A Microcontroller Implementation of Fractional Order Controller

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Abstract: The idea of this paper is to implement a Fractional Order proportional integral PI^λ on a real electronic system by using the STM32 microcontroller. This controller is founded on an extended version of Hermite-Biehler theorem to determine the complete set stabilizing. The STM32 keil starter kit based on a JTAG interface and the STM32 board was used to implement the proposed fractional controller. So, this controller is implemented using Keil development tools designed for ARM processor-based microcontroller devices and working with C/C++ langage. The performances and the efficiency of the developed strategy are illustrated with simulation results.

Keywords: Fractional-order calculus, fractional order PI controller, STM32 microcontroller, Keil.

1. Introduction

Fractional order calculus is a mathematical discipline with a 300-years-old history [1]. In recent years the fractional calculus has attracted the attention of researchers in many fields such as biology, economics and engineering [2-5]. The fractional order calculus has been found that many physical systems have a dynamic behavior non-integer order. The first non-integer order system to be widely recognized is the thermal system; other fractional order systems that are known are the electromagnetic wave systems, electrode-electrolyte polarization, viscoelastic, and many others [6]. Also, the fractional dynamic system appears in the system industries, such as control application [7]. The first fractional order control to regulator the physical systems is proposed by Oustaloup in 1988 [8, 9]. Later, Podlubny developed the fractional PID controller in 1994 [1]. Recently, Rhouma is proposed the fractional model predictive control to fractional order system [8]. In [9], the authors propose a new PI^λ tuning method for first order systems with time delay. Some results on the control of integrating systems with time delay using fractional order PD controllers were obtained. Recently, Monje et al. [10] give a new tuning method called F-MIGO for PI^λ extended from the MIGO method, this tuning rules is used to determine the best fractional and the best PI^λ gains. In [11], the authors propose two sets of tuning rules for fractional PID similar to those of the first set of Ziegler–Nichols. A frequency approach for the auto-tuning of fractional-order PID is proposed in [12], where PI is used to cancel the slope of the curvephase of a position servo system with time delay around a frequency point and the $PI^\lambda D^\mu$ controller is designed to fulfill the specifications of gain crossover frequency.

A microcontroller is described as a computer on a chip because it contains all the features of a full computer including central processor output and input ports with special features, serial communication, digital-analog conversion, analog-digital conversion and signal processing.

In this paper, we propose a Fractional Order proportional integral PI^λ controller for a performed STMicroelectronics microcontroller (STM32). The control application could benefit from the power features and flexibility of the STM32F103xB devices. The proposed framework was developed using Keil development tools designed for ARM processor-based microcontroller devices.

The outline of this paper is organized as follows. In section 2, a problem formulation and some definitions of fractional order systems are introduced, and the G-L definition used to approximate the fractional order system is detailed. Section 3 states the hardware as well as the software development tools used in this application. The section 4 is reserved to focus on the

necessary steps in finding the design method proposed for the PI^λ. Simulation results are given in the section 5. Finally, a conclusion is given.

2. Preliminary of fractional calculus

Fractional calculus is a generalization of differentiation and integration to non-integer orders fundamental operator $_{t0}D_t^\alpha f(t)$ which is defined as:

$$_{t0}D_t^\alpha f(t) = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \displaystyle\int_{t0}^{t} (d\tau)^\alpha & \alpha < 0 \end{cases} \quad (1)$$

Where α is the order, $\alpha \in R$, t and t_0 are the upper and lower limits of the operation, respectively. There are several definitions for fractional order calculus, the most popular definitions used are the Grunwald-Letnikov (G-L) and Riemann-Liouville (R-L) definitions [13]. The R-L definition of function $f(t)$ is defined as:

$$_{t0}D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_{t0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, n-1 < \alpha < n \quad (2)$$

The G-L definition of function $f(t)$ is defined as:

$$_{t0}D_t^\alpha f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[(t-t_0/h)\right]} (-1)^j \frac{\Gamma(\alpha+1)}{\Gamma(\alpha-j+1)\Gamma(j+1)} f(t-jh)$$

(3)

Where $\Gamma(\ )$ is the Gamma function, h is the sampling period, n is an integer.

The laplace transform of R-L and G-L definitions for zero initial conditions can be given as:

$$L\left[_0D_t^\alpha f(t) \right] = s^\alpha L\left[f(t) \right] = s^\alpha F(s) \quad (4)$$

In general, a fractional model can be described by a fractional differential equation characterized of the following form:

$$\sum_{l=0}^{L} a_l D^{\alpha_{al}} y(t) = \sum_{m=0}^{M} b_m D^{\alpha_{bm}} u(t) \quad (5)$$

Using the laplace transform in equation (5), the fractional-order system can be represented by the following transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\displaystyle\sum_{m=0}^{M} b_m s^{\alpha_{bm}}}{\displaystyle\sum_{l=0}^{L} a_l s^{\alpha_{al}}} \quad (6)$$

Where

$(a_l, b_m) \in R^2$, $(\alpha_{al}, \alpha_{bm}) \in R_+^2$, $\forall l=0,1,\cdots L$,
$\forall m=0,1,\cdots M$

3. Fractional-order PI design

The aim of this section is to present the system which will be controlled by a fractional order PI controller and to present the design of the fractional controller. This controller is based on the unit return configuration shown in the following figure.
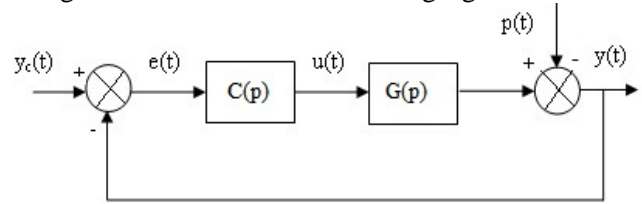

Figure1: Closed-loop fractional system

The first fractional order systems can be described by:

$$G(p) = \frac{K}{1+Tp} \quad (7)$$

Our tuning strategy, is based on Hermite-Biehler theorem and the Pontryagin condition to determine the k_p and k_i parameters.

The fractional PI^λ controller transfer function C(s) is given by the following equation:

$$C(p) = K_p + \frac{K_i}{p^\lambda} \quad (8)$$

The control input of the PI^λ controller is:

$$u(t) = K_p\left(r(t) - y(t) \right) + K_i D_t^{-\alpha}\left(r(t) - y(t) \right) \quad (9)$$

Where r(t) is the reference input or the setpoint signal, e(t) is the error, u(t) is the control, y(t) is the output signal and $D_t^{-\alpha}$ is the fractional differential/integral operators.

The control design method proposed in this paper is based on a Hermite-Biehler and Pontryagin theorem which consist on interlacement property of the real roots of the polynomial characteristic.

The closed-loop characteristic polynomial of a first order system is given by:

$$\delta(p) = (KK_i + KK_p p^\lambda) + (1+Tp)p^\lambda$$

However, we present our theorem [29] useful to compute the stability region of a first order system. Based on the first property of Hermite-Biehler [30]

which consist that all the roots of the polynomial characteristic of the closed loop equation are real.

Theorem1: [14]

We consider a first order plant given by the following transfer function:

$$G(s) = \frac{K}{1+Ts} e^{-Ls}$$

where the parameters T, L and K are positive.

We can determine the set of all stabilizing (k_p, k_i) values for the given plant using the fractional order controller PI^{λ}

$$C(s) = K_p + \frac{K_i}{s^{\lambda}}$$

4. Microcontroller STM32 Star Kit

In this section, an overview of the hardware and the software development tools is presented.

The choice to adopt STM32 is based on tradeoffs between price and performance. So, the STM32 performance line family incorporates the high-performance ARM Cortex-M4, 72 MHz frequency, an extensive range of enhanced I/Os and high-speed embedded memories. the devices of STM32 offer two 12-bit ADCs. Moreover, the STM32 has configurable and flexible power management features that allow to choose the power option to fit application [15-16].

The STM32 starter kit presented in Figure 2 was used to implement the Fractional PI program and The STM32F100 architecture is presented in Figure 3.

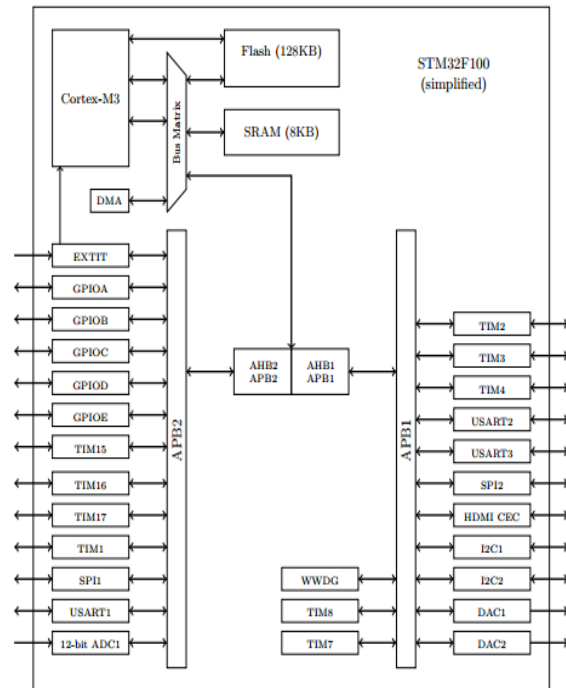Figure2. STM32F4 starter kit

Figure 3. STM32F4 architecture.

Keil is a software development tool includes C/C++, simulation models, Real-Time Operating System, debuggers, integrated environments and evaluation boards for ARM, Cortex-M, Cortex-R4 families. The used version, in this paper, is the μVision 4. This version screen supplies a menu bar for command entry, windows for source files, and a toolbar where we can choose command buttons dialog boxes.

The μVision 4 has two operating modes: Build Mode and Debug Mode.

- *Build Mode:* It's the standard working mode. It lets us to convert all the application files and to generate executable programs.

- *Debug Mode:* It supplies an effective debugger fo testing applications.

5. Experiment results

In order to illustrate the effectiveness and performances of the fractional order controller developed in this paper, we make a practical implementation of the proposed controller on an electronic system by using an STM32 microcontroller. The practical system used in this paper is a first order system as shown in Figure 4.
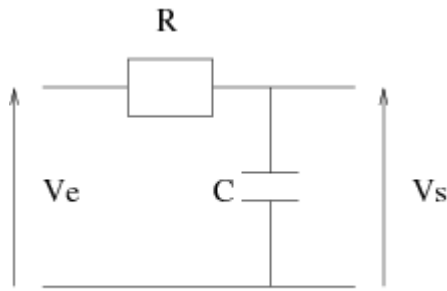
Figure 4. First order system.

In this work we choose R= 3.2K Ω and C=470μF. So, by adopting these values, the system transfer function will be
given by:

$$G(p) = \frac{Y(p)}{U(p)} = \frac{1}{1 + 0.01504\,p} \qquad (10)$$

The output of the system will be injected into the STM32 card through pin 6 of port A (PA6) which will be used as the input of the analog converter "ADC". This converter consist to converts the analog value of the output voltage to a digital value which can be used in the program of the Fractional PI regulator already implemented on the STM32 board. Then, the control delivered by the corrector passes through the pin PA4 of the STM32 card (figure 5).
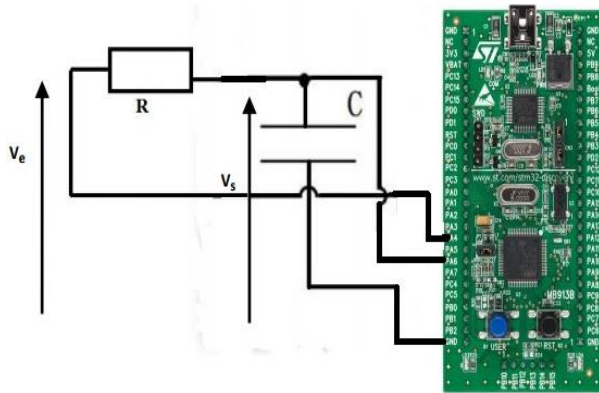

Figure 5. Connecting the STM32 with the system

To implement the fractional-order PI^{λ} of the thermal system we have used the model given by equation (26). So, the thermal system is defined as a first order system with time delay. Therefore, we proceed the design of the controller by exploiting the approach exposed in section 4.

The designed PI^{λ} parameters are fixed as follows:
$K_p = 2$, $K_i = 0.6$ and $\lambda = 0.9$

The evolutions of the output, the set point and the control signal, obtained by applying the Fractional Order proportional integral PI^{λ} controller to the real system, are shown in Figures 6 and 7 respectively.
Based on these results, we note that the output signal meets the desired requirements and the control signal obtained provide a small variation.
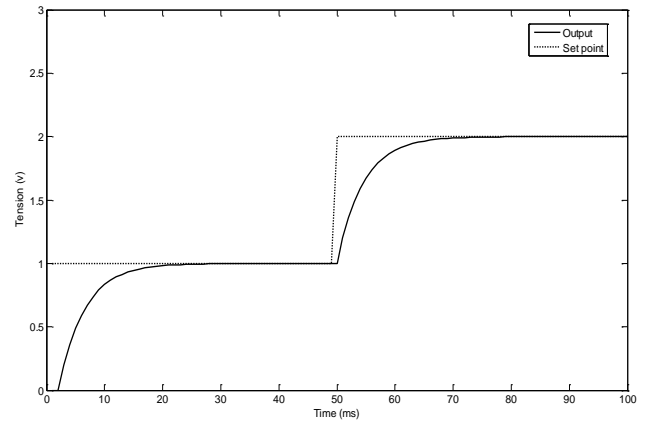

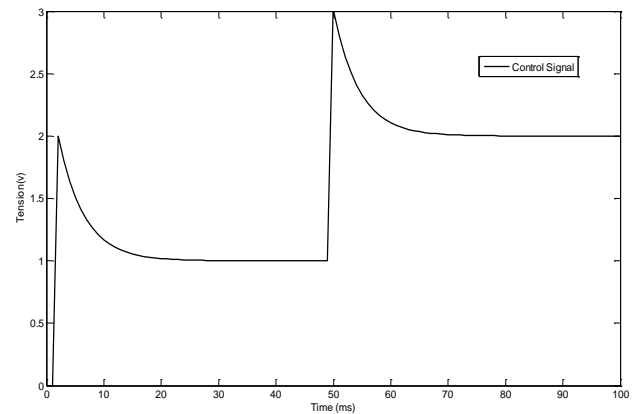Figure 6. Set point and output signals with fractional PI

Figure 7. Control signal with fractional PI

A comparison of the closed-loop performances of the pro-posed approach is established with classic Proportional Integral (PI) controller.
The designed PI parameters are fixed as follows:
$K_p = 2$, $K_i = 0.6$

Figures 8 and 9 exhibits the output, the set point and the control signal obtained by applying the classic PI controller.
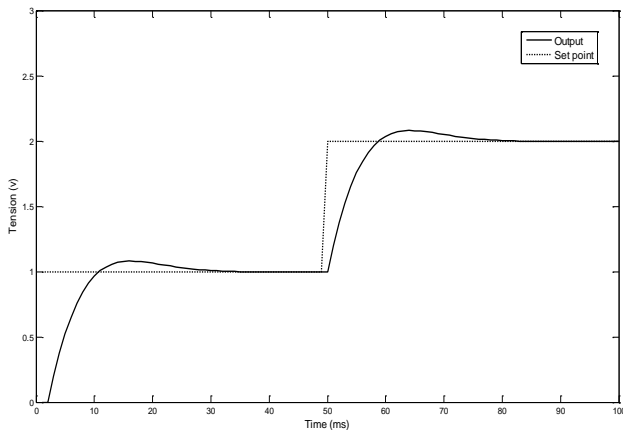
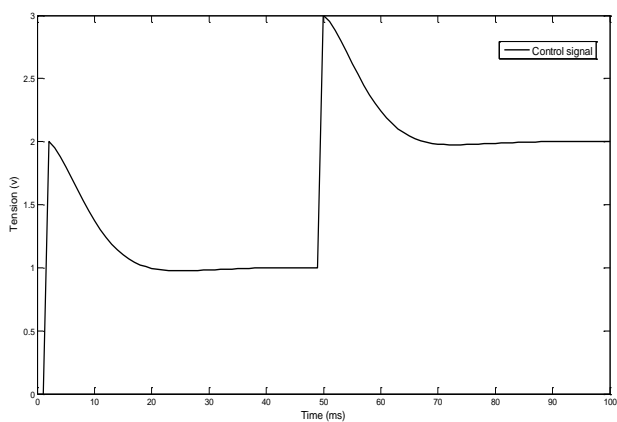Figure 8. Set point and output signals with classic PI

Figure 9. Control signal classic PI

Comparing the results obtained by the proposed PI^λ and the classic PI , we deduce that the first controller reaches the desired reference despite the change of the system dynamic whereas the second controller presents oscillations at the set point variations. We have also remark that the control law obtained by the PI^λ is smoother than one obtained by PI .

6. Conclusion

The implementation of frcational PI controller by the use of the STM32 microcontroller has been the object of this work. This controller is designed so as to ensure certain closedloop performances. The classic PI controller has been tested with the same microcontroller and compared with the fractional PI controller. Practical results show the effectiveness of the latter.

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