Robust Trajectory Tracking Control of Omni-Wheeled Mobile Robots

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Abstract: - In this paper, the robust trajectory tracking control problem for a dynamical model of a mobile robot with three omnidirectional wheels is considered. The motion of the robot is carried out by means of three independent electric motors. Based on the Lyapunov vector function method, a new discrete controller is presented which takes into account the presence of sliding friction forces and parametric variations. The estimations of the region of initial deviations and the unknown part of the inertia matrix are obtained.

Key-Words: - wheeled mobile robot, omni-wheel, slip, trajectory tracking control, Lyapunov vector function, dynamical model, comparison system

1 Introduction

Mobile robots with omnidirectional wheels have an advantage over the mobile robots with differential drive wheels. This is a high maneuverability of a mobile robot which is ensured by the design features of the wheels, on which the rollers are fixed with the axes of rotation lying in the plane of the wheels. Due to the presence of omni-wheels, it is possible to move the robot in any direction without a prior turn. In [1–4] the kinematic and dynamic models describing the motions of a robot with omni-wheels were constructed as well as the stabilization problem of stationary motions was considered. In [5, 6] the control problems for kinematic models of robots were considered. The robust controller applied to dynamic models should ensure that the robot follows a given trajectory and the closed-loop system of the error dynamics is asymptotically stable in the context of incomplete information about the mass-inertial parameters of the system and the action of sliding friction forces along the surface of the wheels. In [7, 8] a nonlinear model of a four-wheeled mobile robot was constructed taking into account the action of dry and viscous friction forces, and a predictive model controller was proposed. In [9] in order to solve the trajectory tracking control problem of a four-wheeled robot, a controller was obtained for a dynamic model taking into account a sliding friction based on the method of calculated torque also called the feedback linearization method. Such a controller contains the torques of all forces acting on the system, the term which is the product of the system's inertia matrix and program acceleration, and the PD controller, which makes it possible to obtain a linear asymptotically stable stationary error dynamic system. Note that the use of this method also requires the complete information on the parameters of the dynamic model of the system.

The topical issues are the development of methods and algorithms for controlling mobile robots with omni-wheels based on dynamic models with unknown mass-inertial characteristics, taking into account the existing sliding friction forces. These methods and algorithms should be robust, i.e. they have to provide a solution to the control problem for a whole class of dynamic robot models whose parameters satisfy the specified conditions. In [10] an adaptive control law was constructed based on the backstepping design procedure using the Lyapunov scalar function which solves the trajectory tracking control problem of the three-wheeled mobile robot with unknown mass-inertial parameters under the acting of sliding friction forces. Note that the relay controller obtained in [10] has a complicated structure and requires the on-line calculations of the estimates of unknown parameters.

In this paper, a discrete controller is constructed that ensures the stabilization of the non-stationary program motion of a mobile robot with three omni-wheels under the condition that the sliding friction forces act and the platform mass is inaccurately known. Based on the comparison method with the Lyapunov vector function, constraints on the system parameters and the estimation of the region of initial deviations are obtained. The results of numerical simulation confirming the theoretical calculations are presented.

2 Problem Formulation

Consider a model of a robot with three omni-wheels, moving on a horizontal surface under the action of the moments developed by three DC motors installed in the axles of the wheels (Fig. 1). Independent control of the rotation of each of the three wheels leads to the fact that inevitably there is a sliding friction along the surface of the movement of the wheels, which must be taken into account when building a dynamic model of the robot. When the omni-wheel moves the sliding friction arises both in the direction along the wheel surface and perpendicular to it due to the slipping of the rollers.

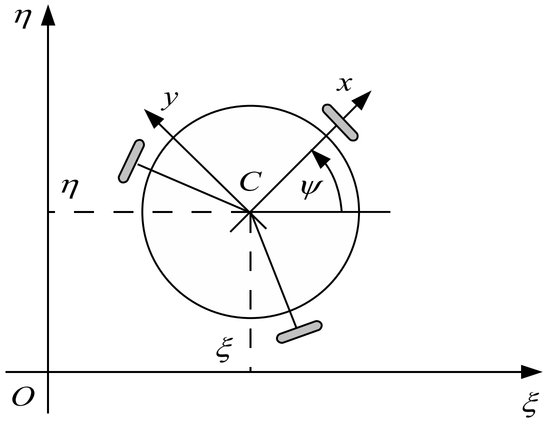

Fig. 1: The model of an omni-wheeled mobile robot

Equations (1) with inaccurately known mass-inertial parameters can be written in the following form:

$$\begin{cases} (m+\Delta m)\ddot{\xi} + h\dot{\xi} + \left(m_d\dot{\psi} - \dfrac{2n}{R}\mu_F\right)\dot{\eta} = \cos\psi u_1 - \sin\psi u_2 \\ (m+\Delta m)\ddot{\eta} + h\dot{\eta} - \left(m_d\dot{\psi} - \dfrac{2n}{R}\mu_F\right)\dot{\xi} = \sin\psi u_1 + \cos\psi u_2 \\ (I+\Delta I)\ddot{\psi} + 2L^2 h\dot{\psi} = u_3 \end{cases} \tag{2}$$

where $u = (u_1, u_2, u_3)^{\mathrm{T}}$ is the vector of control torques; m, I, and m_d are the known components of the mass-inertial parameters of the system,

$$m = \frac{2n^2 J_0}{R^2} + m_0,\ h = \frac{2n}{R}\mu_T + \frac{2n^2 k_2 k_3}{R^2 R_a}$$

$$m_d = \frac{2n^2 J_0}{R^2},\ I = \frac{4n^2 J_0}{R^2} L^2 + I_z$$

m_0 is the mass of the robot; I_z is the momentum of inertia of the robot relative to the vertical axis passing through the center of mass; R is the wheel radius; L is the distance from the center of mass of the robot to the center of the wheel; n is the gear ratio; J_0 is the total momentum of inertia of the actuator, gearbox, and wheel relative to the rotor axis of the electric motor; R_a is the resistance in the motor circuit; k_2 is the motor torque constant; k_3 is the constant of electromotive force; μ_T is the coefficient of sliding friction arising in the direction of the plane of the wheel; μ_F is the coefficient of sliding friction arising in the direction perpendicular to the wheel plane due to the rotation of the rollers; Δm and ΔI are the unknown components of the mass of the platform and its momentum of inertia, respectively satisfying the inequalities: $|\Delta m| < \Delta m_0$, $|\Delta I| < \Delta I_0$, where $\Delta m_0 < m$, $\Delta I_0 < I$.

Let there exist three functions $\xi_0(t),\ \eta_0(t),\ \psi_0(t)$ bounded and twice continuously differentiable for all $t \geq 0$, and let the positive constants $\xi^1_{\max}$, $\eta^1_{\max}$, $\psi^1_{\max}$, $\xi^2_{\max}$, $\eta^2_{\max}$, and $\psi^2_{\max}$ exist such that for all $t \geq 0$ the following inequalities hold:

$$|\dot{\xi}_0(t)| \leq \xi^1_{\max},\ |\dot{\eta}_0(t)| \leq \eta^1_{\max},\ |\dot{\psi}_0(t)| \leq \psi^1_{\max}$$
$$|\ddot{\xi}_0(t)| \leq \xi^2_{\max},\ |\ddot{\eta}_0(t)| \leq \eta^2_{\max},\ |\ddot{\psi}_0(t)| \leq \psi^2_{\max}$$

Consider the formulation of the trajectory tracking control problem. Let the reference trajectory be given by

$$\xi = \xi_0(t),\ \eta = \eta_0(t),\ \psi = \psi_0(t) \tag{3}$$

We introduce the errors:

$$x_1 = \xi - \xi_0(t),\quad x_2 = \eta - \eta_0(t),\quad x_3 = \psi - \psi_0(t) \tag{4}$$

The problem consists in both the finding the feedback controller

$$\begin{aligned} u_1 &= u_1(t, x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3) \\ u_2 &= u_2(t, x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3) \\ u_3 &= u_3(t, x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3) \end{aligned} \tag{5}$$

and the indicating the constraints on the system parameters and trajectories under which the relations are satisfied

$$\lim_{t\to+\infty} x_i(t) = 0,\quad i = 1,2,3,$$

if the initial values $x_1(t_0), x_2(t_0), x_3(t_0)$ and $\dot{x}_1(t_0), \dot{x}_2(t_0), \dot{x}_3(t_0)$ belong to some given neighborhood of the zero point $x_1 = x_2 = x_3 = \dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$.

3 Problem Solution

To solve the trajectory tracking control problem, we consider the following controller with a relay function:

$$
\begin{aligned}
& u_1(t,x_1,x_2,x_3,\dot{x}_1,\dot{x}_2) = \\
& = -\sigma_1 \operatorname{sign}(\nu x_1 + \mu_1 \dot{x}_1)\cos(\psi_0(t) + x_3) - \\
& -\sigma_2 \operatorname{sign}(\nu x_2 + \mu_1 \dot{x}_2)\sin(\psi_0(t) + x_3) \\
& u_2(t,x_1,x_2,x_3,\dot{x}_1,\dot{x}_2) = \\
& = \sigma_1 \operatorname{sign}(\nu x_1 + \mu_1 \dot{x}_1)\sin\left(\psi_0(t) + x_3\right) - \\
& -\sigma_2 \operatorname{sign}(\nu x_2 + \mu_1 \dot{x}_2)\cos\left(\psi_0(t) + x_3\right) \\
& u_3(t,x_3,\dot{x}_3) = -\sigma_3 \operatorname{sign}(\nu x_3 + \mu_2 \dot{x}_3)
\end{aligned}
\tag{7}
$$

where $\nu > 0,\ \mu_i > 0\ (i = 1,2),\ \sigma_j > 0\ (j = 1,2,3)$ are some positive reals.

Theorem 1. Let some positive reals $\delta_1 > 0$, $\delta_2 > 0$, $\nu > 1,\ \mu_i > 0\ (i = 1,2)$, $\sigma_j > 0\ (j = 1,2,3)$ exist such that the following inequalities hold:

$$
\begin{aligned}
& (\nu\mu_2(m + \Delta m_0) - h\mu_1\mu_2 + \nu\,|\,\nu\mu_2(m \pm \Delta m_0) - h\mu_1\mu_2\,| + \\
& + m_d\mu_1[(1+\nu)^2\delta_2 + \mu_2(1+\nu)\psi^1_{\max}] + \\
& + 2n\mu_1\mu_2(1+\nu)\mu_F / R)\delta_1 + \mu_1^2 m_d(1+\nu)\eta^1_{\max}\delta_2 + \\
& + \mu_1^2\mu_2((m + \Delta m_0)\xi^2_{\max} + h\xi^1_{\max} + \\
& + (m_d\psi^1_{\max} + 2n\mu_F / R)\eta^1_{\max} - \sigma_1) \le -\varepsilon_1 < 0
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
& (\nu\mu_2(m + \Delta m_0) - h\mu_1\mu_2 + \nu\,|\,\nu\mu_2(m \pm \Delta m_0) - h\mu_1\mu_2\,| + \\
& + m_d\mu_1[(1+\nu)^2\delta_2 + \mu_2(1+\nu)\psi^1_{\max}] + \\
& + 2n\mu_1\mu_2(1+\nu)\mu_F / R)\delta_1 + \mu_1^2 m_d(1+\nu)\xi^1_{\max}\delta_2 + \\
& + \mu_1^2\mu_2((m + \Delta m_0)\eta^2_{\max} + h\eta^1_{\max} + \\
& + (m_d\psi^1_{\max} + 2n\mu_F / R)\xi^1_{\max} - \sigma_2) \le -\varepsilon_2 < 0
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
& (\nu(I + \Delta I_0) - 2a^2h\mu_2 + \nu\,|\,\nu(I \pm \Delta I_0) - 2a^2h\mu_2\,|)\delta_2 + \\
& + \mu_2^2((I + \Delta I_0)\psi^2_{\max} + 2a^2h\psi^1_{\max} - \sigma_3) \le -\varepsilon_3 < 0
\end{aligned}
\tag{10}
$$

Then, the controller (7) solves the trajectory tracking control problem of the system (2), i.e. the closed-loop system is asymptotically stable, if the set of the initial errors

$$
x_1(0), x_2(0), x_3(0), \dot{x}_1(0), \dot{x}_2(0), \dot{x}_3(0) \tag{11}
$$

satisfies the following inequalities:

$$
\begin{aligned}
& \max\{|\,\nu x_1(0) + \mu_1\dot{x}_1(0)\,|, |\,\nu x_2(0) + \mu_1\dot{x}_2(0)\,|, \\
& |\,x_1(0)\,|, |\,x_2(0)\,|\} < \delta_1 \\
& \max\{|\,\nu x_3(0) + \mu_2\dot{x}_3(0)\,|, |\,x_3(0)\,|\} < \delta_2
\end{aligned}
\tag{12}
$$

Proof. Let introduce the variables:

$$
z_1 = \nu x_1 + \mu_1\dot{x}_1,\ z_2 = \nu x_2 + \mu_1\dot{x}_2,\ z_3 = \nu x_3 + \mu_2\dot{x}_3 \tag{13}
$$

Then, the system (2) can be written as follows:

$$
\begin{aligned}
& \dot{x}_1 = -\frac{\nu}{\mu_1}x_1 + \frac{1}{\mu_1}z_1,\ \dot{x}_2 = -\frac{\nu}{\mu_1}x_2 + \frac{1}{\mu_1}z_2 \\
& \dot{x}_3 = -\frac{\nu}{\mu_2}x_3 + \frac{1}{\mu_2}z_3 \\
& \dot{z}_1 = \left(-\frac{\nu^2}{\mu_1} + \frac{h\nu}{m + \Delta m}\right)x_1 + \frac{m_d\nu}{m + \Delta m} \times \\
& \times\left(-\frac{\nu}{\mu_2}x_3 + \frac{1}{\mu_2}z_3 + \dot{\psi}_0(t)\right)x_2 - \\
& - \frac{2n\nu\mu_F}{(m + \Delta m)R}x_2 + \\
& + \frac{\mu_1 m_d \nu\dot{\eta}_0(t)}{\mu_2(m + \Delta m)}x_3 + \left(\frac{\nu}{\mu_1} - \frac{h}{m + \Delta m}\right)z_1 - \\
& - \frac{m_d}{m + \Delta m}\left(-\frac{\nu}{\mu_2}x_3 + \frac{1}{\mu_2}z_3 + \dot{\psi}_0(t)\right)z_2 + \\
& + \frac{2n\mu_F}{(m + \Delta m)R}z_2 - \\
& - \frac{\mu_1 m_d \dot{\eta}_0(t)}{\mu_2(m + \Delta m)}z_3 - \frac{\mu_1\sigma_1}{m + \Delta m}\operatorname{sign} z_1 - \mu_1\ddot{\xi}_0(t) - \\
& - \frac{\mu_1 h}{m + \Delta m}\dot{\xi}_0(t) - \\
& - \frac{\mu_1(m_d\dot{\psi}_0(t) - 2n\mu_F / R)}{m + \Delta m}\dot{\eta}_0(t)
\end{aligned}
$$

$$
\begin{aligned}
\dot{z}_2 = & -\frac{m_d \nu}{m+\Delta m}\left(-\frac{\nu}{\mu_2}x_3+\frac{1}{\mu_2}z_3+\dot{\psi}_0(t)\right)x_1+ \\
& +\frac{2n\nu\mu_F}{(m+\Delta m)R}x_1+\left(-\frac{\nu^2}{\mu_1}+\frac{h\nu}{m+\Delta m}\right)x_2- \\
& -\frac{\mu_1 m_d \nu\dot{\xi}_0(t)}{\mu_2(m+\Delta m)}x_3+ \\
& +\frac{m_d}{m+\Delta m}\left(-\frac{\nu}{\mu_2}x_3+\frac{1}{\mu_2}z_3+\dot{\psi}_0(t)\right)z_1- \\
& -\frac{2n\mu_F}{(m+\Delta m)R}z_1+ \\
& +\left(\frac{\nu}{\mu_1}-\frac{h}{m+\Delta m}\right)z_2+\frac{\mu_1 m_d\dot{\xi}_0(t)}{\mu_2(m+\Delta m)}z_3- \\
& -\frac{\mu_1\sigma_1}{m+\Delta m}\operatorname{sign} z_2-\mu_1\ddot{\eta}_0(t)-\frac{\mu_1 h}{m+\Delta m}\dot{\eta}_0(t)+ \\
& +\frac{\mu_1(m_d\dot{\psi}_0(t)-2n\mu_F/R)}{m+\Delta m}\dot{\xi}_0(t) \\
\dot{z}_3 = & \left(-\frac{\nu^2}{\mu_2}+\frac{2a^2h\nu}{I+\Delta I}\right)x_3+\left(\frac{\nu}{\mu_2}-\frac{2a^2h}{I+\Delta I}\right)z_3- \\
& -\frac{\mu_2\sigma_3}{I+\Delta I}\operatorname{sign} z_3-\mu_2\ddot{\psi}_0(t)-\frac{2a^2h\mu_2}{I+\Delta I}\dot{\psi}_0(t)
\end{aligned}
\tag{14}
$$

Choose the Lyapunov vector function candidate $\mathrm{V}=\mathrm{V}(x_1,x_2,x_3,z_1,z_2,z_3)$, $\mathrm{V}=(V_1,V_2)^T$ such as follows

$$
\begin{aligned}
\mathrm{V}_1 &= \max\{|x_1|,|x_2|,|z_1|,|z_2|\} \\
\mathrm{V}_2 &= \max\{|x_3|,|z_3|\}
\end{aligned}
\tag{15}
$$

The right-hand time derivative of the Lyapunov vector function candidate (15) satisfies the following inequalities:

$$
\begin{aligned}
\dot{V}_1 \le \max\Big\{ & \frac{1-\nu}{\mu_1}V_1, \frac{1}{\mu_1\mu_2(m+\Delta m)}((\nu\mu_2(m+\Delta m)- \\
& -h\mu_1\mu_2+\nu|\nu\mu_2(m+\Delta m)-h\mu_1\mu_2|+ \\
& +m_d\mu_1[(1+\nu)^2V_2+\mu_2(1+\nu)\psi^1_{\max}]+ \\
& +2(1+\nu)n\mu_1\mu_2\mu_F/R)V_1+\mu_1^2 m_d(1+\nu)\eta^1_{\max}V_2)+ \\
& +\frac{\mu_1}{m+\Delta m}((m+\Delta m)\xi^2_{\max}+h\xi^1_{\max}+ \\
& +(m_d\psi^1_{\max}+2n\mu_F/R)\eta^1_{\max}-\sigma_1), \\
& \frac{1}{\mu_1\mu_2(m+\Delta m)}((\nu\mu_2(m+\Delta m)-h\mu_1\mu_2+ \\
& +\nu|\nu\mu_2(m+\Delta m)-h\mu_1\mu_2|+ \\
& +m_d\mu_1[(1+\nu)^2V_2+\mu_2(1+\nu)\psi^1_{\max}]+ \\
& +2(1+\nu)n\mu_1\mu_2\mu_F/R)V_1+\mu_1^2 m_d(1+\nu)\xi^1_{\max}V_2)+ \\
& +\frac{\mu_1}{m+\Delta m}((m+\Delta m)\eta^2_{\max}+ \\
& +h\eta^1_{\max}+(m_d\psi^1_{\max}+2n\mu_F/R)\xi^1_{\max}-\sigma_2)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_2 \le \max\Big\{ & \frac{1-\nu}{\mu_2}V_2, \frac{1}{\mu_2(I+\Delta I)}(\nu(I+\Delta I)- \\
& -2L^2h\mu_2+\nu|\nu(I+\Delta I)-2L^2h\mu_2|)V_2+ \\
& +\frac{\mu_2}{I+\Delta I}((I+\Delta I)\psi^2_{\max}+2L^2h\psi^1_{\max}-\sigma_3)\Big\}
\end{aligned}
$$

Consider the behavior of the Lyapunov vector function along the solution of system (14), satisfying the initial condition (12), which, given the notation (13), has the form:

$$
\begin{aligned}
\max\{|z_1(0)|,|z_2(0)|,|x_1(0)|,|x_2(0)|\} &< \delta_1 \\
\max\{|z_3(0)|,|x_3(0)|\} &< \delta_2
\end{aligned}
\tag{16}
$$

If inequalities (8) - (10) are fulfilled, then we obtain the estimate:

$$
\begin{aligned}
\dot{V}_1 &\le \max\left\{\frac{1-\nu}{\mu_1}V_1, \frac{-\varepsilon_1}{\mu_1\mu_2(m+\Delta m_0)}, \frac{-\varepsilon_2}{\mu_1\mu_2(m+\Delta m_0)}\right\} \\
\dot{V}_2 &\le \max\left\{\frac{1-\nu}{\mu_2}V_2, \frac{-\varepsilon_3}{\mu_2(I+\Delta I_0)}\right\}
\end{aligned}
\tag{17}
$$

From the inequalities (17) it follows that there is a time moment $t^*>0$ such that for all $t\ge t^*$ the following inequalities hold

$$
\dot{V}_1 \le \frac{1-\nu}{\mu_1}V_1,\ \dot{V}_2 \le \frac{1-\nu}{\mu_2}V_2 \tag{18}
$$

From inequalities (18), we obtain an exponentially stable comparison system

$$\dot{u}_1 = \frac{1-\nu}{\mu_1} u_1, \ \dot{u}_2 = \frac{1-\nu}{\mu_2} u_2$$

Then, using the comparison theorem on asymptotic stability [11, 12], we obtain that $V_1 \to 0, \ V_2 \to 0$ as $t \to +\infty$. Using (15), we obtain that the errors (4) asymptotically tend to zero, i.e. $x_1 \to 0, \ x_2 \to 0, \ x_3 \to 0$ as $t \to +\infty$. In other words, the controller (7) solves the trajectory tracking control problem of the system (2) if the set of initial deviations (11) satisfies the condition (12). This completes the proof.

Remark 1. In order to apply Theorem 1 to solve the trajectory tracking control problem of the mobile robot one has to verify the algebraic conditions (8) - (10) which do not require the calculations of the eigenvalues of the matrices that sufficiently reduces the computation time. In addition, the conditions (8) - (10) have a robust nature, since they allow the trajectory tracking for a large class of mobile robots whose parameters satisfy the given conditions.

4 Numerical Simulation

The controller (7) was applied in the numerical simulation of the motion of the robot with the parameters [9]:

$$m = 20kg, \ I_z = 0,301 kg \times m^2, \ R = 0,08m$$
$$L = 0,2125m, \ n = 14, \ \mu_T = 0,38, \ \mu_F = 0,21$$
$$R_a = 1,11\,\text{Ом}, \ k_2 = 36,4\,\text{mN} \times \text{m/A}, \ k_3 = 0,0363\,\text{B} \times \text{s}$$
$$J_0 = 70,7 \times 10^{-7} kg \times m^2$$

The unknown parts of the mass-inertia parameters are bounded by the following values:

$$\Delta m = 5\,kg, \ \Delta I = 0,1\,kg \times m^2$$

The reference trajectory is chosen as

$$\xi_0(t) = r\sin(\omega t)\,m, \ \eta_0(t) = -r\cos(\omega t)\,m$$
$$\psi_0(t) = \varpi t + \psi_0 \ rad \qquad (26)$$

where

$$r = 0,5 \ \text{m}, \ \omega = 0,8 \ \text{rad/s}$$
$$\varpi = \pi / 6 \ \text{rad/s}, \ \psi_0 = 0 \ \text{rad}$$

Note that equations (26) correspond to the uniform motion of the center of mass of the robot along a given circle with a uniform rotation of the platform around this center.

Using the conditions of Theorem 1, the following control parameters are chosen

$$\mu_1 = 3,5\,\text{s}, \mu_2 = 2\,\text{s}, \nu = 1$$
$$\sigma_1 = 30\,\text{N}, \sigma_2 = 30\,\text{N} \times \text{m}, \sigma_3 = 5\,\text{N}$$

The initial errors are chosen as follows $\delta_1 = 1\,\text{m}, \ \delta_2 = 1\,\text{rad}$.

Figure 2 shows the graphs of the reference trajectory of the center of mass of the robot as well as the actual one.

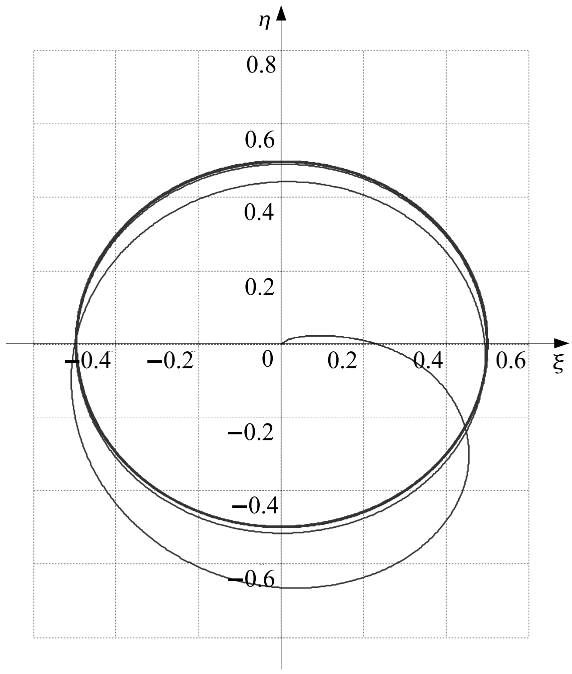

Fig. 2: The model of an omni-wheeled mobile robot

From Fig. 2 one can see that the center of mass of the robot asymptotically approaches to the reference trajectory (26).

5 Conclusion

The paper addresses the trajectory tracking control problem of a dynamic model of mobile robot with three omni-wheels controlled by three independent electric motors. Using the Lyapunov vector function method we have obtained the relay robust feedback tracking controller that provides a solution to the problem under the action of sliding friction forces and with incompletely known mass-inertial parameters of the robot. The advantage of the obtained controller is the simplicity of its practical implementation in comparison with adaptive control laws.

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