Dominant-Dynamics-Based Reduced Order Modeling Using Genetic Algorithm

ADNAN M. AL-SMADI[1], OTHMAN M. AL-SMADI[2], AND MAIS W. MARJI[1]
Yarmouk University[1], Hijjawi Faculty for Engineering Technology, Irbid, Jordan
University of Jordan[2], Department of Electrical Engineering, Amman, Jordan
Smadi98@yahoo.com

Abstract: - Model Order Reduction (MOR) plays a very essential role in reducing the complexity of the models, such as communication systems, transmission lines and most physical systems that impose major difficulties in analysis, simulation and control designs. MOR was developed in control theory areas that study the performance of dynamical systems in order to reduce their complexity. The objective of this paper is to obtain reduced model that is approximated to the original complex high order model with optimal solution and less complexity and with maintaining the same behavior of the original model. This is done using Genetic Algorithm (GA). MATLAB software will be utilized for simulations and testing the achieved results.

Key-Words: - Model Order Reduction; Genetic Algorithm; Eigenvalues; Transfer Function.

1 Introduction

Modeling of linear dynamical systems is encountered in many fields including financial markets, environmental sciences, control engineering, and many other fields. Mathematically modeling for a real system in the area of Engineering, a high order model of the system under consideration is obtained from theoretical concepts. The primary goal of modeling of physical and real-life problems is for the purpose either controlling the process or performing future forecasts. The modeling process can be achieved depending on the complete understanding of the physical process that leads to the derivation of the governing differential equations describing the process. In this case, the model might be fully known in terms of the order and the parameters or might be partially known where some or all of the parameters are unknown [1].

In 1966, Model Order Reduction (MOR) started when Davison [8][9] presented "The Model Analysis" approach using state space techniques. Then several modifications had offered to Davison's approach by Chidambara [10][12]. Later on, Chen and Shieh [13] started to add their imprints using frequency domain expansions. Gibilaro and Lees [14] matched the moments of the impulse response. Then, Hutton and Friedland [15] used the Routh approach for high frequency approximation that was modified by Langholz and Feinmesser [16]. Later on, Pinguet [17] showed that all those methods have state space reformulations.

The classical approach to model order reduction dealt only with eigenvalues[17]. However, Moore [18] presented a revolutionary way of looking at model reduction by showing that the ideal platform to work from is that when all states are as controllable as they are observable. This gave birth to "Balanced Model Reduction", that the concept of dominance is no longer associated with eigenvalues, but rather with the degree of observability and controllability of a given state.

El-Attar and Vidyasagar [19] presented new procedures for model order reduction based on interpreting the system impulse response (transfer function) as an input-output map. Hakvoort [20] noted that in L1 robust control design and model uncertainty can be handled if an upper-bound on the L1 Norm of the model error is known. Hakvoort presented a new L1 Norm optimal reduction approach resulting in a nominal model with minimal upper-bound on the L1 Norm of the error [20].

MOR has been an active research area in design automation over the past two decades. In recent years, MOR has come to be viewed as a method for generating compact models from all sorts of physical systems modeling tools [2][5]. For example, in integrated Circuits (ICs), where increasing package density forces developers to include side effects. Knowing that these devices are often modeled by very large RLC circuits, this would be too demanding computationally and practically due to the detailed modeling of the original system [8]. In control system, in order to obtain an acceptable model of the physical system, a designer does not usually consider all the dynamics of the system [6–10]. The MOR problem has been investigated in the literature extensively [2][3][4][6][9].

Many of searches and optimization methods that initiate optimal solutions are getting more popular. A Genetic Algorithm (GA) is one of these methods and it is used in the application of order reduction of linear system. The purpose of using GA is because of the property of finding a global solution without giving any initial approximation to unknown parameters. GA differs from any other optimization methods. Also, a Genetic Algorithm based on the objective function and corresponding fitness levels used to find the optimal solution. GA operates on a population of potential solutions applying the principle of survival of the fittest function to produce better approximations to a solution.

2 Model Reduction

Many Linear Time Invariant (LTI) systems have fast and slow dynamics, which are referred to as singularly perturbed systems [4][5]. This can be illustrated by considering the following nth order LTI system:

$$\dot{x} = \boldsymbol{A}x+\boldsymbol{B}u$$
$$y = \boldsymbol{C}x+\boldsymbol{D}u \qquad (1)$$

where x is an $n \times 1$ vector, u is a $p \times 1$ vector and y is an $m \times 1$ vector, **A**, **B**, **C,** and **D** are matrices with appropriate dimensions. The aim of the reduction is to obtain an n_r^{th} order reduced model ($n_r^{th} < n$) which can mimic the behavior of the original full model order. Then the n_r^{th} order LTI system is considered by the following:

$$\dot{x}_r = \boldsymbol{A}_r x_r+\boldsymbol{B}_r u$$
$$y_r = \boldsymbol{C}_r x_r+\boldsymbol{D}_r u \qquad (2)$$

The reduced system $G_r(s)$ is obtained by applying Genetic Algorithm with an optimal and approximated solution to the original high order system G(s) and the responses for two system is as small as possible.

$$\mathbf{E(s) = y(s) - y_r(s)} \qquad (3)$$

where E(s) is the error between the original response and the reduced response as shown in Fig(1).

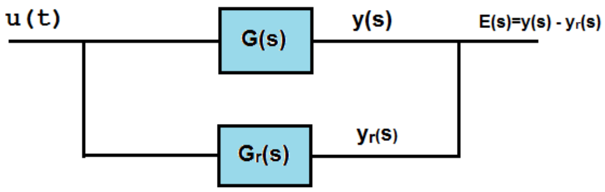

Fig (1): MSE block diagram

The aim of this paper is to obtain reduced model that is approximated to the original complex high order model with optimal solution and less complexity and with maintaining the same behavior of the original model using Genetic Algorithm (GA). GA has been applied to solve several engineering problems that are complex and not easy to solve by conventional optimization methods. GA maintains a population of solutions and implements a survival of the fittest strategy in their search for better solutions. GA Implementation requires the specifications of six basic issues:

(1) Chromosome representation.
(2) Selection function.
(3) The genetic operators.
(4) Initialization.
(5) Termination.
(6) Evaluation function.

The basic mechanism of the GA is provided by the genetic operators. There are two fundamental types of operators (1) crossover (2) mutation. These operators are used to generate new solutions based on existing solutions in the population. The GA moves from generation to generation until a stopping criterion is met. The stopping criterion could be maximum number of generations, population convergence criteria, lack of improvement in the best solution over a specified number of generations or target value for the objective function. Evaluation functions or objective functions of many forms can be used in a GA so that the function can map the population into a partially ordered set [11]. The procedure of the algorithm is as follows:

1. Identifying Genetic Algorithm and how it is capable of obtaining a reduced model with optimal solutions for complex systems with substructure preservation.
2. Calculating the fitness of the system using L1 Norm.

3. Identifying how GA can help to simplify and automate the model reduction.
4. Analyzing the resultant reduced system for both SISO and MIMO in terms of:
    a. System stability
    b. Substructure preservation
    c. Steady state.
5. Optimizing a reduced order controller.
6. Verifying suitability of the controlled system performance.

3 Example

Let us consider the system described by the 4th order transfer function [6][7]

$$G(s) = \frac{S^3+7s^2+24s+24}{(s+1)(s+2)(s+3)(s+4)}$$

$$= \frac{S^3+7S^2+24s+24}{S^4+10S^3+35S^2+50s+24}$$

The state space model represented as follows:

$$A = \begin{bmatrix} -10 & -35 & -50 & -24 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$B = [1 \quad 0 \quad 0 \quad 0]^T$

$C = [1 \quad 7 \quad 24 \quad 24]$

D= 0

with eigenvalues given as {-1,-2,-3,-4} as seen in Fig (2).Hence, the model is of 4th order.

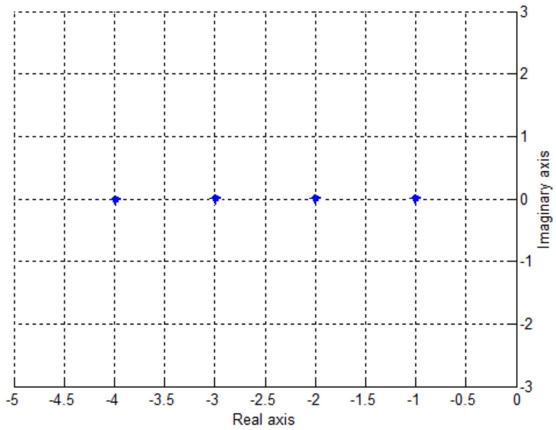

Fig 2: Eigenvalues for the 4th order system

First of all, the reduced order model is chosen according to the dominant eigenvalues of the original system. Here, the dominant eigenvalues is chosen to be {-1,-2} in order to obtain smaller error between the reduced system and the original system outputs and assuming that the reduced matrix D_r is {0}. Therefore the reduced state matrix A_r has the eigenvalues that placed according to their contributions to the system behavior.

The reduced matrix A_r is given by:

A_r = [-1 0 ; 0 -2]

Reduced matrix B_r and reduced matrix C_r will be found using GA taking into account that the reduced system should have a performance approximated to the original one as shown in Fig (3). The GA parameters for the Example are shown in Table 1.

Table 1: GA parameters for the Example

Parameter	Value
Population size	700
Fraction of Crossover	0.8
Fraction of Migration	0.2
Number of best individuals to survive to next generation	30
Maximum tolerable error or norm	1e -6
Maximum Number of Generations	200

$B_r = [1.0870 \quad -0.6323]^T$

$C_r = [1.1262 \quad 0.7038]$

$D_r = [0]$

The system is reduced to 2nd order and the transfer function considered as follows:

$$G_r(s) = \frac{0.7791s + 2.003}{S^2+3s+2}$$

Where the steady state Error is: 0.0252100 and the L1-Norm of Reduced Model is: 1.1863320.

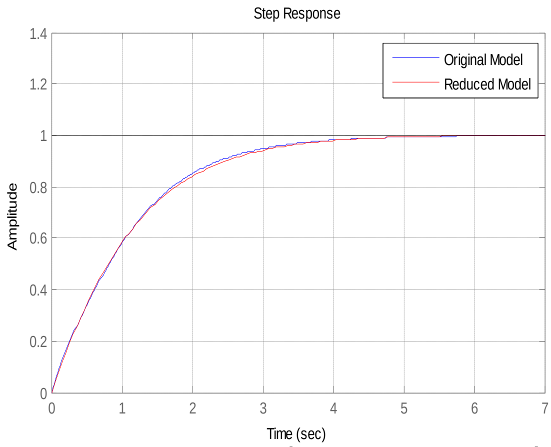

Fig 3: step response for 4th order model and 2nd reduced order

Simulation results are given in Table 2.

Table 2: Simulation results for the Example

System	Steady state value
Original system	1.0000
Reduced system	1.0015
SSE	0.0015

Comparisons of Order reduction of linear systems using an error minimization technique [7] and Clustering Method for Reducing Order of Linear System using Pade Approximation [6] are shown in Table 2.

Table 2: Comparison of methods

Method	Reduced transfer function	SP	RMSE
Genetic Algorithm	$G_r(s) = \frac{0.7791s+2.003}{S^2+3s+2}$	Achieved	2.3713×10^{-4}
Error Minimization Technique	$G_r(s) = \frac{0.8000003s+2}{s^2+3s+2}$	Not achieved	2.3804×10^{-4}
Pade Approximation	$G_r(s) = \frac{-0.1897625s+4.5713}{s^2+4.76185s+4.5713}$	Not achieved	5.12×10^{-2}

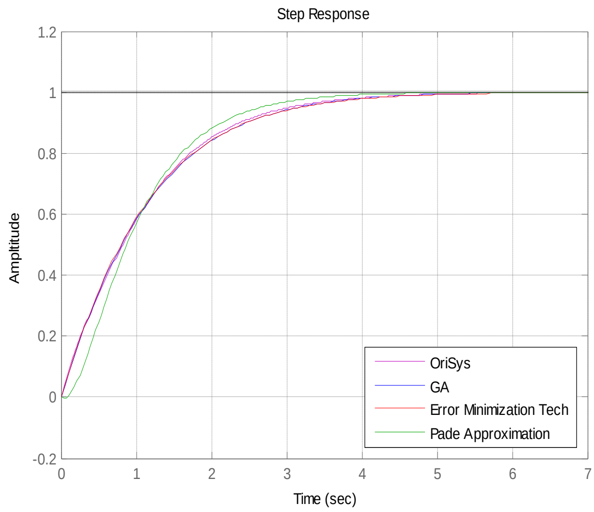

Fig 4: Comparison of step responses

4 Conclusion

In this paper, Genetic Algorithm was used to obtain a reduced model with less complexity comparing with high order original model that has approximated response to the original response. Results are represented in this paper using GA optimization based on singular perturbation approximation which is a well known MOR method. GA differs from normal optimization and search procedures in working with a coding of the parameter set, searching from a population of points, not a single point, and using probabilistic transition rules, not deterministic rules.

References:

[1] Z. Abo-Hammour, O. Alsmadi, S. Momani, and O. Abu Arqub, “A genetic algorithm approach for prediction of linear dynamical systems,” *Mathematical Problems in Engineering*, Vol. 2013, Article ID 831657.

[2] Michele B, Fernando C, Marco G. Phase model reduction and synchronization of periodically forced nonlinear oscillators. *Journal of Circuits, Systems, and Computers*, Vol. 19, No. 4, 2010, pp.749–762.

[3] Zeng X, Yang F, Su Y, Cai W. NHAR: a non-homogeneous Arnoldi method for fast simulation of RCL circuits with a large number of ports. *International Journal of Circuit Theory and Applications* 2009; DOI: 10.1002/CTA.611. Published on line.

[4] Kokotovic P, Khalil H, O’Reilly J. Singular Perturbation Methods in Control: Analysis and Design. Academic: New York, 1986.

[5] Fridman E. Robust sampled-data H∞ control of linear singularly perturbed systems. *IEEE Transactions on Automatic Contro*l, Vol. 52, No. 3, 2006, pp. 470–475.

[6] C. B. Vishwakarma and R.Prasad, “Clustering Method for Reducing Order of Linear System using Pade Approximation*”, IETE Journal of Research*, Vol. 54, Issue 5, 2008, pp. 326-330.

[7] S.Mukherjee, and R.N.Mishra, Order reduction of linear systems using an error minimization technique, Journal of Franklin Inst. Vol. 323, No. 1, pp. 23-32, 1987.

[8] Davison, E.J. Further Reply by E.J. Davison. *IEEE Transaction on Automatic Control*. AC-12, 800, 1967.

[9] Davison, E.J., A Method for Simplifying Linear Dynamic Systems. *IEEE Transaction on Automatic Control*, AC-11, 93-101, 1966.

[10] Chidambara, M.R., Further Comments by M.R. Chidambara, *IEEE Transaction on Automatic Control*, AC-12, 1967, pp. 799-800.

[11] S. Panda, J. S. Yadav, N. P. Patidar and C. Ardil. Evolutionary Techniques for Model Order Reduction of Large Scale Linear Systems. *International Journal of Electrical, Computer, Electronics and Communication Engineering*, Vol. 6, No. 9, 2012.

[12] Chidambara, M.R. (1969), Two Simple Techniques for the Simplification of Large Dynamic Systems. *Proceedings of the JACC*, 1969, pp. 669-674.

[13] Chen, C.F. and Shieh, L.S., A Novel Approach to Linear Model Simplification, *International Journal on Control*, Vol. 8, 1968, pp. 561-570.

[14] Gibilaro, L.G. and Lees, F.P. The Reduction of Complex Transfer Function Models to Simple Models Using the Method of Moments. *Chemical Engineering Science*, Vol. 24, Issue 1, 1969, pp. 85-93.

[15] Hutton, M. and Friedland, B., Routh Approximations for Reducing Order of Linear, Time-Invariant Systems. *IEEE Transaction on Automatic Control*, AC-20 (3): 329-337, 1975.

[16] Langholz, G. and Feinmesser, D., Model Reduction by Routh Approximations. *Int. J. Systems Sci*., Vol. 9, No. 5, 1978, pp. 493-496.

[17] Pinguet, P.J.M., State Space Formulation of a Class of Model Reduction Methods. Master Thesis, University of Cambridge, September 1978.

[18] Moore, B.C., Principal Component Analysis in Linear Systems: Controllability, Observability and Model Reduction. *IEEE Transaction on Automatic Control*, AC-26, 17-32, 1981

[19] El-Attar, R.A. and Vidyasagar, M., Order Reduction by l1 and l∞-Norm Minimization, *IEEE Transaction on Automatic Control*, AC-Vol. 23, No. 4, 1978, pp. 731-734.

[20] Hakvoort, R.G., Worst-Case System Identification in L1: Error Bounds, Optimal Modelsvand Model Reduction. Proceedings of the *31st Conference on Decision and Control*, pp. 499-504, 1992.