An Application of Fuzzy Numbers to Assessment of Human Activities

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Abstract: - Two methods utilizing Triangular and Trapezoidal Fuzzy Numbers together with the Centre of Gravity (COG) defuzzification technique are developed in this paper as tools for assessing human skills. Examples are also presented concerning student and basket-ball player performance, illustrating the applicability of our methods in practice. The advantages and disadvantages of each method are finally discussed and some hints are presented for future research on the subject.

Keywords: - Fuzzy Numbers (FNs), Triangular (TFNs) and Trapezoidal (TpFNs) FNs, Centre of Gravity (COG) defuzzification technique, Fuzzy Assessment methods

1. Introduction

Fuzzy Logic (FL), due to its nature of characterizing the ambiguous real life situations with multiple values, offers rich resources for assessment purposes. This gave several times in past the impulse to the present author to apply principles of FL for assessing human skills using as tools the corresponding system's ***uncertainty*** (e.g. [1], [2], Chapter 5, [3], etc.), the ***Centre of Gravity (COG) defuzzification technique*** (e.g. [2], Chapter 6, [4], etc.) and its ***variations*** constructed for assessment purposes (e.g. [2], Chapter 7, [5], etc.).

In the paper in hands ***Fuzzy Numbers (FNs)*** are used for assessing human activities. The rest of the paper is organized as follows: In Section II the background on FNs and in particular on ***Triangular (TFNs)*** and ***Trapezoidal (TpFNs)*** FNs is presented, which is necessary for the understanding of the rest of the paper. In Section III two methods are developed in which TFNs and TpFNs are used respectively for assessing human skills. Two examples concerning student and basket-ball player assessment are developed in Section IV, wile the last Section V is devoted to our final conclusions

2. Fuzzy Numbers

For general facts on ***Fuzzy Sets (FS)*** we refer to the book [6]. FNs play an important role in fuzzy mathematics analogous to the role played by the traditional numbers in crisp mathematics. A FN is a special form of FS on the set ***R*** of real numbers defined as follows:

Definition 1: A FN is a FS A on the set ***R*** of real numbers with membership function m_A: ***R*** $\rightarrow$ [0, 1], such that:

- A is ***normal***, i.e. there exists x in ***R*** such that $m_A(x) = 1$.
- A is ***convex***, i.e. all its ***a-cuts***

$$A^a = \{x \in U: m_A(x) \geq a\},$$

with a in [0, 1], are closed real intervals.

- Its ***membership function*** $y = m_A(x)$ is a ***piecewise continuous*** function.

For a better understanding of the above definition we give the following counter example:

Example 2: The graph of the membership function of a FS A on ***R*** is represented in Figure 1.

Obviously A is a normal FS with a continuous membership function. However, A is not a convex FS, because, for example, the a-cut

$$A^{0.4} = [5, 8.5] \cup [11, 13]$$

is not a closed interval. Consequently A is not a FN.

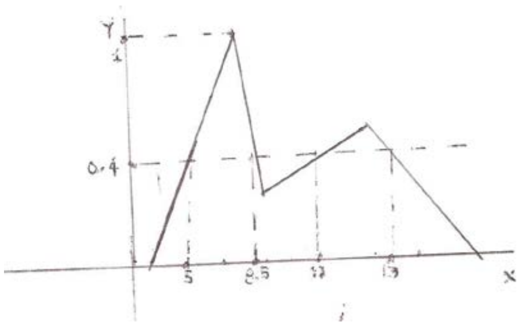

Figure 1: *Graph of a non convex fuzzy set*

Since a FN A is a convex FS, all its *x*-cuts A^x, *x* in [0, 1] can be written in the form $A^x = [A_l^x, A_r^x]$, where A_l^x, A_r^x are real numbers. The following statement defines a ***partial order*** on the set of all FNs:

Definition 3: Given the FNs A and B we write A $\leq$ B (or $\geq$) if, and only if, $A_l^x \leq B_l^x$ and $A_r^x \leq B_r^x$ (or $\geq$) for all x in [0, 1]. Two FNs for which the above relation holds are called ***comparable***, otherwise they are called ***non comparable*** (e.g. see Remark (i) of Section IV below, attached to Example 15).

Note also that one can define the ***four arithmetic operations on FNs*** as we do for the ordinary numbers. For this, there are two different, but equivalent to each other methods reported in the literature:

1. With the help of the x-cuts of the corresponding FNs, which, as we have already seen, are ordinary closed intervals of ***R***. Therefore, in this way the fuzzy arithmetic has been actually based on the arithmetic of the real intervals.
2. By applying the ***Zadeh's extension principle*** ([6], Section 1.4, p.20), which provides the means for any function f mapping the crisp set X to the crisp set Y to be generalized so that to map fuzzy subsets of X to fuzzy subsets of Y.

However, none of the above methods is frequently used in practical applications, because both of them are laborious, involving complicated calculations. What is usually preferred is the use of simpler forms of FNs instead of their general form, where these operations can be performed in a simple way.

For general facts on FNs we refer to the book [7].

The simplest form of FNs are the TFNs. Roughly speaking a TFN (*a, b, c*), with *a, b* and *c* real numbers, expresses mathematically the fuzzy statement ***"approximately equal to b"*** or equivalently that ***"b lies in the interval [a, c]"***. The membership function's ***graph*** of a TFN (*a, b, c*) in the interval [*a, c*] is the union of two straight line segments forming a triangle with the X-axis, while out of [*a, c*] it value is constantly zero (Figure 2).

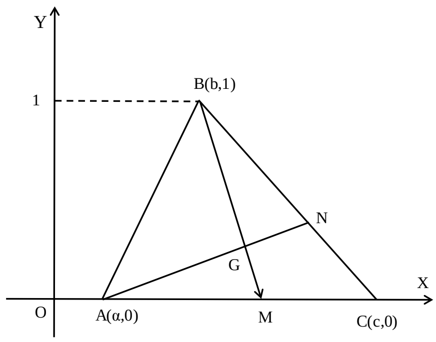

Figure 2: Graph of the TFN (*a, b, c*)

Consequently, the analytic definition of a TFN is given as follows:

Definition 4: Let *a, b* and *c* be real numbers with *a* < *b* < *c*. Then the TFN A = (*a, b, c*) is a FN with membership function:

$$y = m(x) = \begin{cases} \frac{x-a}{b-a}, & x \in [a,b] \\ \frac{c-x}{c-b}, & x \in [b,c] \\ 0, & x < a \text{ and } x > c \end{cases}$$

Obviously we have that *m(b)=1*. Note also that *b* need not be in the "middle" of *a* and *c*.

It is easy to calculate the x-cuts of a TFN. Namely, we have:

Proposition 5: The x-cuts A^x of a TFN A = (*a*, *b*, *c*), with x in [0, 1], are calculated by the formula

$$A^x = [A_l^x, A_r^x] = [a + x(b-a), c - x(c-b)] \ .$$

Proof: Since $A^x = \{y \in \mathbf{R}: m(y) \geq x\}$, applying Definition 4 for $y = A_l^x$ one gets that

$$\frac{y-a}{b-a} = x \Leftrightarrow y = a + x(b-a)$$

Similarly for A_r^x we have that

$$\frac{c-y}{c-b} = x \Leftrightarrow y = c - x(c-b) \ .\text{-}$$

The TpFNs, which are actually generalizations of the *TFNs,* is another simple form of FNs. A TpFN (*a*, *b*, *c*, *d*) with *a*, *b*, *c*, *d* in ***R*** expresses mathematically the fuzzy statement ***"approximately in the interval [b, c]"***. Its membership function *y=m(x)* is constantly 0 out of the interval [*a*, *d*], while its graph in [*a*, *d*] is the union of three straight line segments forming a trapezoid with the X-axis (Figure 3),

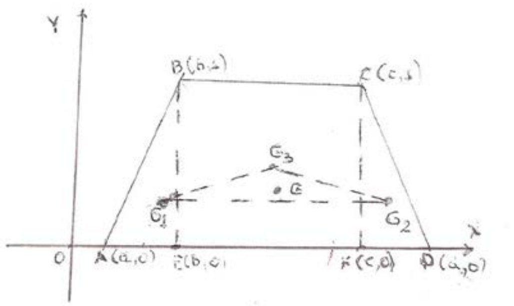

Figure 3: Graph of the TpFN *(a, b, c, d)*

Consequently, the analytic definition of a TpFN is given as follows:

Definition 6: Let $a < b < c < d$ be real numbers. Then the TpFN (*a*, *b*, *c*, *d*) is a FN with membership function:

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ x = 1, & x \in [b,c] \\ \dfrac{d-x}{d-c}, & x \in [c,d] \\ 0, & x < a \ and \ x > d \end{cases}$$

It is easy to observe that the TFN (*a*, *b*, *d*) can be considered as a special case of the TpFN (*a, b, c, d)* with *c=b*..

It can be shown]7] that the two general methods for performing operations on FNs mentioned above lead to the following simple rules for the ***addition*** and ***subtraction*** of TpFNs, while the same rules hold also for the TFNs (e.g. [8], Section 3.2):

Definition 7: Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ be two TpFNs. Then

- The ***sum***

$$A + B = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4).$$

- The ***difference***

$$A - B = A + (-B) = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1),$$

where

$$-B = (-b_4, -b_3, -b_2, -b_1)$$

is defined to be the ***opposite*** of B.

Nevertheless, whereas the sum and the difference of two TpFNs/TFNs as well as the opposite of a TpFN/TFN are also TpFNs/TFNs, their ***product*** and ***quotient,*** although they are FNs, ***they are not always*** *TpFNs/TFNs* (for more details see Section 3.2 of [8]).

One can define also the following two ***scalar operations*** on TpFNs/FNs:

Definition 8: Let $A = (a_1, a_2, a_3, a_4)$ be a TpFN and let k be a real number. Then:

$$k + A = (k+a_1, k+a_2, k+a_3, k+a_4)$$

$$kA = (ka_1, ka_2, ka_3, ka_4),$$

if k>0 and

$$kA = (ka_4, ka_3, ka_2, ka_1),$$

if k<0.

We introduce now the following definition, which will be used later in this paper for assessing, with the help of TpFNs/TFNs, the overall performance of human groups during several activities:

Definition 9: Let $A_i = (a_{1i}, a_{2i}, a_{3i}, a_{4i})$, i = 1, 2, …, n be a finite number of TpFNs/TFNs, where n is a non negative integer, $n \geq 2$. Then we define their ***mean value*** to be the TpFN/TFN:

$$A= \frac{1}{n}(A_1 + A_2 + \ldots + A_n).$$

We shall close our brief account on FNs by using the popular in fuzzy mathematics ***COG technique*** for defuzzifying TFNs/TpFNs. It is recalled here that according to the COG technique the defuzzification of a FS is obtained by calculating the coordinates of the COG of the level's section contained between the graph of the FS's membership function and the OX axis [9]. We start with the case of TFNs:

Proposition 10: The coordinates (X, Y) of the COG of the graph of a TFN (a, b, c) are calculated by the formulas:

$$X = \frac{a+b+c}{3}, Y = \frac{1}{3}.$$

Proof: The graph of the TFN (a, b, c) is the triangle ABC of Figure 2, where A (a, 0), B (b, 1) and C (c, 0). Then, the COG, say G, of ABC is the intersection point of its medians AN and BM, where N ($\frac{b+c}{2}$, $\frac{1}{2}$) and M ($\frac{a+c}{2}$, 0).

Therefore the equation of the straight line on which AN lies is

$$\frac{x-a}{\frac{b+c}{2}-a} = \frac{y}{\frac{1}{2}}$$

or

$$x + (2a - b - c)y = a \quad \textbf{(1)}.$$

In the same way one finds that the equation of the straight line on which BM lies is

$$2x + (a + c - 2b)y = a + c \quad \textbf{(2)}.$$

Since

$$D = \begin{vmatrix} 2 & a+c-2b \\ 1 & 2a-b-c \end{vmatrix} = 3(a-c) \neq 0,$$

the linear system of (1) and (2) has a unique solution with respect to the variables x and y determining the coordinates of the triangle's COG.

But

$$D_x = \begin{vmatrix} a+c & a+c-2b \\ a & 2a-b-c \end{vmatrix} = a^2 - c^2 + ba - bc$$
$$= (a+c)(a-c) + b(a-c) = (a-c)(a+c+b)$$

and

$$D_y = \begin{vmatrix} 2 & a+c \\ 1 & a \end{vmatrix} = a - c.$$

The result follows by applying the Cramer's rule for calculating x and y.-

Next, Proposition 10 will be used as a Lemma for the defuzzification of TpFNs. The corresponding result is the following:

Proposition 11: The coordinates (X, Y) of the COG of the graph of the TpFN (a, b, c, d) are calculatedby the formulas

$$X = \frac{c^2+d^2-a^2-b^2+dc-ba}{3(c+d-a-b)}, \; Y = \frac{2c+d-a-2b}{3(c+d-a-b)}.$$

Proof: We divide the trapezoid forming the graph of the TpFN (a, b, c, d) in three parts, two triangles and one rectangle (Figure 3). The coordinates of the three vertices of the triangle ABE are (a, 0), (b, 1) and (b, 0) respectively, therefore by Proposition 9 the COG of this triangle is the point G_1 ($\frac{a+2b}{3}$, $\frac{1}{3}$).

Similarly one finds that the COG of the triangle FCD is the point G_2 ($\frac{d+2c}{3}$, $\frac{1}{3}$). Also, it is easy to check that the COG of the rectangle BCFE, being the point G_3 of the intersection of its diagonals, has coordinates G_3($\frac{b+c}{2}$, $\frac{1}{2}$). Further, the areas of the two triangles are equal to $S_1 = \frac{b-a}{2}$ and S_2 =

$\frac{d-c}{2}$ respectively, while the area of the rectangle is equal to S_3 = c- b.

It is well known [10] then that the coordinates of the COG of the trapezoid, being the resultant of the COGs $G_i\,(x_i, y_i)$, for i=1, 2, 3, are calculated by the formulas

$$X = \frac{1}{S}\sum_{i=1}^{3} S_i x_i, \quad Y = \frac{1}{S}\sum_{i=1}^{3} S_i y_i \qquad \textbf{(3)}$$

where $S = S_1 + S_2 + S_3 = \frac{c+d-b-a}{2}$ is the area of the trapezoid.

The proof of the Proposition is completed by replacing the above found values of S, S_i, x_i and y_i, i = 1, 2, 3, to formulas (3) and by performing the corresponding operations.

Remark: As we have seen above the TFN (*a*, *b*, *d*) can is a special case of the TpFN (*a*, *b*, *c*, *d*) for $c = b$. In fact, putting $c = b$ in the formulas of Proposition 11 one gets that

$$X = \frac{d^2 - a^2 + db - ba}{3(d-a)} = \frac{(d-a)(d+a+b)}{3(d-a)} = \frac{a+b+d}{3}$$

and

$$Y = \frac{d-a}{3(d-a)} = \frac{1}{3},$$

i.e. he/she finds again the formulas of Proposition 10 for the defuzzification of the TFN (*a, b, d*)

Let us now go back to Figure 3, where the COGs G_1, G_2 and G_3 are the balancing points of the triangles AEB, CFD and of the rectangle BCFE respectively. Therefore, ***the COG of the COGs*** G_1, G_2, G_3, i.e. the COG G of the triangle $G_1 G_2 G_3$, being the balancing point of those COGs, could be considered instead of the COG of the trapezoid ABCD for defuzzifying the TpFN (*a, b, c, d*). The following Corollary calculates the coordinates of G:

Corollary 12: The COG G of the COGs G_1, G_2 and G_3 in Figure 3 has coordinates

$$x = \frac{2(a+d)+7(b+c)}{18}, \quad y = \frac{7}{18}$$

Proof: In the proof of Proposition 11 we have found that $G_1\ (\frac{a+2b}{3}, \frac{1}{3})$, $G_2\ (\frac{d+2c}{3}, \frac{1}{3})$ and $G_3(\frac{b+c}{2}, \frac{1}{2})$.

The y – coordinates of all points of the straight line containing the line segment $G_1\, G_2$ are equal to $\frac{1}{3}$, therefore the point G_3, having y – coordinate equal to $\frac{1}{2}$, does not belong to this line.

Then, by Proposition 10, the COG G of the triangle $G_1G_2G_3$, has coordinates

$$x = \left(\frac{a+2b}{3} + \frac{d+2c}{3} + \frac{b+c}{2}\right) : 3$$

$$= \frac{2(a+d)+7(b+c)}{18}$$

and

$$y = \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{2}\right) : 3 = \frac{7}{18}.$$

3. Using the TFNS and the TPFNS as Assessment Tools

In this section we are going to develop methods of using the TFNs and the TpFNs as tools for assessing the performance of groups of individuals participating in certain activities. For this, let us consider a group, say G, of *n* individuals, where *n* is a natural number grater than 1. The performance of each individual is evaluated by assigning to him/her a score within the climax 0-100 and it is characterized as follows:

- A (85 - 100) = Excellent, B (75 - 84) = Very good, C (60 – 74) = Good, D (50 – 59) = Fair and F (0, 49) = Unsatisfactory.(Failed).

The above linguistic characterizations, although they are compatible to the common sense, could not be considered as being uniquely determined, as they depend on the user's personal goals. For example, in a more strict evaluation one could take A (90 -100), B (80 - 89), C (70 – 79), D (60 – 69) = F (59 – 0), etc. In other words, the above characterizations are actually ***fuzzy linguistic labels***.

i) Use of TFNs: We assign to each of the above fuzzy characterizations a TFN denoted, for reasons of simplicity of our notation, with the same letter as follows:

- A (85, 92.5, 100), B (75, 79.5, 84), C (60, 67, 74), D (50, 54.5, 59) and F (49, 24.5, 0).

Observe that the middle entry of each of the above TFNs is equal to the mean value of the other two entries. In other words, if T (*a*, *b*, *c*) is anyone of the above TFNs, then

$$b = \frac{a+c}{2} \quad \textbf{(4)}.$$

In this way a TFN of the form T (*a*, *b*, *c*) corresponds to each person of the group G assessing his/her individual performance. It is logical therefore to consider the ***mean value*** (Definition 9) of all those TFNs, denoted for simplicity by the same letter G, as a means for assessing the group's ***mean performance.***

Consequently, there are two alternative methods for comparing the performance of two different groups, say G_1 and G_2, with respect to a common activity of them:

- If the TFNs G_1 and G_2 are comparable, by applying Definition 3
- In all cases by defuzzifying the TFNs G_1 and G_2 (Proposition 10).

Observe that the GOGs of the graphs of G_1 and G_2 lie in a rectangle with sides of length 100 units on the X-axis (individual scores from 0 to 100) and one unit on the Y-axis (normal fuzzy sets). Consequently, one obtains the following ***assessment criterion***:

Proposition 13: The nearer the x-coordinate of the COG to 100, the better the corresponding group's performance.

Further, note that, since the mean values G_1 and G_2 are obviously linear combinations of the TFNs A, B, C, D, F with non negative rational coefficients, the following Lemma facilitates their defuzzification:

Lemma 14: Let $M(a, b, c) = k_1A+k_2B+k_3C+k_4D+k_5$ be a TFN, with k_i real numbers, i=1, 2, 3, 4, 5. Then the x-coordinates of the COG of the graph of *M* is equal to

$$X(M) = b$$

Proof: If *M* is one of the TFNs A, B, C, D, F, then combining Proposition 10 with equality (4) one finds that $x(M) = \frac{a + \frac{a+c}{2} + c}{3} = \frac{3(a+c)}{6} = b$.

In general, if A (a_1, b_1, c_1), B (a_2, b_2, c_2),…., F(a_5, b_5, c_5) and M(a, b, c), then

$$M = \sum_{i=1}^{5} k_i(a_i, b_i, c_i) = (\sum_{i=1}^{5} k_i a_i, \sum_{i=1}^{5} k_i b_i, \sum_{i=1}^{5} k_i c_i).$$

Therefore,

$$X(M) = \frac{\sum_{i=1}^{5} k_i a_i + \sum_{i=1}^{5} k_i b_i + \sum_{i=1}^{5} k_i c_i}{3}$$

$$= \sum_{i=1}^{5} k_i \frac{a_i + b_i + c_i}{3} = \sum_{i=1}^{5} k_i b_i = b$$

The above Lemma obviously remains true, if we consider TpFNs instead of TFNs.

Remark: An alternative way for defuzzify a TFN *T* = (*a*, *b*, *c*) is to use the ***Yager Index*** *Ya* (T), introduced in [11] in terms of the a-cuts of T, a in [0, 1], in order to help the ordering of fuzzy sets. It can be shown ([12], p. 62) that

$$Ya(T) = \frac{2b+a+c}{4}.$$

Observe now that

$$X(T) = Ya(T) \Leftrightarrow \frac{a+b+c}{3} = \frac{2b+a+c}{4}$$

$$\Leftrightarrow 4(a+b+c) = 3(2b+a+c) \Leftrightarrow a+c = 2b.$$

The last equality is not true in general for $a<b<c$; e.g. take a=1, b=2.5 and c=3. In other words we have in general that

$$X(T) \neq Ya(T).$$

Nevertheless, by (4) the above equality holds for the TFNs. A, B, C, D and F. Therefore, it obviously holds also for any linear combination of those TFNs. Thus, the above two defuzzification techniques provide the same outcomes when used in our assessment method with TFNs described above.

ii) Use of TpFNs: We assign to each member P of the group G a TpFN, denoted for simplicity by the same letter, as follows:

Assume that the performance of P was evaluated by a numerical score, say s, lying in the subinterval $[a_1, b_1]$ of [0, 100]. Consequently a_1 and b_1 are numerical scores assigned to the fuzzy labels, say Q and T, which are equal to one (the same or different) of the fuzzy labels A, B, C, D, F. Then, we choose P to be equal to the TpFN (a, a_1, b_1, b), where a is the lower numerical score assigned to Q and b is the upper numerical score assigned to T. For example, if s lies in the interval [73, 87], then Q = C and T= A, therefore P = (60, 73, 87, 100).

The ***mean value*** S (Definition 9) of all those TpFNs, could be also considered here as a representative measure for assessing the group's mean performance, which, after defuzzifying S with the help either of Proposition 11 or of Corollary 12, can be characterized in terms of the linguistic labels A, B, C, D and F. The defuzzification criterion for TFNs (Proposition 13) obviously holds for TpFNs too.

Note that, if the number *n* of the elements of G is big enough, the use of TpFNs could lead to laborious calculations. Therefore, in such a case it may be better to use the assessment method with TFNs.

On the other hand, an advantage of using the TpFNs is that by defuzzifying each of them one can define a ***total order*** among the ***individual performances*** of the members of G. On the contrary, this cannot be done in case of the TFNs. For example, for two members of G to whom the same TFN was assigned, we don't know who performed better, if their numerical scores are unknown.

4. Examples

The examples presented here illustrate the fuzzy assessment methods developed in the previous section.

Example14: Table 1 depicts in terms of the linguistic grades A, B, C, D and F the student performance of two Departments of the School of Management and Economics of the Graduate Technological Educational Institute (T. E. I.) of Western Greece at their common progress exam of the course "Mathematics for Economists I".

Table 1: Student performance

Grade	D_1	D_2
A	60	60
B	40	90
C	20	45
D	30	45
F	20	15
Total	170	255

Considering the TFNs A, B, C, D, F introduced in the previous section one observes that in Table 1 they actually appear 170 in total TFNs representing the performance of the students of D_1 and 255 TFNs representing the performance of the students of D_2.

Then, calculating the mean values D_1 and D_2 of those TFNs for each Department one finds that

$$D_1=\frac{1}{170}(60A+40B+20C+30D+20F)$$
$$\approx (63.53,71.7,79.95)$$

and

$$D_2=\frac{1}{255}(60A+90B+45C+45D+15F)$$
$$\approx (65.89,72.71,79.$$

Further, by Lemma 14 one finds that $X\,(D_1) \approx 71.74$ and $X\,(D_2) \approx 72.71$, which, according to Proposition 13, shows that both Departments demonstrated a good (C) mean performance, with the performance of D_2 being better.

Remarks: (i) By Proposition 5 one finds that the x-cuts of the TFNs D_1 and D_2 are

$$D_1^x = [63.53+8.21x,\ 79.95.-8.21x]$$

and

$$D_2^x = [65.89+6.82x,\ 79.53-6.82x]$$

respectively. But

$$63.53+8.21x \leq 65.89+6.82x \Leftrightarrow 1.39x \leq 2.36 \Leftrightarrow x \leq 1.67.$$

But the last inequality is true, since x is in [0, 1]. On the contrary,

$$79.95-8.21x \leq 79.53-6.82x \Leftrightarrow 0.42 \leq 1.39x \Leftrightarrow 0.2 \leq x,$$

which is not true for all the values of x in [0, 1].

Therefore, according to Definition 3 the TFNs D_1 and D_2 are not comparable, which means that without defuzzifying them, as we did above, one can not decide which of the two Departments demonstrated the better performance.

(ii) The evaluation of the student mean performance using TFNs, as it has been described in the previous section, it is obtained from the linguistic characterizations of each student's individual performance and no more information is given about the student numerical scores. But, even if we had the necessary information, the application of the method with TpFNs could be difficult in practice, involving in general laborious calculations with a great number of different TpFNs.

Example 15: The performance of the five players of a basket-ball team, who started a game, was individually assessed by six different athletic journalists using a scale from 0 to 100 as follows:
P_1 (player 1)**:** 43, 48, 49, 49, 50, 52,
P_2: 81, 83. 85, 88, 91, 95,
P_3: 76, 82, 89, 95, 95, 98,
P_4: 86, 86, 87, 87, 87, 88 and
P_5: 35, 40, 44, 52, 59, 62.

Here we shall use both methods with TFNs and TpFNs for assessing the mean player performance.

Use of TFNs: Inspecting the given data one observes that the 30 in total scores assigned to the five players by the six journalists correspond to 14 characterizations of excellent (A) performance, to 4 for very good (B), to 1 for good (C), to 4 for fair (D) and to 7 characterizations for unsatisfactory (F) performance. Therefore, the mean player performance can be assessed by calculating the TFN

$$M = \frac{1}{30}(14A + 4B + C + 4D + 7F) \approx (58.33, 68.98, 79.63)$$

But X(M) = 68.98, which shows that the players demonstrated a good (C) mean performance

Use of TpFNs: We assign to each basket-ball player a TpFN as follows: **P_1** = (0, 43, 52, 59), **P_2** = (75, 81, 95, 100), **P_3** = (75, 76, 98, 100), **P_4** = (85, 86, 88, 100) and **P_5 =** (0**,** 35, 62, 74).

We calculate the mean value of the TpFNs **P_i**, i =1, 2, 3, 4, 5, which is equal to

$$P = \frac{1}{5}\sum_{i=1}^{5} P_i = (47, 64.2, 79, 86.6).$$

Applying Proposition 11 one finds that

$$X(P) = \frac{79^2 + (86.6)^2 - (64.2)^2 - 47^2 + 79*86.6 - 47*(64.2)}{3(79+86.6-47-64.2)} \approx 68.84$$

which shows that the players demonstrated a good (C) mean performance.

Alternatively, applying Corollary 12 one finds that

$$x(p) = \frac{2(47+86.6)+7(64.2+79)}{18} \approx 70.53,$$

showing again that the five players demonstrated a good mean performance.

Remark: Defuziffying the TpFNs **P_i**, i =1, 2, 3, 4, 5, as we did above for P, one defines a total order among the individual performances of the five players. On the contrary, this cannot be done when using TFNs instead of TpFNs.

5. Conclusion

The COG defuzzification technique was combined in this paper with the use of TFNs / TpFNs to develop two fuzzy methods for assessing human skills. The second method with TpFNs, although, in contrast to the first method with TFNs, is not always applicable in practice, it has the advantage that through it one can define a total order among all the individual performances of the members of a group.

Both methods have a general character, which means that they can be applied for the assessment of many other human activities, apart from student and basket-ball player assessment done here, as well as for several machine activities (e.g. CBR or decision making with the help of computers, etc.).

This is actually our main proposal for further future research on the subject

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