Simulations of Average Simplified Predictive Control

[1]Samer Mansour and [2] Jalal Karam
[1] Alfaisal University, smansour@alfaisal.edu, [2] Emirates College for Advanced Education, jkaram@ecae.ac.ae

Abstract: This work presents an alternative to the least squares optimization used in Dynamic Matrix Control (DMC). Instead of calculating future moves by minimizing the sum of the squares of the future errors (least squares), each future error is individually minimized. Each minimization results in an individual recommendation for the lone future move and the actuated move would be an average of all the individual recommendations. The work presents an analytical study of the closed-loop dynamics of the method and it is used here mainly to prove the ability of the method to perform control as well as estimate the closed-loop time constants. The performance of the method is illustrated and compared to a benchmark DMC via simulation.

Keywords: Model Predictive Control; Dynamic Matrix Control; Least Squares Alternative.

1. Introduction

Linear Model Predictive Control (MPC) has been a popular feedback control methodology since the 1970s mainly in process control. In general, MPC uses the plant linear open-loop dynamics to solve an on-line constrained optimization problem of finding future control moves that minimize a certain cost function of predicted future errors (differences between the predicted plant output and a set-point profile). The plant dynamics are usually quantified by a normalized open-loop test p(t). The plant is excited by a step input and the normalized plant output p(t) is recorded. At current time t, assuming that the unknown controller output (future control moves) will be applied over a future period T c (control horizon), the plant output, hence the future errors, can be predicted over a future period T (Called prediction horizon) as a function of the future control moves by invoking linearity and using the open-loop test p(t). Actual values of the future control moves are calculated by minimizing a cost function involving the future errors. The formulation of the cost function and the optimization scheme vary with different predictive control methods.

- In Dynamic Matrix Control (DMC) [1] the cost function is the sum of the squares of all the future moves over the prediction horizon and the optimization formulation yield an ill-conditioned Least Squares problem. Different approaches were derived to resolve the ill-conditioning. The control horizon in DMC can involve one or more control moves.
- In Simplified Predictive Control (SPC) (single-parameter SPC [2] and two-parameter SPC [3]) a single future error is minimized. The control horizon involves a single control move and it is a well-conditioned method.

In this paper a new method that utilizes all the future moves as in DMC but does not suffer from ill-conditioning as in SPC is presented. It is dubbed here “Averaged SPC”. In this method, every future error over the prediction horizon is minimized individually in SPC fashion. Each future error individual minimization yields a recommended value for the lone control move. The actuated control move is the average of all the recommendations. The paper presents an analytical study of the performance of the new method based on the work published in [4]. Simulation results illustrating the implementation of the method are presented. There is no claim here that the new method delivers a better performance than the long existing DMC. However, in the author’s humble opinion, this method provides a tangible analytical and algorithmic demonstration on how the essence and power of predictive control really resides in the model while the control optimization (formulations and algorithms) and the design of the cost functions are secondary and allow a very large margin for maneuvering. This would allow for a better understanding of MPC and would open the door for comparisons studies between different types of control ([3], [5]). Future work would provide new control formulations by collecting different subsets of future errors to minimize as well as different cost functions, something that might prove useful in nonlinear problems such as in [6].

1.1. Formulation

A simple first-order linear time-invariant plant is considered here without any loss of generality since the results found are easily transferred to a more general case. The plant open-loop test $p(t)$ that satisfies $\dot{p}(t)+p(t)=1$ $p(0)=0$. So

$$p(t)=1-e^{-t}$$

Let Δt be the sampling period for both the open-loop test and the closed-loop feedback control.

Let $P_k=p(k\Delta t)=1-e^{-k\Delta t}$. Let N be the ratio of the prediction horizon and the sampling period: $N=\frac{T}{\Delta t}$.

The step change in the plant input that is applied between $n*\Delta t$ and $(n+1)*\Delta t$ is denoted Δu_n. The measured output at time $t=n*\Delta t$ can be calculated as follow

$$y_m^n=\sum_{i=0}^{n-1}\Delta u_i\, p\big((n-i)\Delta t\big)+\text{noise}(t)$$

For the sake of simplifying the analytical study and since the performance of the method under noise is not in question here, let $noise(t)=0$.

At the current time t= $n\Delta t$, all control moves Δu_i for $0\le i\le n-1$ were applied. Thus the predicted output, for a future time instant $(t+j\Delta t)$, denoted as $\hat{y}_j^n$ is due to all previous control moves Δu_i and is calculated in the following convolution formulation

$$\hat{y}_j^n=\sum_{i=0}^{n-1}\Delta u_i\, p\big((n-i+j)\Delta t\big)\quad j=0,1,2\cdots n$$

2. The Method of Averaged SPC

The method calls for calculating the control move Δu_n at time $(t=n\Delta t)$ by first minimizing each future error e_n^k. Each of the minimizations leads to a recommendation for the control move equal to $(e_k^n/\ P_k)$. The actuated value of Δu_n is a weighted average of all the recommendations as follow

$$\Delta u_n=\frac{1}{N}\sum_{k=1}^{N}\frac{\gamma e_k^n}{P_k}$$

where γ is a weighting parameter. Normally γ is in the vicinity of 1. The more γ is increased above 1 the more aggressive the control becomes.

For simplicity let the set-point $y_{sp}=1$ the error is then

$$e_k^n=y_{sp}-\hat{y}_k^n=1-\sum_{i=0}^{n-1}\Delta u_i\, p\big((n-i+k)\Delta t\big)$$

$$\Delta u_n=\frac{\gamma}{N}\sum_{k=1}^{N}\frac{1-\sum_{i=0}^{n-1}\Delta u_i\, p\big((n-i+k)\Delta t\big)}{p(k\Delta t)}$$
$$=\frac{\gamma}{N}\sum_{k=1}^{N}\frac{1}{p(k\Delta t)}-\sum_{k=1}^{N}\frac{\sum_{i=0}^{n-1}\Delta u_i\, p\big((n-i+k)\Delta t\big)}{p(k\Delta t)}$$

To get the continuous form use the continuous variables

$$t=n\Delta t, y=i\Delta t, z=k\Delta t, T=N\Delta t$$

So $\Delta y=\Delta z=\Delta t$ and Equation (6) becomes

$$\Delta u_n=\frac{\gamma}{N}\left[\sum_{z=\Delta t}^{T}\frac{1}{p(z)}-\sum_{z=\Delta t}^{T}\frac{\sum_{t=0}^{T-\Delta t}\Delta u_i\, p(t+z-y)}{p(z)}\right]$$

Multiplying both sides by Δz and rearranging the inner sum

$$\Delta u_n\Delta z=\frac{\gamma}{N}\left[\sum_{z=\Delta t}^{T}\frac{1}{p(z)}\Delta z-\right.$$
$$\left.\sum_{z=\Delta t}^{T}\frac{1}{p(z)}\left(\sum_{t=0}^{T-\Delta t}\frac{\Delta u_i}{\Delta y}p(t+z-y)\Delta y\right)\Delta z\right]$$

Let $\Delta t\rightarrow dt\rightarrow 0$, then $\Delta y\rightarrow dy$, $\Delta z\rightarrow dz$, and $\Delta u_n\rightarrow du$ and note that $dy=dz=dt\rightarrow 0$. And the continuous form of Equation (8) is

$$\frac{N}{\gamma}dudz$$
$$=\overbrace{\int_{dt}^{T}\frac{1}{p(z)}dz}^{A}-\underbrace{\int_{dt}^{T}\frac{1}{p(z)}\left(\int_{0}^{T-dt}\dot{u}(y)p(t+z-y)dy\right)dz}_{D(t)}$$

Let A be the constant

$$A=\int_{dt}^{T}\frac{1}{p(z)}dz=\int_{dt}^{T}\frac{1}{1-e^{-z}}dz=\int_{dt}^{T}\frac{-e^{-z}}{-e^{-z}+(e^{-z})^2}dz$$

And it is easy to verify

$$A=T-dt+\ln\frac{p(T)}{p(dt)}=T-dt+\ln\frac{P_N}{P_1}$$

Let $D(t)$ be

$$D(t)=\int_{dt}^{T}\frac{1}{p(z)}\left(\int_{0}^{T-dt}\dot{u}(y)p(t+z-y)dy\right)dz$$
$$=\int_{dt}^{T}\frac{1}{p(z)}\left(\int_{0}^{T-dt}\dot{u}(y)\big(1-e^{-(t+z-y)}\big)dy\right)dz$$
$$=\int_{dt}^{T}\frac{1}{p(z)}\left(\int_{0}^{T-dt}\dot{u}(y)dy\right)dz$$
$$-\int_{dt}^{T}\frac{1}{p(z)}\left(\int_{0}^{T-dt}\dot{u}(y)e^{-(t+z-y)}dy\right)dz$$
$$=[u(t)-u(0)]\overbrace{\int_{dt}^{T}\frac{1}{p(z)}dz}^{A}-$$

$$\left[\underbrace{\int_{dt}^{T} \frac{e^{-z}}{p(z)} dz}_{B}\right]\left[\underbrace{\int_{0}^{t-dt} \dot{u}(y)e^{-(t-y)}\, dy}_{I(t)}\right]$$

Let $u(0) = 0,\ B = \int_{dt}^{T} \frac{e^{-z}}{p(z)} dz,$ and $I(t) = \int_{0}^{t-dt} \dot{u}(y)e^{-(t-y)}\, dy.$ $D(t)$ becomes $D(t) = u(t)A - BI(t)$. It is easy to verify that $B = \ln\frac{P_N}{P_1}$ and A becomes $T - dt + B$. In $I(t),\ t - dt \approx t$ thus

$$I(t) = e^{-t}\int_{0}^{t} \dot{u}(y)e^{y}\, dy$$

And it is easy to verify that

$$\dot{I}(t) = -I(t) + \dot{u}(t)$$

So Equation 9 becomes

$$\frac{Ndz}{\gamma} du = A - [u(t) - u(0)]A + BI(t)$$

Starting the system from rest $(u(0) = 0)$ and multiplying and dividing by dt the left hand side lead to

$$\frac{\overbrace{Ndzdt}^{q}}{\gamma}\frac{du}{dt} = A - u(t)A + BI(t)$$

Deriving both sides and using Equation 15 for $\dot{I}(t)$, lead to the second order ordinary differential equation in u

$$q\ddot{u} + (A + q - B)\dot{u} + Au = A$$

A steady-state of $u_{ss}(t) = 1$ in the last equation shows that the new formulation does provide control to the set-point. In the zone where the differential equation yields and overdamped solution we can approximate the time constants and the analytical solution. The time constants are found from the quadratic characteristic equation $q\gamma^2 + (T - dt + q)\gamma + A = 0$ whose discriminant is $\Delta = (T - dt + q)^2 - 4qA = (T - dt + q)^2 - 4q(T - dt + B)$. As $dt \ll 1$, by taking $T - dt + B \approx T + B,\ \Delta$ can be approximated by

$$\Delta = T^2[1 + \frac{Tdt^2}{\gamma^2} - \frac{2dt}{\gamma}\left(1 + \frac{2B}{T}\right)]$$

As $\left[\frac{Tdt^2}{\gamma^2} - \frac{2dt}{\gamma}(1 + \frac{2B}{T})\right] \ll 1,\ \sqrt{\Delta}$ can be approximated as

$$\sqrt{\Delta} = T\left[1 + \frac{1}{2}\left[\frac{Tdt^2}{\gamma^2} - \frac{2dt}{\gamma}(1 + \frac{2B}{T})\right]\right]$$

Approximate solutions of the quadratic are

$$\lambda_1 = \frac{\gamma}{2T} - \frac{q}{4} - \frac{\gamma}{dt} - \frac{B}{T}$$
$$\lambda_2 = \frac{\gamma}{2T} + \frac{q}{4} - 1 - \frac{B}{T}$$

This leads to two time constants one slow and one fast. $\tau_{fast} = 1/\lambda_1$ and $\tau_{slow} = 1/\lambda_2$. The solution of Equation (18) is then

$$u(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

Where

$$c_1 = \frac{-\dot{u}(0) - \lambda_2}{\lambda_2 - \lambda_1}$$
$$c_2 = \frac{\dot{u}(0) + \lambda_1}{\lambda_2 - \lambda_1}$$

And $\dot{u}(0)$ can be approximated by $\frac{\Delta u_0 + \Delta u_1}{\Delta t}$. It is easy to verify that the very first control move $\Delta u_0 = \frac{\gamma A}{T}$. The second control move Δu_1 can be approximated as follow

$$\begin{aligned}\Delta u_1 &= \frac{\gamma}{N}\sum_{k=1}^{N}\frac{1 - \Delta u_0 p((k+1)\Delta t)}{p(k\Delta t)} \\ &= \frac{\gamma}{N}\sum_{k=1}^{N}\frac{1}{p(k\Delta t)} \\ &\quad - \frac{\gamma\Delta u_0}{N}\sum_{k=1}^{N}\frac{p\big((k+1)\Delta t\big)}{p(k\Delta t)} \\ &\approx \Delta u_0 - \frac{\gamma\Delta u_0}{N}\sum_{k=1}^{N} 1 = \Delta u_0(1 - \gamma)\end{aligned}$$

And an approximation of $\dot{u}(0)$ is then

$$\dot{u}(0) = \frac{(2 + \gamma)\Delta u_0}{dt}$$

3. Simulation Results

The Averaged SPC control was simulated in Matlab. Figures (1) and (2) show the performance of the simulated Avg. SPC as well as the corresponding results from the analytical study. Figure (3) shows the similarity in the performance of the Averaged SPC and the Dynamic Matrix Control.

4. Conclusion.

The Averaged SPC method was presented and proven to yield a control performance comparable to DMC. An analytical continuous study of the dynamics of the method was also presented and shown to match closely the results from the simulation of the Averaged SPC control. Future work will elaborate further on the analytical study to adequately understand the influence of each parameter in order to provide systematic tuning of the control and to also draw quantifiable comparison with other MPC methodologies.

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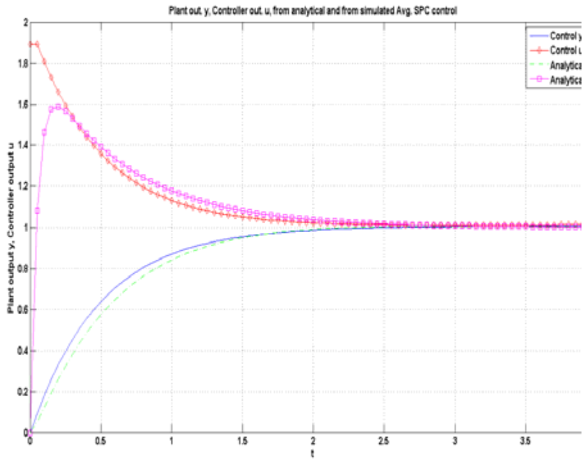

Figure 1. Plant output y and controller output u from the analytical study and the Avg. SPC control simulation for $dt = 0.05$ and $\gamma = 0.5$. For these values the analytical study yields the time constants $\tau_{fast} = 0.061$ and $\tau_{slow} = 0.63$. The slow time constant of the plant output from the simulation appears to be around 0.55.

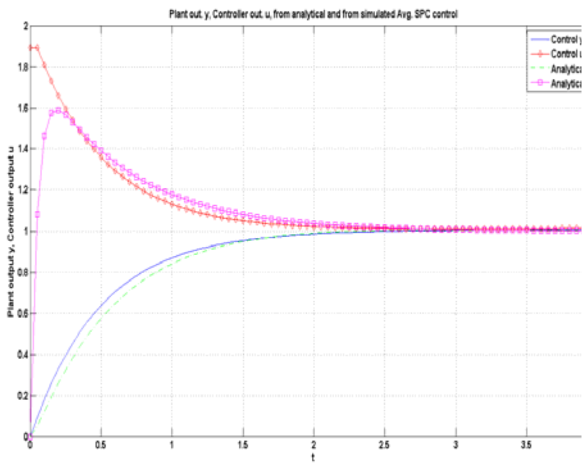

Figure 2. Plant output y and controller output u from the analytical study and the Avg. SPC control simulation for $dt = 0.05$ and $\gamma = 1$. For these values the analytical study yields the time constants $\tau_{fast} = 0.05$ and $\tau_{slow} = 0.635$. The slow time constant of the plant output from the simulation appears to be around 0.55.

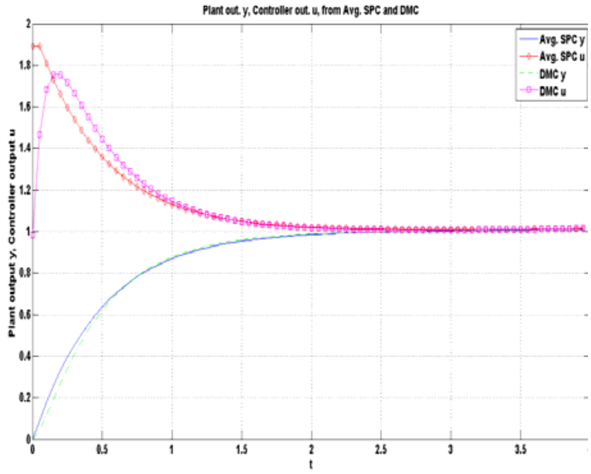

Figure 3. Plant output y and controller output u from theAvg. SPC and the DMC control simulations. dt = 0.05, γ = 1, and the DMC move suppression is 1.01.

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