Optimized Fuzzy Control of a Two-Wheeled Mobile Pendulum System

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Abstract: - This paper investigates the performance of an optimized fuzzy control structure developed for the stabilization of a naturally unstable mechatronic system. The mechatronic system is a so-called mobile wheeled pendulum consisting of two actuated coaxial wheels and an inner body which oscillates (as a pendulum) around the wheel axis during planar motion. This motion is controlled in closed loop ensuring both the stabilization of the inner body as well as the planar motion of the wheels. The control structure comprises three fuzzy logic controllers whose input-output ranges and membership functions had been defined heuristically in an earlier study. This paper analyzes the achievable control performance through the formulation of a complex performance index and application of the particle swam optimization on the parameters of the control structure. The complex performance index took into account the reference tracking errors and the extent of inner body oscillation. The optimized fuzzy logic controllers showed a remarkable 29% overall performance improvement in the closed loop dynamics compared to the performance of the initial fuzzy control parameters. The simulation results proved that the optimized closed loop behavior protects more the electro-mechanical structure of the plant since the fast reference tracking performance was achieved along with effectively limited inner body oscillations.

Key-Words: - fuzzy control, optimization, self-balancing robot, mobile robot

1 Introduction

Fuzzy logic provides an easy, expert oriented way to establish control structures through the definition of heuristic IF-THEN rules based on the observations collected of the system dynamics [1]. This approximate reasoning covers model imprecision as well as uncertainties, and through the broadly defined fuzzy sets robust and smooth control action is achieved which in many cases provides superior control performance compared to the linear solutions [2]-[6]. Although, the heuristically defined inference machine roughly meets the design requirements it usually provides a suboptimal control performance. This suboptimal control performance can be further improved by trial and error tuning, however, the engineering intuition based iterative tuning becomes rather difficult if complex nonlinear systems with high order dynamics are controlled, moreover, this way the best control performance cannot be guaranteed. The tuning procedure can be realized with numerical optimization as well, which replaces the designer's tedious, iterative task and optimizes the control parameters by locating the minimum of the formulated fitness function.).

This paper describes the results related to the optimization of a fuzzy control structure developed with empirical rules and trial and error tuned membership functions for the stabilization of a naturally unstable mechatronic system, a so-called mobile wheeled pendulum (MWP) [7], [8]. This mechatronic system composed of two coaxial wheels (no additional caster), and an inner body (hereinafter IB) that forms a pendulum between the wheels, as illustrated in Fig. 1. Since the system has only two contact points with the supporting surface, the IB tends to oscillate when the wheels are actuated. This motion leads to a control challenging problem, which is the simultaneous stabilization of the IB and the predefined control of the planar motion of the wheels.

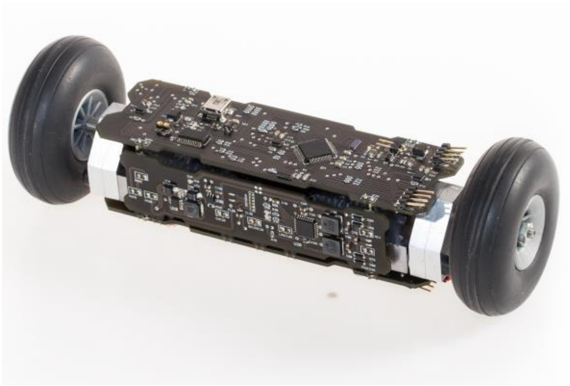

Fig. 1. Photograph of the mobile pendulum system.

Linear controllers had been elaborated and analyzed for this type of systems in [9]-[11], however the design of the controllers was based on trial and error procedures and the achievable control performance has not been investigated, which motivated this study.

The electro-mechanical properties of the system along with its mathematical model was described in [12] in detail. A fuzzy logic based anti-sway speed control structure was elaborated in [13], where simulation and implementation results showed that the proposed controllers successfully ensured both the planar motion and the stabilization of the IB. This fuzzy inference machine was characterized by heuristically defined membership functions and IF-THEN rules collected from observations of the system dynamics. Moreover, the control performance achieved with the preceding fuzzy control structure was analyzed by evaluating the overall control performance as well as different transient responses. The study in [2] compared the fuzzy anti-sway speed control structure with a linear-quadratic-Gaussian (LQG) controller elaborated in [3]. The measurement results of the real system dynamics showed that the proposed fuzzy control structure provided better overall control performance, however, the LQG control strategy showed faster system dynamics for transient events. A video demonstration of the system dynamics is available on the website [14].

In order to enhance the control performance and find the most suited control parameters this paper proposes an objective (or fitness) function for control quality evaluation and utilizes the particle swarm optimization (PSO) on the parameters (related to the ranges and shape of membership functions) of the fuzzy inference machine. The fitness function was formulated such a way to make the optimized fuzzy control structure provide fast system dynamics as well as reduce the IB oscillations and jerks in the mechanics. The goal of this paper is to investigate and measure the achievable control performance based on the application of numerical optimization, moreover, to provide the interested reader an example of how to elaborate an optimization procedure of control structures developed for mechatronic systems.).

The rest of this paper is organized as follows. Section 2 introduces the mechatronic system and its nonlinear mathematical model. In section 3 the fuzzy anti-sway speed control structure and its initial properties are discussed. Section 4 introduces the optimization method, formulates a complex fitness function and discusses the optimization results. Finally, section 5 contains the conclusions and the future work recommendations.

2 Mechatronic system

2.1 Electro-mechanical properties

The mechanical structure of the MWP consists of two coaxial wheels and a steel IB. As it can be seen in Fig. 1, no caster wheel is attached to the body, therefore the MWP has only two contact points with the supporting surface. The wheels are actuated through DC motors that form the connection between the electrical and mechanical sides. The motors are attached to-, while the embedded electronic parts are placed around the IB. The torque produced by the motors is transferred to the wheels through rolling bearings [12].

The embedded electronics is built around two 16-bit ultra-low-power Texas Instruments microcontrollers. The system dynamics is measured with three-dimensional MEMS accelerometers and gyroscopes as well as incremental encoders are attached to the shafts of the motors. The DC motors are driven through H-bridges with pulse width modulation (PWM) signals. The embedded electronics also contains a wireless module that enables the recording of the measurement results [3].

2.2 Mathematical model

The simulation environment consists of the mathematical model of the plant and the control structure forming together the closed loop. The mathematical model had been derived earlier in [12], the main parameters are depicted in Fig. 2. The

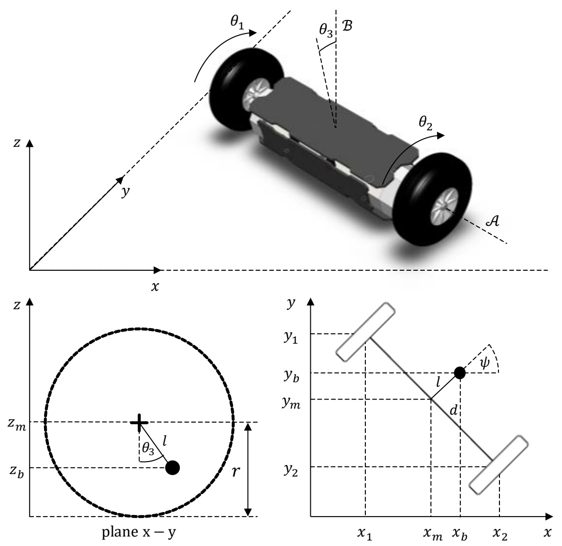

Fig. 2. Plane and side view of the MWP and its spatial coordinates.

angular position of the wheels are denoted with θ_1 and θ_2, while θ_3 indicates the oscillation angle of the IB. The distance between the center of mass of the IB and the wheel axis is marked with l, moreover, the radius of the wheels and the distance between them are denoted with r and d, respectively.

The nonlinear state-space representation $\dot{x}(t) = h(x,u)$ of the MWP dynamics is described as [12]:

$$\dot{x}(t) = \begin{bmatrix} \dot{q} \\ M(q)^{-1}\left(\tau_a - \tau_f - V(q,\dot{q})\right) \\ \frac{1}{L}\left(u - k_E k \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \dot{q} - RI\right) \end{bmatrix}, \quad (1)$$

$$y(t) = Cx(t),$$

where $x_{8\times1} = (q, \dot{q}, I)^T$ is the state vector, $q_{3\times1} = (\theta_1, \theta_2, \theta_3)^T$ contains the configuration variables, $I_{2\times1} = (I_1, I_2)^T$ and $u_{2\times1} = (u_1, u_2)^T$ denote the currents and voltages of the motors, respectively. Moreover, $M(q)$ is the 3-by 3 inertia matrix and $V(q,\dot{q})$ is the 3-dimensional vector term including the Coriolis, centrifugal and potential force terms, while τ_a and τ_f indicate the torques transmitted to the wheels and the effect of friction. The applied DC motors are characterized by the rotor resistance R and inductance L, the back-EMF constant k_E, and the gear ratio of the gearbox k as well. The output matrix C of the state space equation is selected such a way to produce the $y_{5\times1} = (v, \theta_3, \omega_3, \xi, I_{avg})$ output, where $v = r(\theta_1 + \theta_2)/2$ is the linear speed of the plant, $\omega_3 = \dot{\theta}_3$ is the oscillation rate of the IB, while $\xi = r(\dot{\theta}_2 - \dot{\theta}_1)/d$ and $I_{avg} = (I_1 + I_2)/2$ denote the yaw rate and the average current consumption, respectively.

3 Fuzzy control structure

The objective of the control structure is to simultaneously ensure the planar motion of the MWP and suppress the resulting IB oscillations. The fuzzy control structure elaborated in [2], [13] in detail is a so-called anti-sway speed control structure that ensures the following control objectives:

- $\lim_{t\to\infty} v(t) = v_d$ for the linear speed,
- $\lim_{t\to\infty} \xi(t) = \xi_d$ for the yaw rate,
- $\lim_{t\to\infty} \omega_3(t) = 0$ for the IB stabilization,

where v_d and ξ_d denote the desired linear speed and yaw rate values.

The control structure depicted in Fig. 3a consists of three cascade connected fuzzy logic controllers (FLC), where FLC1 and FLC3 are P-type controllers and are responsible for the control of linear speed and yaw rate of the plant, respectively, while a PD-type FLC2 is applied for the suppression of the IB oscillations. Fig. 3b shows the membership functions of each controller, and the corresponding rule bases are well detailed in [2]. The input (antecedent of each rule) of FLC1 is the speed error $e_v(i) = v_d(i) - v(i)$, while its output (or consequent) is the control voltage $u_v(i)$. Similarly, $e_\xi(i) = \xi_d(i) - \xi(i)$ yaw rate error forms the input, while the $u_\xi(i)$ control action is the output of FLC2. Regarding FLC3, its inputs are the oscillation error $e_{\theta_3}(i)$ and its time derivative $e_{\omega_3}(i)$, while the output of the controller is denoted with $u_{\theta_3}(i)$.

The simulation of the closed loop behavior was performed in MATLAB/Simulink environment. The state space equation (1) was implemented using an S-function block, while the FLCs were designed with the Fuzzy Logic Toolbox of MATLAB. In the simulation the $T_s = 0.01$ sec sampling time was also taken into account, which equals to the sampling time of the sensors on the MWP.

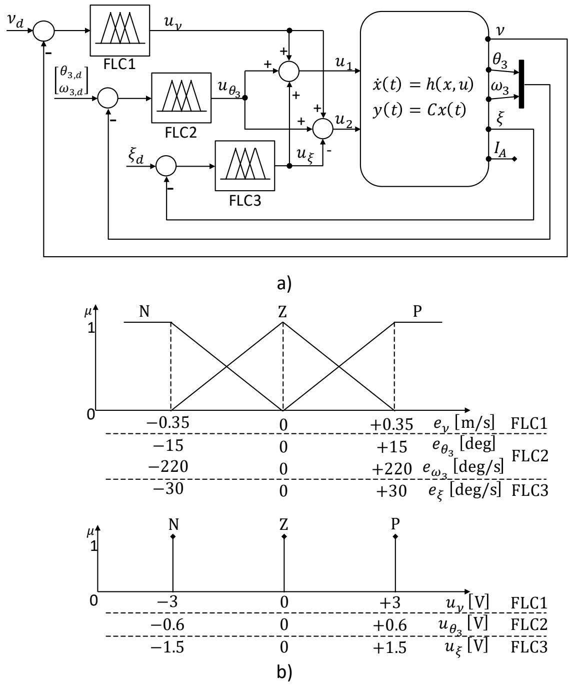

Fig. 3. (a) Block diagram of the control structure and (b) fuzzy parameters.

4 Tuning of the controllers

The control structure introduced in the previous section along with the membership functions fulfilled the control objectives, however, the parameters of the controllers were selected based on trial-end-error method. The trial-and-error procedure does not guarantee the best control performance, rather a compromise solution. This section describes the application of numerical optimization on the FLC parameters in order to enhance the control performance. The optimization algorithm results the best possible FLC parameters by locating the minimum of the formulated fitness function.

4.1 FLC parameters

The main parameters that determine the fuzzy inference are related to the shapes and ranges of the applied membership functions. Varying the shape, position and range of these functions different control performance is achieved. The triangular membership functions and the singleton consequents depicted in Fig. 3b are characterized by three parameters and a gain, respectively, summarized in the third column of Table 1. These parameters were selected to be tuned by means of numerical optimization whose initial values are given in the fourth column based on Fig. 3b.

4.2 Fitness function

The control performance is measured with the fitness function. In [2] different formulas were recommended for the quality measurement of reference tracking and suppression of IB oscillations. Among these error integrals, the combination of three mean absolute errors (MEA) was chosen for the fitness function that qualifies the overall control performance. Therefore, both the quality of reference tracking (by evaluating the errors $e_v = v_d - v$ and $e_\xi = \xi_d - \xi$) and the quality of IB oscillation suppression (by evaluating the error $e_{\omega_3} = 0 - \omega_3$) was implemented in a complex fitness function:

$$F = \sqrt[3]{\left(\frac{\sum|e_{v,j}|}{N}\right)^{\alpha}\left(\frac{\sum|e_{\omega_3,j}|}{N}\right)^{\beta}\left(\frac{\sum|e_{\xi,j}|}{N}\right)^{\gamma}}, \qquad (2)$$

where $j = 1 \ldots N$, N denotes the length of the measurement, while $\alpha = 1.5$, $\beta = 0.5$ and $\gamma = 0.85$ weights represent the preferences between the three control objectives. The aim of the optimization problem is to find the control parameters (p_i, d_i in Table 1) that correspond to the minimum fitness function value.

4.3 Particle swarm optimization

The simulation environment was considered as a black box object, its inputs and outputs are the desired speeds (v_d, ξ_d) and reference tracking errors (e_v, e_{ω_3}, e_ξ), respectively, moreover, it is characterized by the FLC parameters (p_i, d_i). The particle swarm optimization (PSO) was applied for the tuning of the control parameters, since it is a robust heuristic method that has already proven its fast convergence property [4].

Table 1. Notation of the FLC parameters: initial and optimized values.

FLC1

Fuzzy set	Meaning	Parameters	Initial values	**Optimized values**
N (in)	negative	$\Gamma(-\infty, -p_{11}, -p_{12})$	$p_{11} = 0.35$ and $p_{12} = 0$	$p_{11} = 0.424$ and $p_{12} = 0.005$
Z (in)	zero	$\Gamma(-(p_{11} - p_{13}), 0, (p_{11} - p_{13}))$	$p_{13} = 0$	$p_{13} = 0.141$
P (in)	positive	$\Gamma(p_{12}, p_{11}, \infty)$	–	–
U (out)	consequent gain	u_1	$u_1 = 3$	$u_1 = 3.165$

FLC2

Fuzzy set	Meaning	Parameters	Initial values	**Optimized values**
N (in1)	negative	$\Gamma(-\infty, -p_{21}, -p_{22})$	$p_{21} = 15$ and $p_{22} = 0$	$p_{21} = 14.847$ and $p_{22} = 0.018$
Z (in1)	zero	$\Gamma(-(p_{21} - p_{23}), 0, (p_{21} - p_{23}))$	$p_{23} = 0$	$p_{23} = 2.190$
P (in1)	positive	$\Gamma(p_{22}, p_{21}, \infty)$	–	–
N (in2)	negative	$\Gamma(-\infty, -d_{21}, -d_{22})$	$d_{21} = 220$ and $d_{22} = 0$	$d_{21} = 257.463$ and $d_{22} = 89.699$
Z (in2)	zero	$\Gamma(-(d_{21} - d_{23}), 0, (d_{21} - d_{23}))$	$d_{23} = 0$	$d_{23} = 84.581$
P (in2)	positive	$\Gamma(d_{22}, d_{21}, \infty)$	–	–
U (out)	consequent gain	u_2	$u_2 = 0.6$	$u_2 = 0.351$

FLC3

Fuzzy set	Meaning	Parameters	Initial values	**Optimized values**
N (in)	negative	$\Gamma(-\infty, -p_{31}, -p_{32})$	$p_{31} = 30$ and $p_{32} = 0$	$p_{31} = 16.629$ and $p_{32} = 0$
Z (in)	zero	$\Gamma(-(p_{31} - p_{33}), 0, (p_{31} - p_{33}))$	$p_{33} = 0$	$p_{33} = 3.970$
P (in)	positive	$\Gamma(p_{32}, p_{31}, \infty)$	–	–
U (out)	consequent gain	u_3	$u_3 = 1.5$	$u_3 = 2.398$

The PSO uses an effective mechanism that mimics the swarm behavior of birds flocking and fish schooling in order to guide the particles searching for the global optimal solution in the search space. Let χ_i and δ_i denote the n-dimensional position and velocity vector of the ith particle in the swarm, while ρ_i and λ_i indicate the personal best position (which gives the best fitness value so far) of the ith particle and the neighborhood best position, respectively. The velocity and position vectors are modified in every generation based on the following equations [15]:

$$\delta_{id} = w_i \delta_{id} + c_1 r_1 (\rho_{id} - \chi_{id}) + c_2 r_2 (\lambda_{id} - \chi_{id}),$$
$$\chi_{id} = \chi_{id} + \delta_{id}, \tag{3}$$

where i denotes the ith particle, $d \in [1, n]$ is the dimension, c_1 and c_2 are positive constants, $r_1, r_2 \in [0,1]$ are random values, and w is the inertia weight. The parameters were selected as $w = 0.9$, $c_1 = 0.5$ and $c_2 = 1.5$ based on previously gained experiences [4]. In the simulation environment the Particle swarm toolbox for MATLAB [16] was utilized, while for the number of generations and populations $n_{gen} = 150$ and $n_{pop} = 150$ were chosen, since the optimization problem is characterized by many variables.

4.4 Results

The optimization results are depicted in Fig. 4, while the optimized FLC parameters are summarized in the fifth column of Table 1. The closed loop behavior was simulated with the following reference (desired) signals:

- $v_d = \{0.4, 0, -0.2, 0\}$ m/sec linear speeds,
- $\xi_d = \{30, 0, -70, 0\}$ deg/sec yaw rate values.

The fitness function value significantly improved after the optimization procedure, from $F = 0.4304$ (related to the initial p_i, d_i parameters in the fourth column of Table 1) to $F = 0.3052$ providing 29% better overall control performance (the smaller the value the better control performance is achieved). Based on the simulation results it can be observed, that the optimized FLC parameters ensured faster closed loop behavior (the reference values were achieved in less than 1 sec), moreover the oscillation of the IB was limited and quickly suppressed (similarly, in less than 1 sec). The optimization resulted an efficient control structure that remarkably enhanced the system behavior (fast and effective reference tracking), moreover, the electro-mechanical parts of the MWP are protected as well, since high peaks and jerks related to inner body oscillation are limited. Regarding the partial fitness function results, it can be remarked that the reference tracking performance was enhanced by 35% and 41% for the linear speed and yaw rate

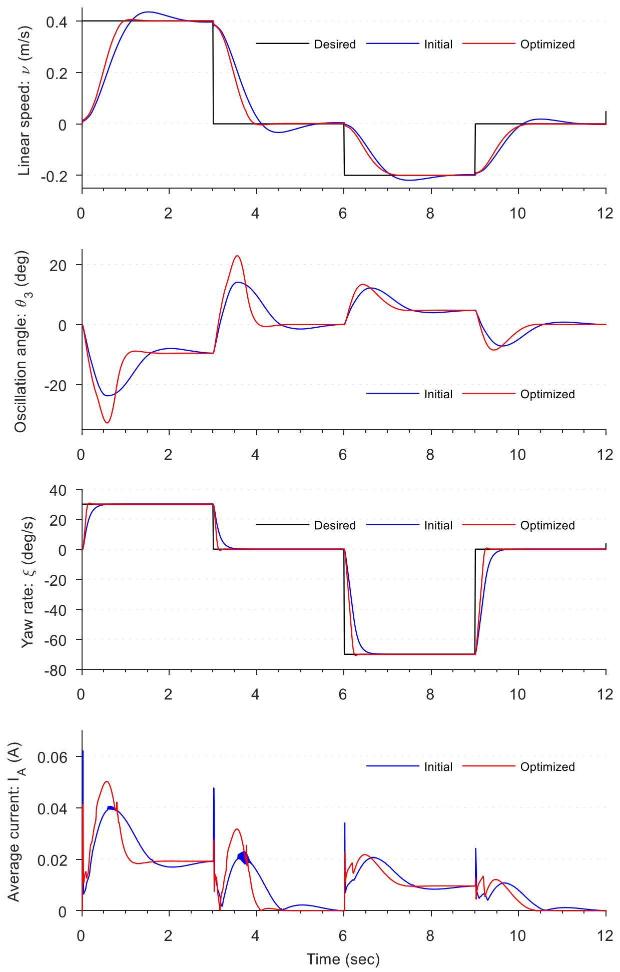

Fig. 4. Simulation results.

control, respectively, while the performance of the suppression of the IB oscillation was enhanced by 6% with the optimized fuzzy control structure.

The simulation results showed that both the flexibility fuzzy logic provides and the application of the effective PSO algorithm allowed to enhance the overall control performance. This performance can be further improved with more sophisticated fuzzy logic controllers that are characterized by bigger rule bases and more linguistic values (e.g., the inputs and outputs of the FLCs could be decomposed into five membership functions in order to define finer and more advanced fuzzy inference machines). The investigation of more advanced FLCs and their implementation on the real mechatronic system are left for another paper.

5 Conclusion

This paper described the application of particle swarm optimization on FLCs in order to enhance the overall control performance of a mechatronic system. The control structure and the FLCs were designed in an earlier paper, these design properties formed the initial closed loop performance. A complex performance index was formulated that took into account the reference tracking errors with certain preferences. This performance index was minimized in the optimization procedure which resulted significantly better FLC parameters. The simulation results proved that the optimized fuzzy control structure enhanced the overall control performance by 29% and provided both faster system dynamics and less inner body oscillations therefore protecting more the electro-mechanical structure of the MWP. Future work will involve the implementation of the optimized control structure in the embedded system of the MWP as well as the analyzes and validation of the control performances.

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