Design of an Augmented Automatic Choosing Control by Weighted Gradient Optimization Automatic Choosing Functions for Nonlinear Systems

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Abstract: In this paper, we present a novel approach of a nonlinear feedback control called augmented automatic choosing control (AACC) using sigmoid type weighted gradient optimization automatic choosing functions for a class of nonlinear systems. When the control is designed, a constant term which arises from linearization of a given nonlinear system is treated as a coefficient of a stable zero dynamics. The controller is a structure-specified type which has some parameters. Parameters of the control are suboptimally selected by extremizing a combination of the Hamiltonian and Lyapunov functions with the aid of the genetic algorithm. This approach is applied to a field excitation control problem of power system, which is Ozeki-Power-Plant of Kyushu Electric Power Company in Japan, to demonstrate the usefulness of the AACC. Simulation results show that the new controller can improve the performance remarkably.

Key–Words: augmented automatic choosing control, nonlinear control, genetic algorithm, weighted gradient optimization automatic choosing function

1 Introduction

A genetic algorithm (GA) is one of evolutionary computing algorithms in engineering sciences[1]. The GA has been used to solve such complicated tasks as nonlinear global optimization problems. The purpose of this paper is to present a nonlinear feedback control called Augmented automatic choosing control (AACC), which is designed by making good use of the GA.

Generally, it is easy to design the optimal control laws for linear systems, but that is not the case for nonlinear systems, though they have been studied for many years[2]$\sim$[7].One of the most popular and practical nonlinear control laws is synthesized by applying a linearization method by Taylor expansion truncated at the first order and the linear optimal control method to a given nonlinear system. This is only effective in a small region around the steady state point or in almost linear systems[2]$\sim$[5].

As one of approaches to overcome these drawbacks, AACC is proposed for nonlinear systems[7]. Its design procedure is as follows.

Assume that a system is given by a nonlinear differential equation. Choose a separative variable, which makes up nonlinearity of the given system. The domain of the variable is divided into some subdomains. On each subdomain, the system equation is linearized by Taylor expansion around a suitable point so that a constant term is included in it. This constant term is treated as a coefficient of a stable zero dynamics. The given nonlinear system approximately makes up a set of augmented linear systems, to which the optimal linear control theory is applied in order to get the linear quadratic (LQ) controls[3]. These LQ controls are smoothly united by sigmoid type weighted gradient optimization automatic choosing functions to synthesize a single nonlinear feedback controller.

This controller is a structure-specified type which has some parameters, such as the number of divisions of the domain, regions of the subdomains, points of the Taylor expansion, gradients of the automatic choosing functions, and so on. These parameters must be selected optimally to be just the controller's fit. Since they lead to a nonlinear optimization problem, we are able to solve it suboptimally and successfully by using the GA, which is one of evolutionary computing algorithms in engineering sciences. In this paper the suboptimal values of these parameters are obtained by acquiring both minimization of the Hamiltonian and maximization of a stable region in the sense of Lyapunov.

This approach is applied to a field excitation control problem of power system, which is Ozeki-Power-Plant of Kyushu Electric Power Company in Japan, to demonstrate the usefulness of the AACC. Computer simulation results show that the new controller using the GA is able to improve the performance remarkably.

2 Augmented Automatic Choosing Control Using Zero Dynamics

Assume that a nonlinear system is given by

$$\dot{x} = f(x) + g(x)u, \ \ x \in \mathbf{D} \tag{1}$$

where $\cdot = d/dt, \quad x = [x[1], \cdots, x[n]]^T$ is an n-dimensional state vector, $u = [u[1], \cdots, u[r]]^T$ is an r-dimensional control vector, $f : \mathbf{D} \to R^n$ is a nonlinear vector-valued function with $f(0) = 0$ and is continuously differentiable, $g(x)$ is an $n \times r$ driving matrix with $g(0) \neq 0$, $\mathbf{D} \subset R^n$ is a domain , and T denotes transpose.

Considering the nonlinearity of f, introduce a vector-valued function $C : \mathbf{D} \to R^L$ which defines the separative variables $\{C_j(x)\}$, where $C = [C_1 \cdots C_j \cdots C_L]^T$ is continuously differentiable. Let D be a domain of C^{-1}. For example, if $x[2]$ is the element which has the higher nonlinearity in f, then

$$C(x) = x[2] \in D \subset R \ \ (L = 1).$$

The domain D is divided into some subdomains: $D = \cup_{i=0}^{M} D_i$, where $D_M = D - \cup_{i=0}^{M-1} D_i$ and $C^{-1}(D_0) \ni 0$. $D_i (0 \le i \le M)$ endowed with a lexicographic order is the Cartesian product $D_i = \Pi_{j=1}^{L} [a_{ij}, b_{ij}]$, where $a_{ij} < b_{ij}$.

Introduce a stable zero dynamics :

$$\dot{x}[n+1] = -\sigma_i x[n+1] \tag{2}$$

$$(x[n+1](0) \simeq 1, \quad 0 < \sigma_i < 1).$$

Eq.(1) combines with (2) to form an augmented system

$$\dot{\mathbf{X}} = \bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u \tag{3}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ x[n+1] \end{bmatrix} \ \ \in \mathbf{D} \times R$$

$$\bar{f}(\mathbf{X}) = \begin{bmatrix} f(x) \\ -\sigma_i x[n+1] \end{bmatrix}, \bar{g}(\mathbf{X}) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix}.$$

We assume a cost function being

$$J = \frac{1}{2} \int_0^\infty \left(\mathbf{X}^T \mathbf{Q} \mathbf{X} + u^T R u \right) dt \tag{4}$$

where $\mathbf{Q} = \mathbf{Q}^T > 0, \ R = R^T > 0$, and the values of these matrices are properly determined based on engineering experience.

On each D_i, the nonlinear system is linearized by the Taylor expansion truncated at the first order about a point $\hat{X}_i \in C^{-1}(D_i)$ and $\hat{X}_0 = 0$ (see Fig. 1):

$$f(x) + g(x)u \simeq A_i x + w_i + B_i u \quad \text{on} \ \ C^{-1}(D_i) \tag{5}$$

where

$$\begin{aligned} A_i &= \partial f(x)/\partial x^T|_{x=\hat{X}_i}, \ w_i = f(\hat{X}_i) - A_i \hat{X}_i, \\ B_i &= g(\hat{X}_i). \end{aligned}$$

Make an approximation of (3) by

$$\dot{\mathbf{X}} = \bar{A}_i \mathbf{X} + \bar{B}_i u \quad \text{on} \ \ C^{-1}(D_i) \times R \tag{6}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma_i \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}.$$

An application of the linear optimal control theory[3] to (4) and (6) yields

$$u_i(\mathbf{X}) = -R^{-1} \bar{B}_i^T \mathbf{P}_i \mathbf{X} \tag{7}$$

where the $(n+1) \times (n+1)$ matrix $\mathbf{P}_i$ satisfies the Riccati equation :

$$\mathbf{P}_i \bar{A}_i + \bar{A}_i^T \mathbf{P}_i + \mathbf{Q} - \mathbf{P}_i \bar{B}_i R^{-1} \bar{B}_i^T \mathbf{P}_i = 0. \tag{8}$$

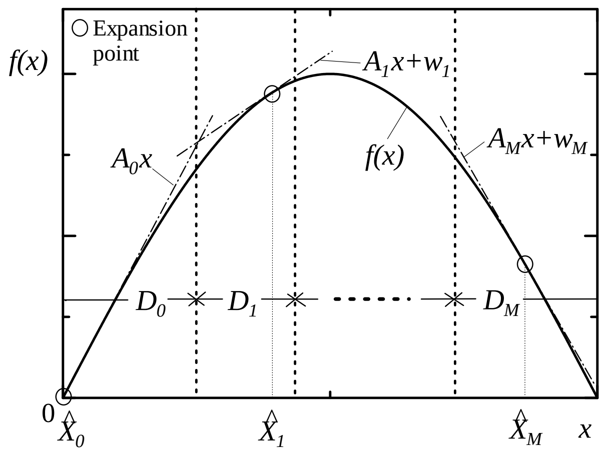

Fig. 1 Sectionwize linearization

Introduce a weighted gradient optimization automatic choosing function of sigmoid type :

$$I_i(x) = d_i \prod_{j=1}^{L} \Big\{ 1 - \frac{1}{1+\exp\left(2N_{1i}\left(C_j(x)-a_{ij}\right)\right)} - \frac{1}{1+\exp\left(-2N_{1i}\left(C_j(x)-b_{ij}\right)\right)} \Big\} \quad (9)$$

where N_{1i} and d_i are positive real values, $-\infty \leq a_{ij}\ ,\ b_{ij} \leq \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero when $d_i = 1$(see Fig. 2).

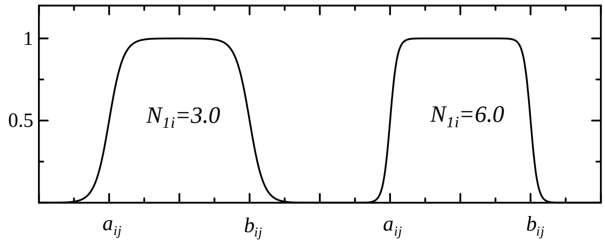

Fig. 2 Automatic Choosing Function(N_{1i}=$3.0, 6.0$)

Uniting $\{u_i(\mathbf{X})\}$ of (7) with $\{I_i(x)\}$ of (9), we have an augmented automatic choosing control

$$u(\mathbf{X}) = \sum_{i=0}^{M} u_i(\mathbf{X}) I_i(x). \quad (10)$$

3 Parameter Selection by GA

The Hamiltonian for Eqs.(3) and (4) is given by

$$H(\mathbf{X}, u, \lambda) = \frac{1}{2}\left(\mathbf{X}^T\mathbf{Q}\mathbf{X} + u^T R u\right) + \lambda^T\left(\bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u\right). \quad (11)$$

Assume that the adjoint vector $\lambda \in R^{n+1}$ is

$$\lambda = \sum_{i=0}^{M} \mathbf{P}_i \mathbf{X} I_i(x). \quad (12)$$

The necessary condition of the optimality is $\partial H/\partial u = 0$ or $u = -R^{-1}\bar{g}(\mathbf{X})^T\lambda$, which derives from Eq.(10) using Eq.(12) and

$$H(\mathbf{X}, u, \lambda) = \frac{1}{2}\mathbf{X}^T\mathbf{Q}\mathbf{X} - \frac{1}{2}u^T R u + \bar{f}^T(\mathbf{X})\lambda \quad (13)$$

using Eq.(11).
Next, introduce a Lyapunov function candidate:

$$V(\mathbf{X}) = \mathbf{X}^T \Pi(\mathbf{X}) \mathbf{X} \quad (14)$$

where

$$\Pi(\mathbf{X}) = \sum_{i=0}^{M} \mathbf{P}_i \Pi_i(x) \ ,$$

$$\Pi_i(x) = \eta_i \prod_{j=1}^{L} \Big\{ 1 - \frac{1}{1+\exp\left(2N_2\left(C_j(x)-a_{ij}\right)\right)} - \frac{1}{1+\exp\left(-2N_2\left(C_j(x)-b_{ij}\right)\right)} \Big\}, \quad (15)$$

N_2 and η_i are positive real values.

By the Lyapunov's direct method[4], the equilibrium point 0 is uniformly stable on a connected set:

$$\mathbf{D}_V = \left\{ x \in \mathbf{D} : V(\mathbf{X}) < \gamma, \dot{V}(\mathbf{X}) < 0 \right\}$$

where

$$\gamma = \inf\left\{ V(\mathbf{X}) : \mathbf{X} \neq 0, \dot{V}(\mathbf{X}) = 0 \right\}. \quad (16)$$

In order to design optimal control by the Hamiltonian and expand the stable region in the sense of Lyapunov as wide as possible, we define a performance

$$PI = \omega_1 \int_{\mathbf{D}} |H(\mathbf{X}, u, \lambda)| / \mathbf{X}^T\mathbf{X} d\mathbf{X} - \omega_2 \gamma \quad (17)$$

where $\omega_i(\omega_i \geq 0; i = 1, 2)$ is weight.

A set of parameters included in the control (10):

$$\bar{\Omega} = \left\{ M, N_{1i}, N_2, d_i, a_{ij}, b_{ij}, \hat{X}_i, \eta_i \right\}$$

is suboptimally selected by minimizing PI with the aid of GA[1] as follows.

<ALGORITHM>
step1:A-priori: Set values $\bar{\Omega}_{apriori}$ appropriately.
step2:Parameter: Choose a subset $\Omega \subset \bar{\Omega}$ to be improved and rewrite it by $\Omega = \{M, N_{1i}, \cdot\cdot\} = \{\alpha_k : k = 1, \cdot\cdot, K\}$.
step3:Coding: Represent each α_k with a binary bit string of $\widetilde{L}$ bits and then arrange them into one string of $\widetilde{L}K$ bits.
step4:Initialization: Randomly generate an initial population of $\widetilde{q}$ strings $\{\Omega_p : p = 1, \cdot\cdot, \widetilde{q}\}$.
step5:Decoding: Decode each element α_k of Ω_p by $\alpha_k = (\alpha_{k,max} - \alpha_{k,min}) A_k / \left(2^{\widetilde{L}} - 1\right) + \alpha_{k,min}$ where $\alpha_{k,max}$:maximum, $\alpha_{k,min}$:minimum, and A_k:decimal value of α_k.
step6:Control: Design $u = u(\mathbf{X})_p \quad (p = 1, \cdots, \widetilde{q})$ for Ω_p by using Eq.(10).
step7:Adjoint: Make $\lambda = \lambda(\mathbf{X})_p \quad (p = 1, \cdots, \widetilde{q})$ for Ω_p by using Eq.(12).
step8:Lyapunov function: Make $\gamma = \gamma_p$ $(p = 1, \cdots, \widetilde{q})$ for Ω_p by using Eq.(16).
step9:Fitness value calculation: Calculate

$$PI_p = \omega_1 \int_{\mathbf{D}} \Big| \frac{1}{2}\mathbf{X}^T\mathbf{Q}\mathbf{X} - \frac{1}{2}u(\mathbf{X})_p^T R u(\mathbf{X})_p + \bar{f}^T(\mathbf{X})\lambda(\mathbf{X})_p \Big| / \mathbf{X}^T\mathbf{X} d\mathbf{X} - \omega_2 \gamma_p \quad (18)$$

by Eqs.(13) and (17), or fitness $F_p = -PI_p$. Integration of (18) is approximated by a finite sum.

step10:Reproduction: Reproduce each of individual strings with the probability of $F_p / \sum_{j=1}^{\widetilde{q}} F_j$.

step11:Crossover: Pick up two strings and exchange them at a crossing position by a crossover probability P_c.

step12:Mutation: Alter a bit of string (0 or 1) by a mutation probability P_m.

step13:Repetition: Repeat step5$\sim$step12 until prespecified G-th generation. If unsatisfied, go to step2.

Fig.3 is the flowchart of the GA.

As a result, we have a suboptimal control $u(\mathbf{X})$ for the string with the best performance over all the past generations.

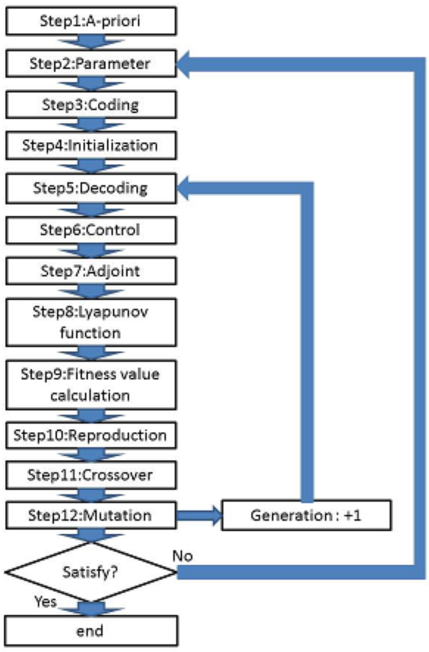

Fig. 3 Flowchart of the GA

4 Numerical Example

Consider a field excitation control problem of power system. Fig.4 is a diagram of Ozeki-Power-Plant of Kyushu Electric Power Company in Japan. This system is assumed to be described[6] by

$$\widetilde{M}\frac{d^2\delta}{dt^2} + \widetilde{D}\frac{d\delta}{dt} + P_e = P_{in}$$
$$P_e = E_I^2 Y_{11}\cos\theta_{11} + E_I\widetilde{V}Y_{12}\cos(\theta_{12} - \delta)$$

$$E_I + T'_{d0}\frac{dE'_q}{dt} = E_{fd}$$
$$E_I = E'_q + (X_d - X'_d)I_d$$
$$I_d = -E_I Y_{11}\sin\theta_{11} - \widetilde{V}Y_{12}\sin(\theta_{12} - \delta)$$
$$\widetilde{D} = \widetilde{V}^2\Big\{\frac{T''_{d0}(X'_d - X''_d)}{(X'_d + X_e)^2}\sin^2\delta + \frac{T''_{q0}(X_q - X''_q)}{(X_q + X_e)^2}\cos^2\delta\Big\},$$

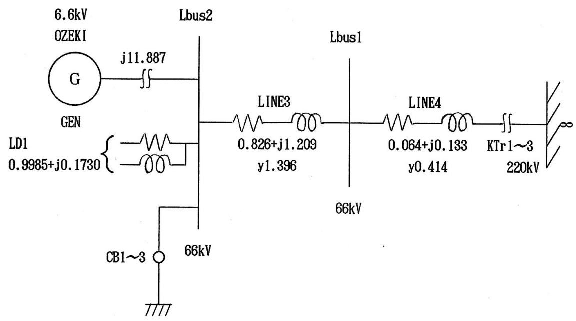

Fig. 4 Diagram of Ozeki-Power-Plant

where δ: phase angle, $\dot{\delta}$: rotor speed, $\widetilde{M}$: inertia coefficient, $\widetilde{D}(\delta)$: damping coefficient, P_{in}: mechanical input power, $P_e(\delta)$: generator output power, $\widetilde{V}$: reference bus voltage, E_I: open circuit voltage, E_{fd}: field excitation voltage, X_d: direct axis synchronous reactance, X'_d: direct axis transient reactance, X_e: external impedance, $Y_{11}\angle\theta_{11}$: self-admittance of the network, $Y_{12}\angle\theta_{12}$: mutual admittance of the network, and $I_d(\delta)$: direct axis current of the machine. Put x=$[x[1], x[2], x[3]]^T$=$[E_I - \hat{E}_I, \delta - \hat{\delta}_0, \dot{\delta}]^T$ and $u = E_{fd} - \hat{E}_{fd}$, so that

$$\begin{bmatrix} \dot{x}[1] \\ \dot{x}[2] \\ \dot{x}[3] \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} + \begin{bmatrix} g_1(x) \\ 0 \\ 0 \end{bmatrix} u \qquad (19)$$

where

$$\begin{aligned} f_1(x) &= -\frac{1}{kT_{d0}}(x[1] + \hat{E}_I - \hat{E}_{fd}) \\ &\quad + \frac{(X_d - X'_d)\widetilde{V}Y_{12}}{k}X_3\cos(\theta_{12} - x[2] - \hat{\delta}_0) \\ f_2(x) &= x[3] \\ f_3(x) &= -\frac{\widetilde{V}Y_{12}}{\widetilde{M}}(x[1] + \hat{E}_I)\cos(\theta_{12} - x[2] - \hat{\delta}_0) \\ &\quad - \frac{Y_{11}\cos\theta_{11}}{\widetilde{M}}(x[1] + \hat{E}_I)^2 - \frac{\widetilde{D}}{\widetilde{M}}x[3] + \frac{P_0}{\widetilde{M}} \\ g_1(x) &= \frac{1}{kT_{d0}}, \quad k = 1 + (X_d - X'_d)Y_{11}\sin\theta_{11}. \end{aligned}$$

Parameters are

Table 1: Performances

Method	$x^{\mathrm{T}}(0)$: initial point			
	$[0, 0.4, 0]$	$[0, 1.3, 0]$	$[0, 1.35, 0]$	$[0, 1.414, 0]$
LOC	0.95375	$\times$	$\times$	$\times$
AACC(Old,ω_2=10)	0.99287	2.47172	$\times$	$\times$
AACC(Old,ω_2=100)	0.99574	2.41060	$\times$	$\times$
AACC(New,ω_2=1)	1.31332	2.93085	2.56388	2.77665
AACC(New,ω_2=10)	1.09785	2.90358	2.56572	2.80244
AACC(New,ω_2=100)	0.94484	3.17066	3.07067	$\times$

$\times$: very large value

$$
\begin{array}{llll}
\widetilde{M} = & 0.016095[pu] & T_{d0} = & 5.09907[sec] \\
\widetilde{V} = & 1.0[pu] & P_0 = & 1.2[pu] \\
X_d = & 0.875[pu] & X'_d = & 0.422[pu] \\
Y_{11} = & 1.04276[pu] & Y_{12} = & 1.03084[pu] \\
\theta_{11} = & -1.56495[pu] & \theta_{12} = & 1.56189[pu] \\
X_e = & 1.15[pu] & X''_d = & 0.238[pu] \\
X_q = & 0.6[pu] & X''_q = & 0.3[pu] \\
T''_{d0} = & 0.0299[pu] & T''_{q0} = & 0.02616[pu] \\
\hat{E}_I = & 1.52243[pu] & \hat{\delta}_0 = & 48.57° \\
\hat{\dot{\delta}}_0 = & 0.0[deg/sec] & \hat{E}_{fd} = & 1.52243[pu].
\end{array}
$$

Set $\mathbf{X} = [x^T, x[4]]^T = [x[1], x[2], x[3], x[4]]^T$, $n = 3$, $\hat{X}_0 = \hat{\delta}_0 = 48.57°$,$d_0 = 1$, $C(x)$=$x[2]$, $L = 1$, $\mathbf{Q}$=diag$(1, 1, 1, 1)$, R=1, $\eta_0 = 1$, $\omega_1 = 1$, $\sigma_i = 0.33294(0 \leq i \leq M)$ and $x[4](0) = 1$. Experiments are carried out for the new control(AACC), and the ordinary linear optimal control(LOC)[3].

1) AACC(New, ω_2=1):
M=1, $\hat{X}_1 = 80°$, ω_2=1, $D_0 = (-\infty, a - \hat{\delta}_0]$, D_1=$[a - \hat{\delta}_0, \infty)$. The parameters are suboptimally selected along the algorithm of section 3. Ω=$\{N_{1i}, N_2, d_1, \eta_1, a\}$,$G$=100, $\widetilde{q}$=100, $\widetilde{L}$=8, P_c=0.8, P_m=0.03. $\mathbf{D}$=[0.0,2.0]×[-0.5,2.0]×[-5.0,5.0]×[0.0,1.5]. The result is that N_{11}=6.66, N_{12}=8.29, N_2=2.16, d_1=0.10, η_1=0.58 and a=$49.24°$.

2) AACC(New, ω_2=10):
The parameters are suboptimally selected by using the same way of the AACC(New, ω_2=1) except the weight ω_2=10. The result is that N_{11}=4.60, N_{12}=5.77, N_2=0.14, d_1=0.10, η_1=2.57 and a=$56.30°$.

3) AACC(New, ω_2=100):
The parameters are suboptimally selected by using the same way of the AACC(New, ω_2=1) except the weight ω_2=100. The result is that N_{11}=9.73, N_{12}=7.98, N_2=0.60, d_1=0.29, η_1=2.73 and a=$69.93°$.

4) AACC(Old, ω_2=10):
The parameters are suboptimally selected by using the same way of the AACC(New,ω_2=10) which uses the fixed weight of the gradient optimization automatic choosing function [7]. Ω=$\{N_{1i}, N_2, \eta_1, a\}$. The result is that N_{11}=7.48, N_{12}=1.11, N_2=0.18, η_1=2.83 and a=$78.90°$.

5) AACC(Old, ω_2=100):
The parameters are suboptimally selected by using the same way of the AACC(Old, ω_2=10) except the weight ω_2=100. The result is that N_{11}=8.06, N_{12}=1.03, N_2=0.10, η_1=2.87 and a=$78.90°$.

Table 1 shows performances by the AACC and the LOC. The cost function of Table 1 is

$$\widetilde{J} = \frac{1}{2}\int_0^{25} \left(\mathbf{X}^T\mathbf{Q}\mathbf{X} + u^T\mathbf{R}u\right) dt.$$

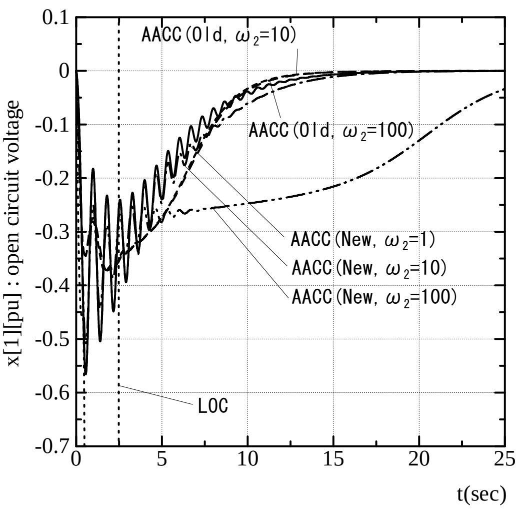

Fig. 5 Responses of LOC, AACC
$(x^T(0) = [0, 1.3, 0])$

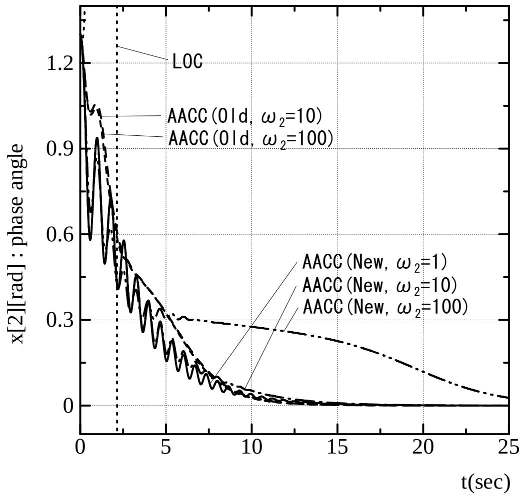

Fig. 6 Responses of LOC, AACC
$(x^T(0) = [0, 1.3, 0])$

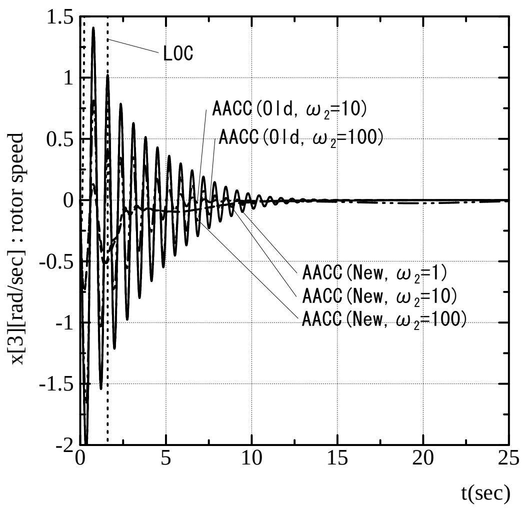

Fig. 7 Responses of LOC, AACC
$(x^T(0) = [0, 1.3, 0])$

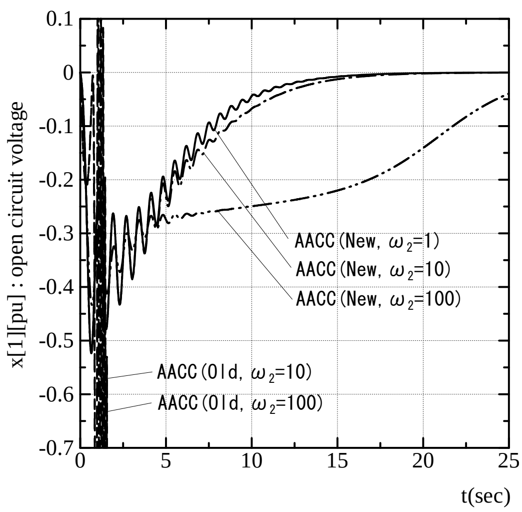

Fig. 8 Responses of AACC
$(x^T(0) = [0, 1.35, 0])$

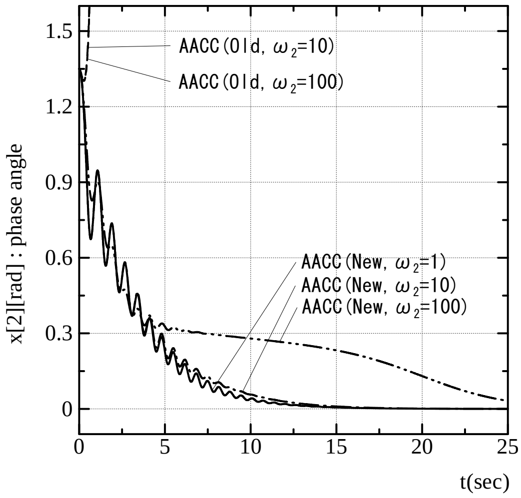

Fig. 9 Responses of AACC
$(x^T(0) = [0, 1.35, 0])$

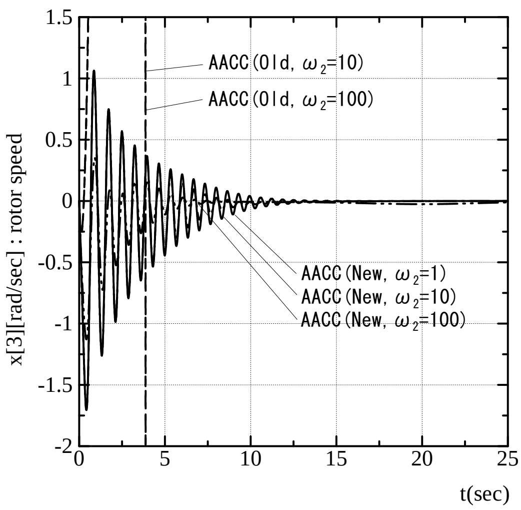

Fig. 10 Responses of AACC
$(x^T(0) = [0, 1.35, 0])$

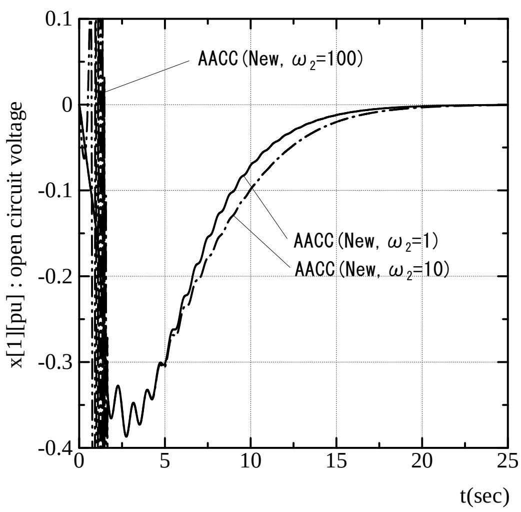

Fig. 11 Responses of AACC
$(x^T(0) = [0, 1.414, 0])$

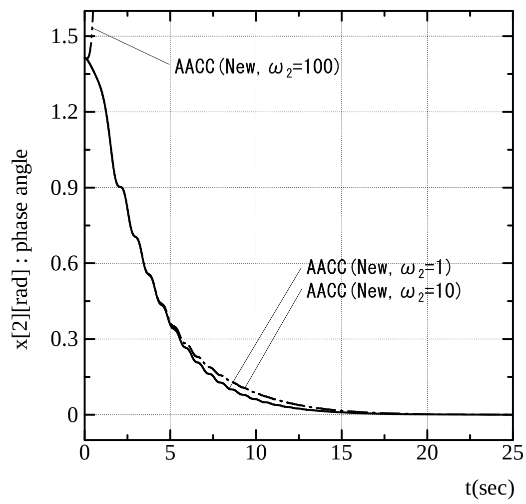

Fig. 12 Responses of AACC
$(x^T(0) = [0, 1.414, 0])$

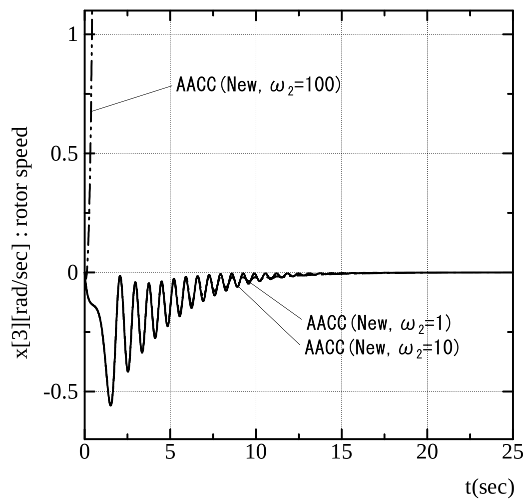

Fig. 13 Responses of AACC
$(x^T(0) = [0, 1.414, 0])$

Figs. 5, 6 and 7 show the responses in the case of $x^T(0) = [0, 1.3, 0]$. Figs. 8, 9 and 10 show the responses in the case of $x^T(0) = [0, 1.35, 0]$. Figs. 11, 12 and 13 show the responses in the case of $x^T(0) = [0, 1.414, 0]$. These results indicate that the stable region of the new AACC is better than the old AACC and the LOC.

5 Conclusions

We have studied an augmented automatic choosing control designed by extremizing a combination of the Hamiltonian and Lyapunov functions using the weighted gradient optimization automatic choosing functions for nonlinear systems. This approach was applied to a field excitation control problem of power system to demonstrate the usefulness of the AACC. Simulation results have shown that this controller could improve the performance remarkably.

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