Two stages nonlinear systems identification

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Abstract: - The problem of identifying nonlinear systems is proposed in the presence of hard nonlinearity. The nonlinear systems considered in this paper is structured by Hammerstein systems. The identification is studied in presence of possibly infinite-order linear dynamics and static nonlinearity. Hammerstein models consist of a series connection including a nonlinear element and a linear subsystem. The Identification problem is addressed in the presence of hard nonlinearity.

Key-Words: - **N**onlinear systems, Hammerstein model, Hard nonlinearity, identification problem, frequency approach.

1 Introduction

The problem of system identification based on different variants of the Hammerstein model has been given a great deal of interest, especially on the last decade, and several solutions are now available.
The Hammerstein model is a series connection of a memoryless nonlinearity and a linear dynamic bloc (Fig.1). Black-box nonlinear system identification is a very wide research area [1]. The considerable diversity of nonlinear systems has led to a large variety of identification problems and a proliferation of identification approaches and methods.
In this paper, the problem of identifying Hammerstein systems is addressed. Hard nonlinearities of known type have been considered in [2]-[4].
Unlike many previous works e.g. [3], the model structure of the linear subsystem is entirely unknown. Furthermore, the system nonlinearity is of arbitrary-shape and can be noninvertible. In most previous works devoted to Hammerstein system identification, the nonlinear element is supposed to be continuous. Moreover, this latter is generally assumed to be a (truncated) polynomial or Fourier series in the variable e.g. [5].
Multi-stage methods, involving two or several stages, have been proposed in most previous works. Then, their consistency was ensured if the inputs are Gaussian and the nonlinearity is odd. Deterministic parameter identification methods consist in reformulating the problem as an optimisation task, generally coped with using various relaxation techniques. Then, local convergence properties ensured in presence of PE inputs.
In this paper, the problem of identifying Hammerstein systems is addressed, for simplicity, in the continuous-time. Unlike many previous works, the model structure of the linear subsystem is entirely unknown. Furthermore, the system nonlinearity is of hard (Figs. 2a-b) type and is not required to be invertible.
The present strategy is allowed to interest a wide range of the system nonlinearity (Figs. 2a-b). The identification problem amounts to determining an accurate estimate of the (nonparametric) frequency response $G(j\omega)$, for a set of frequencies $(\omega_1 \ldots \omega_m)$, and the nonlinearity. The present identification method is a two-stage: the system nonlinearity is identified first, using simple constant inputs, and based upon in the second stage to identify the linear subsystem.
In frequency methods, the linear subsystem frequency response and the nonlinearity map are determined in two or several stages [4].
The paper is organized as follows: the identification problem is formulated in Section 2; the nonlinear operator identification is coped with in Section 3; the linear subsystem frequency response determination is investigated in Section 4.

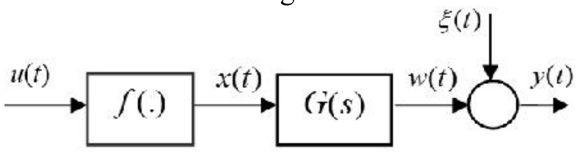

Fig.1. Hammerstein model structure

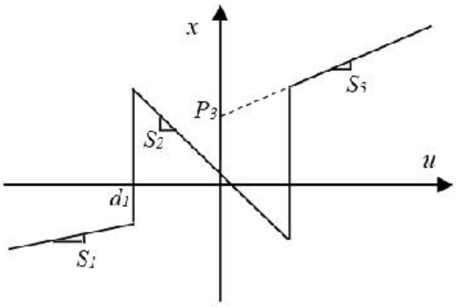

Fig.2a. Hard nonlinearity

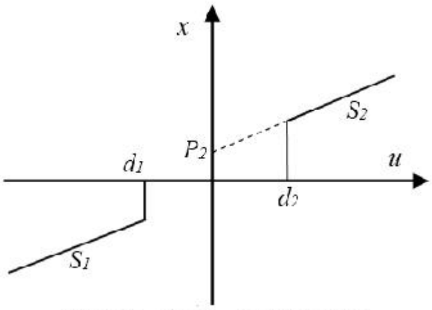

Fig. 2b. Hard nonlinearity with preload and dead zone

2 Problem Formulation

Standard Hammerstein systems consist of a memoryless nonlinear element $f(.)$ followed in series by a linear dynamic subsystem $G(s)$ (Fig.1). This model is analytically described by the following equations:

$$x(t) = f\left(u(t)\right) \tag{1}$$

$$w(t) = g(t) * x(t) \tag{2}$$

$$y(t) = w(t) + \xi(t) = g(t) * x(t) + \xi(t) \tag{3}$$

where $g(t) = L^{-1}\left(G(s)\right)$ is the inverse Laplace transform of $G(s)$; the symbol * refers to the convolution operation; $x(t)$ is the internal signal; $w(t)$ is the undisturbed output.

The only measurable signals are the system input $u(t)$ and output $y(t)$. The equation error $\xi(t)$ is a zero-mean stationary sequence of independent random variables; it accounts for external noise, it is supposed to be ergodic (so that arithmetic averages can be substituted to probabilistic means whenever this is necessary).

Because the system identification is carried out in open loop (Fig.1), the linear block $G(s)$ must satisfy the stability asymptotically. Except for this assumption, the linear subsystem is arbitrary, particularly it may be arbitrary and unknown structure. The system nonlinearity $f(.)$ is of arbitrary-shape and is allowed to be noninvertible.

We aim at designing an identification scheme that is able to provide a model estimate $\left(\hat{f}(.), \hat{G}(j\omega_k)\right)$ that represents well the system when. Since $w(t)$ and $x(t)$ are not measurable, the system identification should be fully based upon measurements of the input $v(t)$ and the output system $y(t)$. Therefore, the considered identification problem does not have a unique solution: if the model $\left(f(u), G(s)\right)$ represents a solution then, any model of the form $\left(f(u)/k, kG(s)\right)$ is also a solution (where k is any nonzero real). This naturally leads to the question: what particular model should we focus on? This question will be answered later. Such a lack of uniqueness, will be exploited (in Section 3) to cope with the uncertainty on the amplitude of the internal signals $x(t)$ and $w(t)$.

3 Nonlinearity System Identification

In this section, we aim establish an identification scheme that is able to provide an accurately estimate of the system nonlinearity $f(.)$. In Section 2 it was shown that, if k is any nonzero real, so any model of the form $\left(f(u)/k, kG(s)\right)$ is representative of the system.

Therefore, without reducing the problem generality, one can assume $G(0) = 1$. Accordingly, the system to be identified is described by the transfer function:

$$\overline{G}(s) = \frac{G(s)}{G(0)} \tag{4a}$$

and the nonlinearity:

$$\overline{f}(u) = G(0) f(u) \tag{4b}$$

It is readily seen that, this model to be identified (i.e. $\overline{f}(.)$ and $\overline{G}(s)$) is the only that satisfies the property: $\overline{G}(0) = 1$ (i.e. the linear subsystem is of unit static gain). To avoid multiplication of notations, the model of interest will still be denoted $\left(f(.), G(s)\right)$ but the description (1)-(3) is completed with the property $G(0) = 1$.

Then, the considered system is excited by simple constant inputs:

$$u(t) = U_j \quad \text{for} \quad j = 1 \ldots n \tag{5}$$

where the number n is arbitrarily chosen by the user, preferably sufficiently large. On the other hand, the internal signal $x(t)$ is constant (i.e. $x(t) \xrightarrow[t\to\infty]{} X_j$). Then, it follows from (1), (4a-b) and (5) that, in the steady-state:

$$X_j = f(U_j) \quad \text{for} \quad j = 1\ldots n \tag{6}$$

Then, it is readily seen using the assumption of asymptotic stability of the linear subsystem, (2), (4a-b) and (6), that the undisturbed output is also constant, in the steady-state, i.e. $w(t) \xrightarrow[t\to\infty]{} W_j$. This latter can be expressed as:

$$W_j = f(U_j) \quad \text{for} \quad j = 1\ldots n \tag{7}$$

Then, the system output is constant up to noise (in the steady-state). Finally, we can conclude using (7) that W_j (for $j = 1\ldots n$) only depends on the system nonlinearity $f(.)$ and the input signal. Therefore, using the fact $\xi(t)$ is zero-mean, it follows from (2) and (7) that, the estimate of the steady-state undisturbed output W_j ($j = 1\ldots n$) can be easily recovered using the following estimator:

$$\hat{W}_j(M) = \frac{1}{M}\sum_{l=1}^{M} y(l) \quad \text{for} \quad j = 1\ldots n \tag{8}$$

where M is any sufficiently large integer. Specifically, W_j can be recovered by averaging $y(t)$ on a sufficiently large interval. Then, a set of points $\left(U_j, f(U_j)\right) = \left(U_j, W_j\right)$ (with $j = 1\ldots n$) belonging to nonlinearity $f(.)$ can be determined. Finally, an accurate estimate $\hat{f}_M(.)$ of $f(.)$ can be easily obtained.

4 Linear Subsystem Identification

The aim in this section is to establish an estimator of the linear subsystem. The complex gain $G(j\omega)$ is characterized by the modulus gain $|G(j\omega)|$ and the phase $\varphi(\omega) = \angle G(j\omega) = \arg\left(G(j\omega)\right)$.

The identification problem under study is dealt using a method based on the frequency approach. Firstly, by successively connecting the estimated points $\left\{\left(U_j, \hat{f}_M(U_j)\right); j = 1\cdots n\right\}$, a set of segments of $f(.)$ is obtained (Figs.2a-b). For reasons of identifiability, at least one segment must have a non-zero slope. Let q designates any segment have a non-zero slope. Then, the nonlinear system described in section 1 is excited with a given sine input:

$$u(t) = u_0 + U\sin(\omega t) \tag{9}$$

where the frequency $\omega > 0$ is kept constant, u_0 is the offset and U the amplitude of sine signal. The choice of couple of parameters (u_o, U) will be made after.

Note that, the internal signal $x(t)$ is periodic with period $2\pi/\omega$.

On the other hand, it is supposed first that the parameters (u_o, U) in (9) are chosen such that $u(t)$ spans only one segment of $f(.)$ having a nonzero slope.

Further, if $u(t)$ spans only the segment q, the following result can be easily obtained:

$$x(t) = S\,u(t) + P = Su_0 + SU\sin(\omega t) + P \tag{10}$$

where (S, P) is the couple parameters of segment q (Figs.2a-b). Precisely, S is the slope of segment q and P is the value of $x(t)$ when $u(t) = 0$. S and P can be determined using the experimental data of nonlinearity estimator.

Accordingly, one immediately gets from (2) and (10) that, the undisturbed output $w(t)$ is also sine signal (in the steady-state) and can be expressed as:

$$w(t) = Su_0 + P + SU\left|G(j\omega)\right|\sin\left(\omega t + \varphi(\omega)\right) \tag{11}$$

Finally, one immediately gets from (3) and (11) that:

$$y(t) = Su_0 + P + SU\left|G(j\omega)\right|\sin\left(\omega t + \varphi(\omega)\right) + \xi(t) \tag{12}$$

This result shows that, the system output in the steady-state is sine signal up to noise.

On the other hand, a judicious choice of the couple $(u_o,\ U)$ in (9) can be performed using the experimental data of nonlinearity estimator, established in section 3. Based upon the curve of the nonlinearity $f(.)$, let choose any segment q of $f(.)$ with nonzero slope. Then, the identified system is submitted to the sine input (9) within the selected segment. The choice of the amplitude U and the offset u_0 in (9) is done in such a way that, $u(t)$ spans only one segment.

If this condition is satisfied, then the system output $y(t)$ is a sine signal (up to noise) after a transient period. The amplitude U and u_0 can be adjusted until

the output becomes sine signal (up to noise).
Then, the choice of U and u_0 in (9) can be performed using practical experiences.
These results imply that, the complex frequency gains of the linear subsystem (i.e. the modulus gain $|G(j\omega)|$ and the phase $\varphi(\omega)$) can be recovered if the undisturbed output $w(t)$ is accessible to measurement. At this stage, $w(t)$ is not measurable. Fortunately, an accurate estimator of $w(t)$ exists, thanks to the (steady-state) periodicity of $w(t)$ and the ergodicity of the noise $\xi(t)$. The estimator, denoted $\hat{w}_N(t)$, is obtained by performing a T-periodic averaging with $T = 2\pi/\omega$ [6]:

$$\hat{w}_N(t) = \frac{1}{N}\sum_{k=1}^{N} y(t + k\frac{2\pi}{\omega}) \quad \text{for} \quad t \in \left[0 \;\; \frac{2\pi}{\omega}\right) \qquad (13a)$$

$$\hat{w}_N(t + k\frac{2\pi}{\omega}) = \hat{w}_N(t) \quad \text{for} \quad k = 1, 2, \ldots \qquad (13b)$$

where N is any sufficiently large integer. The estimator (13a-b) is uniformly consistent i.e. $\hat{w}_N(t)$ converges (w.p.1 as $N \to \infty$) to $w_{U,\omega}(t)$, whatever t.
Finally, using the amplitude and the argument of the sine signal (11), the gain modulus and the phase of the linear subsystem can be easily obtained using (11) and (13a-b).

5 Conclusion

The problem of system identification is addressed for Hammerstein systems where the system nonlinearity is of arbitrary-shape hard type and can be noninvertible. The considered linear subsystem is not necessarily parametric and can be of infinite order.
The identification scheme is performed using two separate stages. First, the system nonlinearity is identified using simple constant inputs. In the second stage, the linear subsystem is determined with sine inputs.
To the author knowledge, unlike many of previous study, the present method has solved the identification problem for a large class of Hammerstein systems. Furthermore, the proposed approach involves easily generated excitation signals.

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