Adaptive Field-Oriented Control with MRAC Regulator for the Permanent Magnet Synchronous Motor

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Abstract: In the industrial world; the applications of Permanent Magnet Synchronous Motor(PMSM) do not cease in spite of nonlinearity of the mathematical model. In this study; one detailed the demonstration of application of the Field-Oriented Control (FOC) with the Model Reference Adaptative Control (MRAC) where there is a regulator in the feedforward and the another chain direct. The results are proven by the SIMULINK/MATLAB software.

Key–Words: Permanent Magnet Synchronous Motor(PMSM); Field-Oriented Control (FOC); Feedforward; Model Reference Adaptative Control (MRAC).

1 Introduction

Thanks to progress of the power electronics, industry was directed towards the use of the machines with AC current in order to benefit from their advantages such as, the variation speed flexibility and the stability operation [1]. Among the machines with AC current used in the drives, the PMSM which presents a certain number of advantages, namely not of rotor losses, a weak inertia, its mass couple high compared to that of the asynchronous motor and traditional synchronous motor. Moreover, they have inductances relatively weak, which involves fast answers of the currents and thus of the couple [2, 3]. To thwart this difficulty and to obtain a situation equivalent to that of the machine with D.C. current with separated excitation, BLASCHKE and HASSE in 1972[4], proposed a technique of control known as control vectorial called too FOC. The fundamental idea of this strategy is to compare the synchronous machine behavior to that of a D.C. machine, i.e. a model linear and uncoupled what makes it possible to improve its dynamic behavior [4]. However, the vectorial control it could not be established and really used because the regulations, at the time, thus rested on component analogical, the establishment of the control was difficult. With the micro-controllers event and thanks to progress electronics numerical and the appearance of the fast processors of digital processing of the signal like (DSP), the realization of their control became increasingly simple. That led to an research explosion and applications relating to the FOC of the PMSM.

To satisfy the needs for this study, we organized our work as follows:

- Mathematical Model of the PMSM.
- Vectorial Control of the PMSM.
- Simulations and Results.

2 Mathematical Model of the PMSM

With the simplifying assumptions relating to the PMSM, the model of the motorexpressed in the reference of PARK, in the form of state is written [5, 6, 7]:

$$\begin{aligned}
\frac{di_d}{dt} &= -\frac{R}{L_d}i_d + \frac{L_q}{L_d}p\Omega i_q + \frac{1}{L_d}V_d \\
\frac{di_q}{dt} &= -\frac{L_d}{L_q}p\Omega i_d - \frac{R}{L_q}i_q - \frac{\Psi}{L_q}p\Omega + \frac{1}{L_q}V_q \\
\frac{d\Omega}{dt} &= \frac{3}{2}\frac{p}{J}\left(\Psi i_q + (L_d - L_q)i_d i_q\right) - \frac{f_v}{J}\Omega - \frac{1}{J}T_l
\end{aligned} \tag{1}$$

Where V_d, V_q are quadrature axis and direct axis voltages, i_d, i_q are quadrature axis and direct axis currents. Ω rotating speed, T_l applied external load torque.

If the motoris with constant air-gap (without polar parts).

In these equations:

R	: stator resistance,
Ψ	: flux linkages of permanent magnet rotor,
L_d	: direct stator inductance,
L_q	: quadrature stator inductance,
f_v	: frictional coefficient,
p	: number of poles.

3 Vectorial Control of the PMSM

The synchronous motor model in the reference frame of PARK led to a system of differential equations where the currents are not independent one of the other, they are connected by nonlinear terms or coefficients. This coupling is eliminated by a method of compensation [8]. The principle of this decoupling amounts defining two new variables of control such as [8]:

$$\begin{cases} V_d &= v_d - e_d \\ V_q &= v_q + e_q \end{cases} \quad (2)$$

Where e_d and e_q are the terms of coupling given by :

$$\begin{cases} e_d &= L_q p\Omega i_q \\ e_q &= L_d p\Omega i_d + \Psi p\Omega \end{cases} \quad (3)$$

and orders it uncoupling

$$\begin{cases} v_d &= L_d \dot{i_d} + R i_d \\ v_q &= L_q \dot{i_q} + R i_q \end{cases} \quad (4)$$

Transfer functions of this system uncoupled while taking as in-puts $v_d,\ v_q$ and as out-puts $i_d,\ i_q$ and :

$$\begin{cases} \dfrac{i_d}{v_d} &= \dfrac{1}{L_d\left(s+\frac{R}{L_d}\right)} \\ \dfrac{i_q}{v_q} &= \dfrac{1}{L_q\left(s+\frac{R}{L_q}\right)} \end{cases} \quad (5)$$

We will present the synthesis of each regulator separately closely connected to clarify the synthesis methodology of each one of them.

i_d-current regulator :
We wish to obtain in closed loop a response of the type 1^{st} order. To achieve this objective, one takes an regulator with MRAC of the type:

$$v_d(s) = \theta_2\left(\theta_1 U_{ic} - i_d\right) \quad (6)$$

We can represent the system in closed loop by the figure (FIG.1)

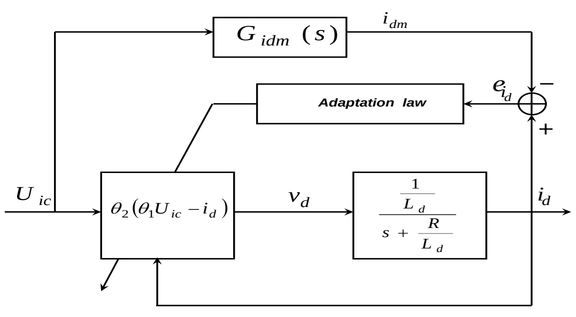

Figure 1: Diagram block in closed loop of MRAC of direct axis current.

The reference model of the system in closed loop is selected with a first-order transfer function:

$$G_{idm}(s) = \frac{b_{im}}{s + a_{im}}$$

That is to say the optimality criterion $J(e)$ of the adjustment loop is expressed by the absolute value [9]:

$$J(e) = |e| \quad (7)$$

Its derivative is :

$$\frac{\partial J(e)}{\partial e} = sign(e) \quad (8)$$

In closed loop, the transfer function is written:

$$G_{BFi}(s) = \frac{\theta_1\theta_2 K_i}{s + \frac{R}{L_d} + \theta_2 K_i} \quad (9)$$

With $K_i = \frac{1}{L_d}$. The error $e = i_d - i_{dm}$, its derivative compared to the parameters gives :

$$\frac{\partial e}{\partial \theta_1} = \frac{\theta_2 K_i}{s + \frac{R}{L_d} + \theta_2 K_i} U_{ic}(s) \quad (10)$$

$$\begin{aligned} \frac{\partial e}{\partial \theta_2} &= \frac{\theta_1 K_i\left(s+\frac{R}{L_d}\right)}{\left(s+\frac{R}{L_d}+\theta_2 K_i\right)^2} U_{ic}(s) \\ &= \frac{s+\frac{R}{L_d}}{s+\frac{R}{L_d}+\theta_2 K_i} . \frac{\theta_1 K_i}{s+\frac{R}{L_d}+\theta_2 K_i} U_{ic}(s) \end{aligned} \quad (11)$$

For $e = 0 \Rightarrow i_d = i_{dm}$ then $\frac{R}{L_d} + \theta_2 K_i = a_{im}$, and $\theta_1\theta_2 K_i = b_{im}$.

$$\begin{aligned} \frac{\partial e}{\partial \theta_1} &= \frac{1}{\theta_1} . \frac{b_{im}}{s + a_{im}} U_{ic}(s) \\ &= \frac{1}{\theta_1} . i_{dm} \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{\partial e}{\partial \rho_2} &= \frac{1}{\theta_2} . \frac{b_{im}}{s + a_{im}} \\ &\quad . \frac{s + a_{im} - \frac{b_{im}}{\theta_2}}{s + a_{im}} U_{ic}(s) \\ &= \frac{1}{\theta_2} . \frac{b_{im}}{s + a_{im}} . \left(U_{ic}(s) - \frac{1}{\theta_2} i_{dm}\right) \end{aligned} \quad (13)$$

Taking into account (8), (12) and (13), one can write the equation of gradient θ_1 and θ_2:

$$\mathcal{L}\left\{\frac{d\theta_1}{dt}\right\} = -\gamma_{i1} \frac{\partial J(e)}{\partial e} \frac{\partial e}{\partial \theta_1} \quad (14)$$

$$\theta_1(s) = -\frac{\gamma_{i1}}{s} sign(e) . \frac{1}{\theta_1} . i_{dm} \quad (15)$$

And

$$\mathcal{L}\left\{\frac{d\theta_2}{dt}\right\} = -\gamma_{i2}\frac{\partial J(e)}{\partial e}\frac{\partial e}{\partial \theta_2} \quad (16)$$

$$\theta_2 = -\frac{\gamma_{i2}}{s}sign(e).\frac{1}{\theta_2}.\frac{b_{im}}{s+a_{im}} .\left(U_{ic}(s)-\frac{1}{\theta_2}i_{dm}\right) \quad (17)$$

Speed regulator :

According to the mechanical equation of the PMSM (1); we have :

$$\Omega = \frac{1}{Js+f_v}\left(T_{em}-T_l\right) \quad (18)$$

From where the expression of the electromechanical torque is given by the formula :

$$T_{em} = \frac{3p}{2}\left(\Psi+(L_d-L_q)i_d\right)i_q \quad (19)$$

While replacing, i_{sq} the system (5) in the torque (19)

$$T_{em} = \frac{3p}{2}\left(\Psi+(L_d-L_q)i_d\right).\frac{1}{L_q\left(s+\frac{R}{L_q}\right)}.v_{sq} \quad (20)$$

Therefore, equation (18) becomes :

$$\Omega = \frac{\frac{3p}{2}\left(\Psi+(L_d-L_q)i_d\right)}{L_q\left(Js+f_v\right)\left(s+\frac{R}{L_q}\right)}.v_{sq} - \frac{1}{Js+f_v}T_l \quad (21)$$

For closed loop speed it was proposed regulator MRAC of the form :

$$v_{sq}(s) = \vartheta_2\left(\vartheta_1 U_{\Omega c}-\Omega\right) \quad (22)$$

The functional diagram is given by the figure (FIG.2)

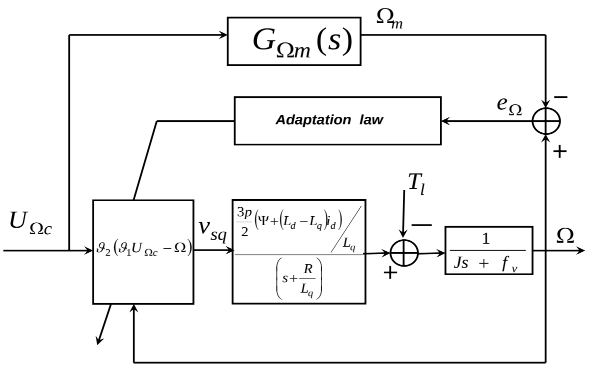

Figure 2: Diagram block in loop closed of MRAC regulator of rotating speed .

The reference model of the loop closed is selected with a second-order transfer function:

$$G_{\Omega m}(s) = \frac{b_{\Omega m}}{s^2+a_{1\Omega m}s+a_{0\Omega m}}$$

In open loop, the transfer function is written:

$$G_{BO\Omega}(s) = \frac{\vartheta_1\vartheta_2 K_\Omega}{\left(s+\frac{R}{L_q}\right)\left(s+\frac{f_v}{J}\right)} \quad (23)$$

with $K_\Omega = \frac{3p}{2L_qJ}\left(\Psi+(L_d-L_q)i_d\right)$

And in closed loop:

$$G_{BF\Omega}(s) = \frac{\vartheta_1\vartheta_2 K_\Omega}{s^2+\left(\frac{R}{L_q}+\frac{f_v}{J}\right)s+\frac{Rf_v}{L_qJ}+\vartheta_2 K_\Omega} \quad (24)$$

The error $e = \Omega - \Omega_m$, its derivative compared to the parameters gives :

$$\frac{\partial e}{\partial \vartheta_1} = \frac{\vartheta_2 K_\Omega}{s^2+\left(\frac{R}{L_q}+\frac{f_v}{J}\right)s+\frac{Rf_v}{L_qJ}+\vartheta_2 K_\Omega}U_{\Omega c}(s) \quad (25)$$

$$\frac{\partial e}{\partial \vartheta_2} = \frac{s^2+\left(\frac{R}{L_q}+\frac{f_v}{J}\right)s+\frac{Rf_v}{L_qJ}}{\left(s^2+\left(\frac{R}{L_q}+\frac{f_v}{J}\right)s+\frac{Rf_v}{L_qJ}+\vartheta_2 K_\Omega\right)^2}U_{\Omega c}(s)$$
$$= \frac{s^2+\left(\frac{R}{L_q}+\frac{f_v}{J}\right)s+\frac{Rf_v}{L_qJ}}{s^2+\left(\frac{R}{L_q}+\frac{f_v}{J}\right)s+\frac{Rf_v}{L_qJ}+\vartheta_2 K_\Omega}U_{\Omega c}(s) .\frac{1}{s^2+\left(\frac{R}{L_q}+\frac{f_v}{J}\right)s+\frac{Rf_v}{L_qJ}+\vartheta_2 K_\Omega} \quad (26)$$

So that $e = 0 \Rightarrow \Omega = \Omega_m$ then $\frac{Rf_v}{L_qJ}+\vartheta_2 K_\Omega = a_{0\Omega m}$; $\frac{R}{L_q}+\frac{f_v}{J} = a_{1\Omega m}$ and $\vartheta_1\vartheta_2 K_\Omega = b_{\Omega m}$.

$$\frac{\partial e}{\partial \vartheta_1} = \frac{1}{\vartheta_1}.\frac{b_{\Omega m}}{s^2+a_{1\Omega m}s+a_{0\Omega m}}U_{\Omega c}(s) \quad (27)$$
$$= \frac{1}{\vartheta_1}.\Omega_m \quad (28)$$

$$\frac{\partial e}{\partial \vartheta_2} = \frac{s^2+a_{1\Omega m}s+a_{0\Omega}-\frac{b_{\Omega m}}{\vartheta_1}}{s^2+a_{1\Omega m}s+a_{0\Omega}}U_{\Omega c}(s) .\frac{1}{s^2+a_{1\Omega m}s+a_{0\Omega m}}$$
$$= \frac{1}{s^2+a_{1\Omega m}s+a_{0\Omega m}}.\left(U_{\Omega c}(s)-\frac{1}{\vartheta_1}\Omega_m(s)\right) \quad (29)$$

Taking into account (8), (28) and (29), one can write the equation of gradient ϑ_1 and ϑ_2:

$$\mathcal{L}\left\{\frac{d\vartheta_1}{dt}\right\} = -\gamma_{\Omega 1}\frac{\partial J(e)}{\partial e}\frac{\partial e}{\partial \vartheta_1} \quad (30)$$

$$\vartheta_1 = -\frac{\gamma_{\Omega 1}}{s}.sign(e).\frac{1}{\vartheta_1}.\Omega_m \quad (31)$$

And

$$\mathcal{L}\left\{\frac{d\vartheta_2}{dt}\right\} = -\gamma_{\Omega 2}\frac{\partial J(e)}{\partial e}\frac{\partial e}{\partial \vartheta_2} \quad (32)$$

$$\vartheta_2 = -\frac{\gamma_{\Omega 2}}{s}sign(e).\left(U_{\Omega c}(s)-\frac{1}{\vartheta_1}\Omega_m(s)\right) .\frac{1}{s^2+a_{1\Omega m}s+a_{0\Omega m}} \quad (33)$$

4 Results and Simulations

To examine practical usefulness, the proposed regulator has been simulated for a PMSM (see [10]), whose parameters are depicted in Table 1.

Parameters	Notation	Value	Unit
p	Pairs of poles	4	
L_d	d-inductance	$9.\ 10^{-4}$	$Kg.m^2$
L_q	q-Inductance	0.02682	H
Ψ_r	Flux linkage	0.1750	Wb
R	Stator resistance	2.875	Ω
f_v	Friction factor	$2.\ 10^{-2}$	$N.m.s$
J	Inertia	$8.\ 10^{-4}$	$Kg.m^2$

Table 1: PMSM parameters used in simulations.

The vector of motor state is initialized whit $\begin{bmatrix} i_{sd} & i_{sq} & \psi_{rd} & \Omega \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0.2 & 0 \end{bmatrix}^T$, and the results are given for the motor of which a direct starting, i.e. a resistive torque null ($T_l = 0$). We conceived simulation by carrying out the diagram general in blocks as the figure shows it (FIG.3). We show a detailed scheme SIMULINK of the control with MRAC regulator in Fig.4.

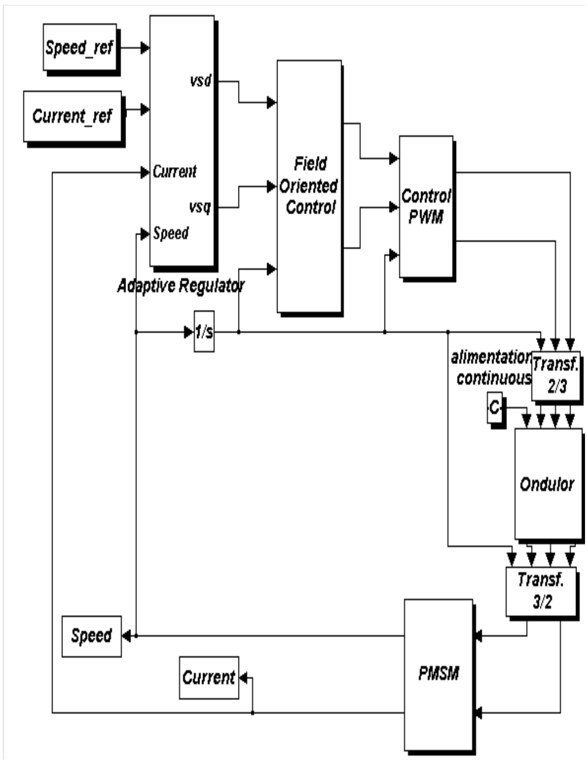

Figure 3: Diagram general of FOC with reference model.

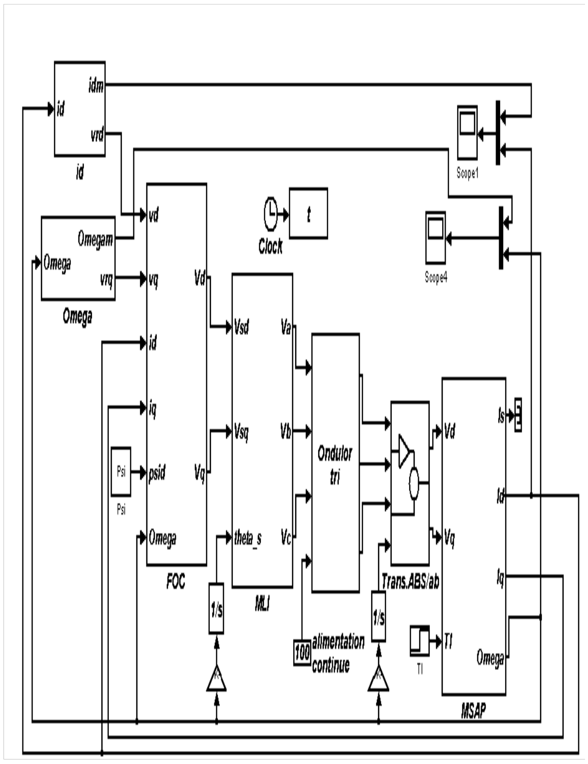

Figure 4: SIMULINK of FOC with reference model.

The current curve is represented by the figure (FIG.5) where it follows the desired trajectory exactly; except at moments $t_1 = 1.2s$, $t_2 = 3s$ and $t_3 = 4.5s$. When there is a change of direction the current or the rotating speed. On will have disturbances with the curve. The figure (FIG.6) gives the error current which equalizes à $-3.2mA$ with a variance of 8.7×10^{-3}.

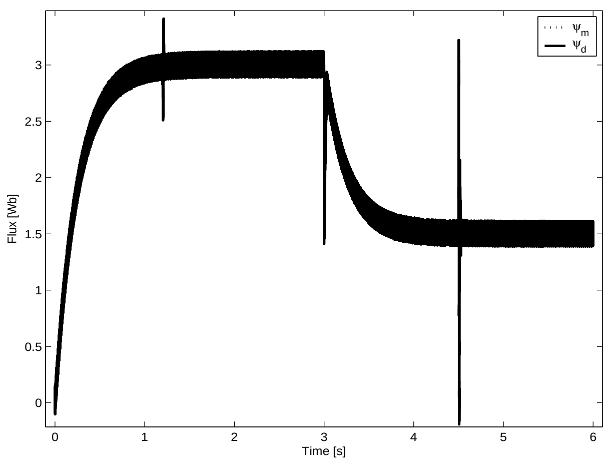

Figure 5: Direct current performance.

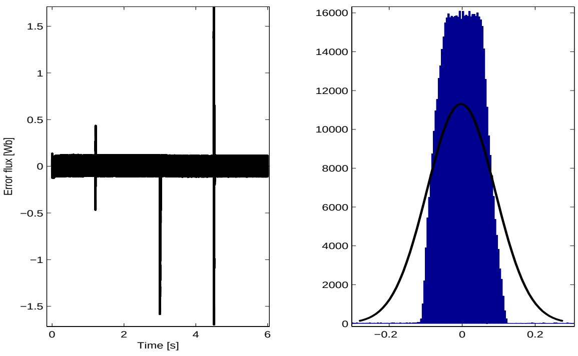

Figure 6: Error direct current performance.

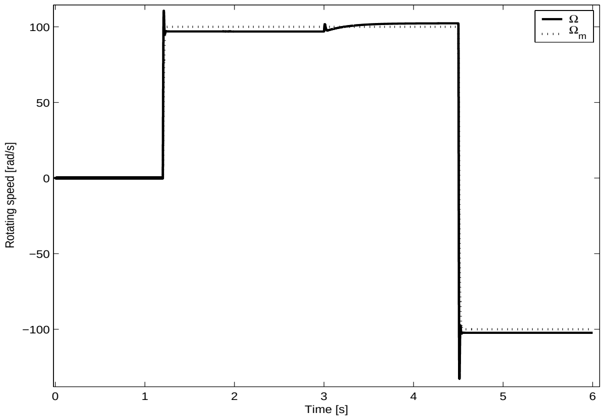

Figure 7: Rotating speed performance.

The rotating speed is shown in the figure (FIG.7) where it to follow perfectly the pattern desired with an error means of $-1.2636 rad/s$ and one variance 26.675 to see the figure (FIG.8).

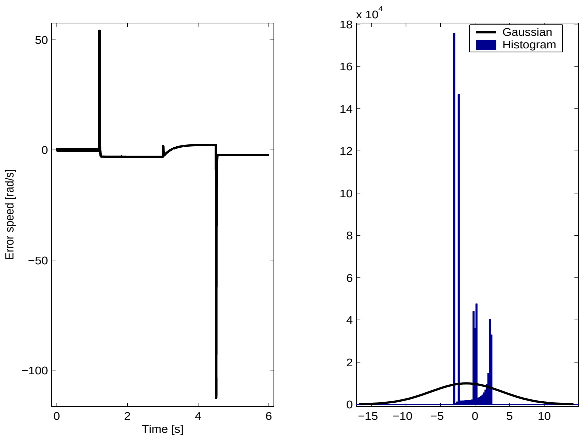

Figure 8: Error rotating speed performance.

The controller parameters evolution θ_1 and θ_2 are given by the figure (FIG.9) the same ones for the ϑ_1 and ϑ_2 are assembled by the figure (FIG.10).

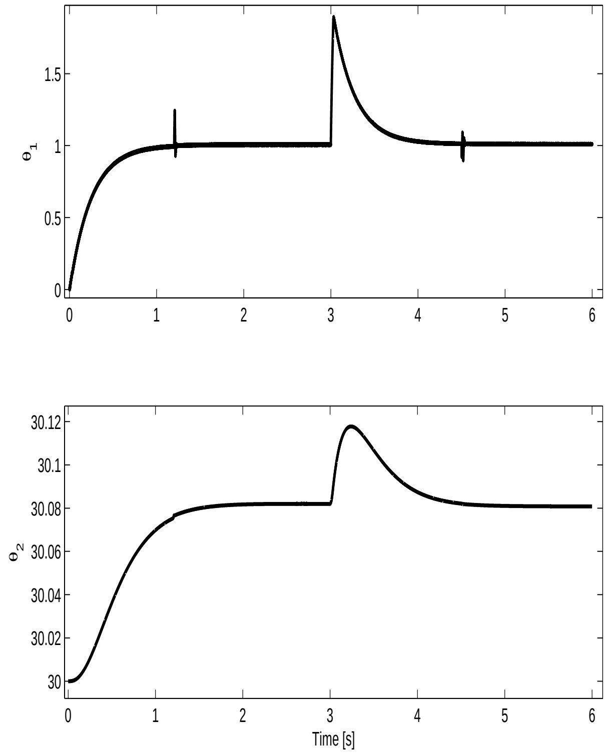

Figure 9: Parameters θ_1 and θ_2.

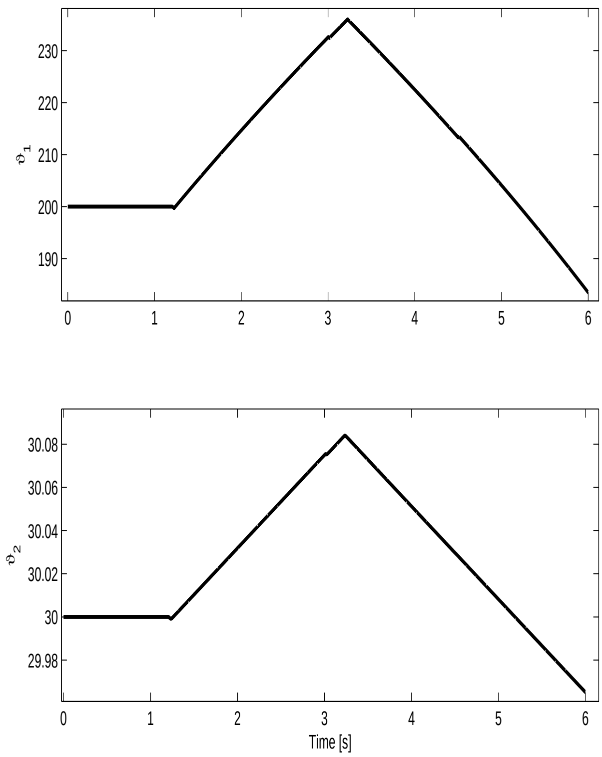

Figure 10: Parameters ϑ_1 and ϑ_2.

5 Conclusion

In this study; one used a regulator in the feedforward and the other in the direct chain. The FOC of PMSM with MRAC is proven analytically and solved by simulation by SIMULINK/MATLAB. The simulation results are perfect.

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