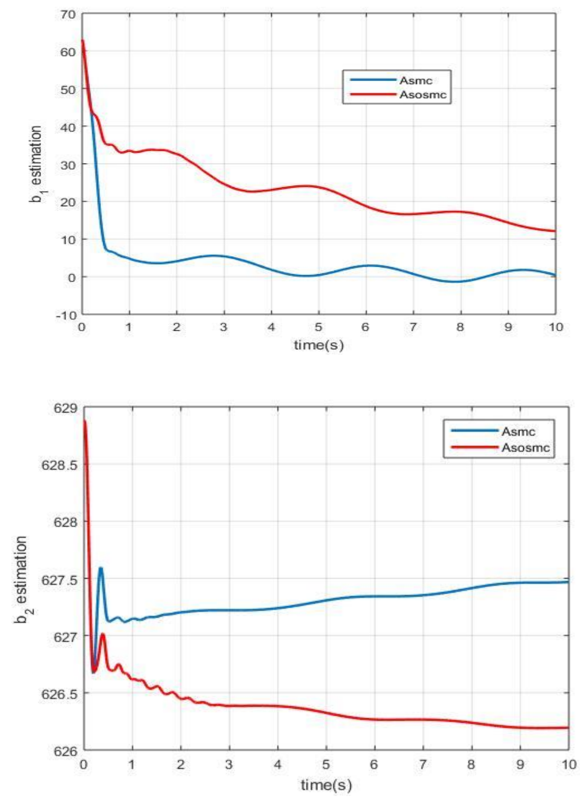
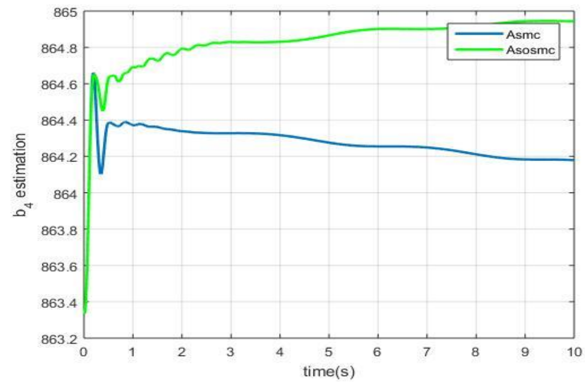
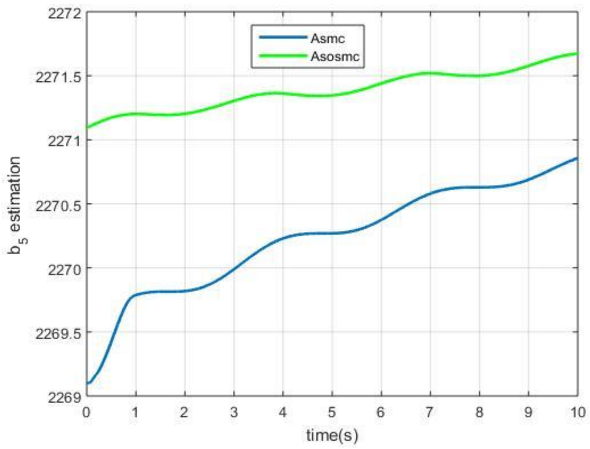
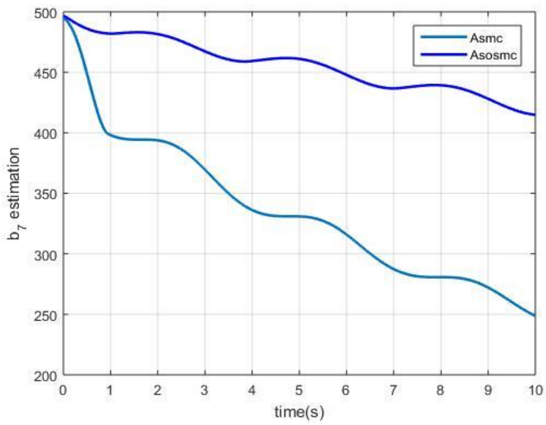


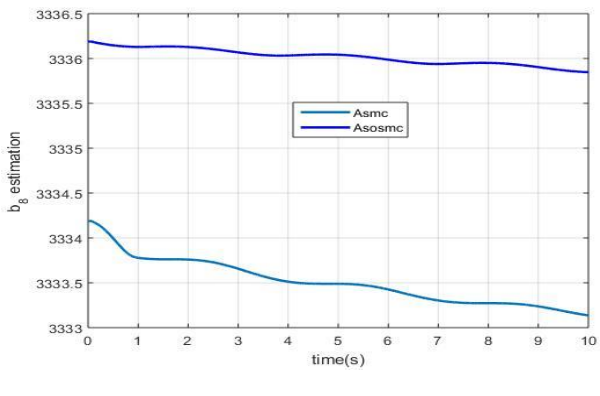
Fig 10: Tracking sliding surfaces

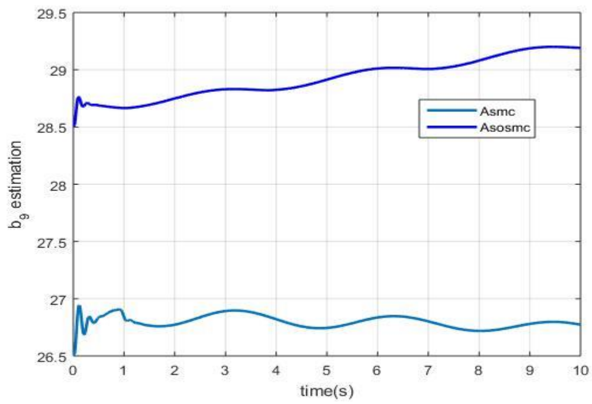












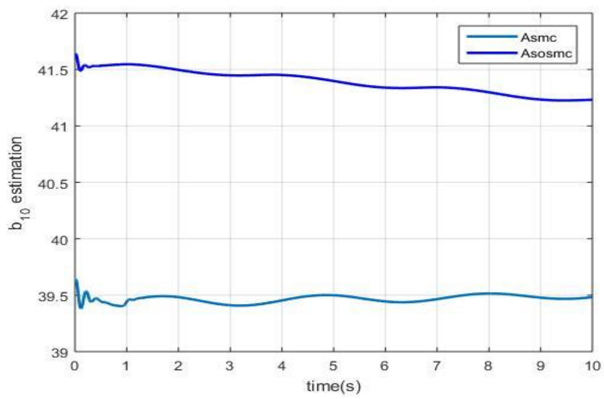


Fig 11: parameter estimation for tracking (b1-b10)

The effectiveness of adaptive second order sliding mode control(ASOSMC) controller changed into compared with conventional, classical or 1st order sliding mode control(SMC), 2nd order SMC, model predictive control(MPC) and adaptive 2nd order SMC by using the six-performance index like integral absolute error(IAE), integral square error(ISE), integral time square error(ITSE), integral time absolute error(ITAE), mean square error(MSE) and mean absolute error(MAE) with percentage improvement a minimum of 31.4127% and a maximum of 99.8490% in case of ASMC and SMC respectively. These results show that the adaptive 2nd order SMC controller outperforms the above listed controllers**.**

5. Conclusions and Recommendations for Future Work

Robotic system is one part of the essential mechatronic system. In this thesis paper, we take some assumptions, the model of serial flexible link system was developed and the four controllers was designed. The Lyapunov principle and equations was proved. MATLAB/Simulink software is used to simulate the work. The simulation shows that the four controllers can regulate the system and make the states follow the reference trajectory even in the presence of external disturbance.

Finally, by comparing the performance index of each controller with external disturbance and uncertainties, the simulation results clearly indicated that ASOSMC has best performance than the four controllers. The reduced value or percentage improvement in error resulted in perfect tracking towards the desired value and hence, the reduced vibrations and chattering felt by the flexible link model. So, the less performance index ensured better comfort to the robotic application. Therefore, the proposed ASOSMC controller was more effective in regulation and better tracking. In the proposed controller design a linear spring model of the plant is used in flexible links to simplify model equation. However, the model may not be linear spring rather it is nonlinear and complex. Hence it may decrease the accuracy of the designed control. An adaptive MPC can be designed to take into account the uncertainty where the parameters of the system updated using RPME algorithm. For future; we will suggest to implement and test ASOSMC practically.

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