Process Control application of an alternative Two degrees of freedom data-driven VRFT controller

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Abstract: In this paper, the Virtual Reference Feedback Tuning (VRFT) is applied in a Two Degrees of Freedom Structure with feedforward path. It is shown that this framework can be applied while preserving the decoupling between the reference tracking response and the disturbance rejection, which is a characteristic of this controller structure, while the design task for both controllers is kept independent. It is applied to a pH neutralization plant in a bench scale for both simulation and in situ.

Key–Words: VRFT, Data-driven control, pH control

1 Introduction

Control based on pure data from the plant has become an active area of research nowadays. There are several examples of control in which the modeling step is skipped and a controller is found directly from data. The Virtual Reference Feedback Tuning (VRFT) ([1, 2, 3, 4, 5]) translates the model reference control problem into an identification problem, the controller being the transfer function to identify. This identification is based on some "virtual signals" computed from a batch of data taken directly from an open-loop experiment. These methodologies are good examples of this new trend in control that attempts to find the controller by skipping the modeling step and instead, based on an optimization problem, tries to find the right parameters for restricted order controllers.

These control strategies are, or can be set, into a two degrees of freedom (2DoF) topology, allowing the tackling of both the reference tracking and the disturbance rejection. But since we are dealing with reduced order controllers, a compromise between these two objectives inevitably arises. In this paper, a two degrees of freedom topology with feedforward action is studied. Theoretically (with no constraints on the controller's structure), this topology is able to totally decouple these two responses and the design task associated with them. To tune the restricted order controller, what is shown here is that the VRFT constitutes a suitable method to be extended to this topology, while the computational effort is similar to the original VRFT.

This methodology was applied to a pH neutralization bench plant, using real data taken from an open loop experiment. The pH neutralization is an important process in different industry applications (for example in pharmaceuticals, as pointed out in [6]) and fundamental in the prevention of corrosion, in the protection of ecological wild life and human welfare in, for example, waste water treatment plants and water recycling, as pointed out in [7]. However, it is already well known that the strong non-linearity of this neutralization is one of the points that makes this process a difficult task to control ([8]) and, at the same time, it is one of the reasons why this process has been so widely studied.

The aim of using the VRFT is to bypass the modeling of this highly non-linear pH plant to find a restricted order linear controller, based only on data taken from the process itself using a Two Degrees of Freedom Controller (2DoF) with feedforward action in such a way that the design of the feedback controller is totally independent of the feedforward controller. The VRFT is a straightforward, easy to implement methodology that was also found to be very flexible when trying to extend the original controller topology. Since only a simple batch of data is needed, the number of experiments on the real plant can be reduced and one set of data can be used to compute different control structures.

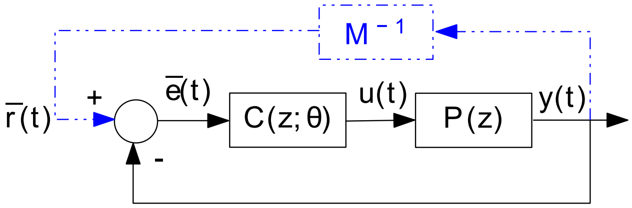


Figure 1: The VRFT set up. The dashed lines represent the "virtual" part of the method. The line-dot represents the target response. The virtual signal $\bar{r}(t)$ is used to identify the controller that would yield $u(t)$ when the input to the controller is $\bar{e}(t) = \bar{r}(t) - y(t)$

2 Virtual Reference Feedback Tuning

The Virtual Reference Feedback Tuning (VRFT) is a one-shot data-based method for the design of feedback controllers. The original idea was presented in [9], and then formalized by Lecchini, Campi, Savaresi and Guardabassi (see [1, 2, 3]). In this section, an outline of the method is presented.
In [1], the method is presented for the tuning of a feedback controller. The control objective is to minimize the model-reference criterion given by:

$$J_{MR}(\theta) = \left\| \left(\frac{P(z)C(z;\theta)}{1+P(z)C(z;\theta)} - M(z) \right) W(z) \right\|_2^2 \quad (1)$$

Where the controller $C(z;\theta)$ belongs to the controller class $\{C(z;\theta)\}$, given by $C(z;\theta) = \beta^T(z)\theta$, where $\beta(z) = [\beta_1(z)\ \beta_2(z) \cdots \beta_n(z)]^T$ is a known vector of transfer functions, and $\theta = [\theta_1\ \theta_2\ \cdots \theta_n]^T$ is the vector of parameters; $M(z)$ represents the target complementary function and $W(z)$ is a weighting factor.
The main idea of the method is that, given a set of input-output data from the plant operating in open-loop (i.e. $u(t)$ and $y(t)$ respectively), the designer should be able to minimize (1), without a model of the plant. This can be achieved by creating a "virtual" signal constructed from the open-loop data. If the real output of the open-loop $y(t)$ had been taken in closed-loop and the reference-to-output transfer function were $M(z)$, a "virtual reference" signal $\bar{r}(t)$ could be found that, if applied to the closed loop system, would yield $y(t)$ as the output. If that is the case, the output of the controller should be equal to $u(t)$ and then, this controller can be found by *identifying* the transfer function which yields the output $u(t)$ when the input $\bar{e}(t) = \bar{r}(t) - y(t)$ is applied to the controller's input as depicted in Fig. 1. In short, the original 1DoF VRFT algorithm, as presented by the authors in [1], is given as follows: Given a set of measured I/O data $\{u(t), y(t)\}_{t=1,\dots,N}$:

1. Calculate:
   - a virtual reference $\bar{r}(t)$ where $y(t) = M(z)\bar{r}(t)$, and
   - the corresponding tracking error $e(t) = \bar{r}(t) - y(t)$
2. Filter the signals $e(t)$ and $u(t)$ with a suitable filter $L(z)$:

$$e_L(t) = L(z)e(t)$$
$$u_L(t) = L(z)u(t)$$

3. Select the controller parameter vector, say, $\hat{\theta}_N$, that minimizes the following criterion:

$$J_{VR}^N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\left(u_L(t) - C(z;\theta)e_L(t)\right)^2 \quad (2)$$

If $C(z;\theta) = \beta^T(z)\theta$, the criterion (2) can be given by:

$$J_{VR}^N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\left(u_L(t) - \varphi_L^T(t)\theta\right)^2 \quad (3)$$

with $\varphi_L(t) = \beta(z)e_L(t)$ and the parameter vector $\hat{\theta}_N$ is given by

$$\hat{\theta}_N = \left[\sum_{t=1}^{N}\varphi_L(t)\varphi_L(t)^T\right]^{-1}\sum_{t=1}^{N}\varphi_L(t)u_L(t) \quad (4)$$

The authors, also showed that the filter $L(z)$ should be the one that approximates the identification criterion (2) to the control criterion (1). These filters turn out to be:

$$|L|^2 = |1-M|^2\,|M|\,|W|^2\,\frac{1}{\Phi_u} \quad (5)$$

where Φ_u is the spectral density of $u(t)$.
In the case of 2DoF, the design methodology is presented in [2] and is similar to the one degree of freedom case. The control structure is presented in Fig. 2 with the virtual signals included. The control criterion for this case incorporates the sensitivity function shaping:

$$J_{MR}(\theta_r,\theta_y) = \|(\Psi_M(z;[\theta_r,\theta_y]) - M(z))W_M(z)\|_2^2 + \|(\Psi_S(z;\theta_y) - S(z))W_s(z)\|_2^2 \quad (6)$$

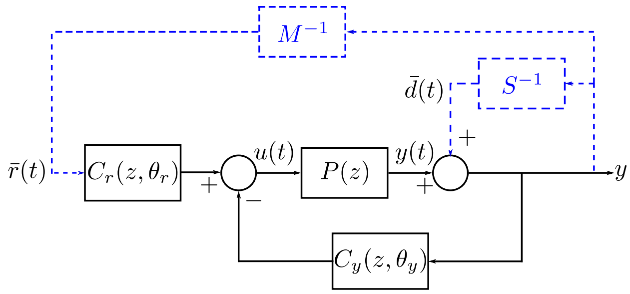


Figure 2: Two degrees of freedom structure

with

$$\Psi_M(z;[\theta_r,\theta_y]) = \frac{P(z)C_r(z;\theta_r)}{1+P(z)C_y(z;\theta_y)}$$
$$\Psi_S(z;\theta_y) = \frac{1}{1+P(z)C_y(z;\theta_y)}$$

and M as before being the target input-to-output transfer function and S the target sensitivity function. The virtual reference is computed as in the one degree of freedom controller. The new virtual signals are the "virtual perturbation" $\bar{d}(t)$ and the virtual perturbed output that are computed as:

$$\begin{aligned} y(t)+\bar{d}(t) &= S(z)\bar{d}(t) \\ \bar{y}(t) &= y(t)+\bar{d}(t) \end{aligned} \tag{7}$$

On the basis of these signals, the controller's parameters are found by minimizing the following alternative identification cost function:

$$\begin{aligned} J_{VR}^N(\theta_r,\theta_y) = &\frac{1}{N}\sum_{t=1}^{N}\left[\Gamma_M(t;[\theta_r,\theta_y])\right]^2 \\ &+\frac{1}{N}\sum_{t=1}^{N}\left[\Gamma_S(t;[\theta_r,\theta_y])\right]^2 \end{aligned} \tag{8}$$

where,

$$\begin{aligned} \Gamma_M(t;[\theta_r,\theta_y]) &= L_M(z)(u(t)-C_r(z;\theta_r)\bar{r}(t) \\ &\quad + C_y(z;\theta_y)y(t)) \\ \Gamma_S(t;[\theta_r,\theta_y]) &= L_S(z)(u(t)+C_y(z;\theta_y)\bar{y}(t)) \end{aligned}$$

and $L_M(z)$ and $L_S(z)$ are appropriate filters to be chosen in such a way that (8) becomes an approximation to (6). If the controllers are linear in the parameter ($C_r(z;\theta_r) = \beta_r(z)^T\theta_r$ and $C_y(z;\theta_y) = \beta_y(z)^T\theta_y$), the cost criterion (8) becomes a standard quadratic optimization problem. One of the interesting things about the VRFT framework is that it has been used in several applications and has even been extended for other cases. In [4] the Output Error (OE) method is suggested as a way to find the parameters of the controllers without the constraint of having to parameterize them linearly in the parameters. In [10], the VRFT framework is applied and the identification problems are adapted to the MIMO case. In [11], the framework was adapted for the tuning of a PID controller with an adaptive design. From a more practical point of view, in [12, 13] the VRFT is used in a Functional Electrical Stimulation, to find feedback controllers that can help paraplegics to stand up, by means of electrical signals applied to the muscles. A non-linear controller structure is also presented and used with the VRFT framework. All these examples show how the method has been well received thanks to its flexibility and the simplicity of its implementation.

3 Description of the 2DoF with Feed-forward action structure

The 2DoF alternative structure under consideration is presented in Fig. 3. This structure was originally presented in [14] in a generic fractional representation approach. The distinctive point of this proposal is that it guarantees a complete separation and independence (not always guaranteed on other structures) between the reference and feedback designs. This independence is understood as the ability to change the reference tracking specification (given by a transfer function M) without having to change the feedback controller C_s when dealing with restricted order controllers.

In addition, as was shown in [15], if we optimize over a generic configuration with respect to a quadratic cost index, the underlying controller obeys this particular choice. Therefore, as within the VRFT approach the control cost functional is formulated as a 2-norm problem, reformulating the 2-DoF VRFT on this alternative structure makes sense.

Originally, this structure was intended to be used in a model-based control, because the plant model should be factorized as $P = ND^{-1}$ where N and D are stable and proper transfer functions. It can be found that if $C_{ff} = DQ$ and $C_x = NQ$, the input-output transfer function becomes

$$y = NQr \tag{9}$$

Suppose that the desired reference to the output transfer function is given by M, in that case, the relation in (9) leads to

$$Q = N^{-1}M \tag{10}$$

If (10) holds, the relationship in (9) is totally independent of the controller C_s. In fact, this feedback

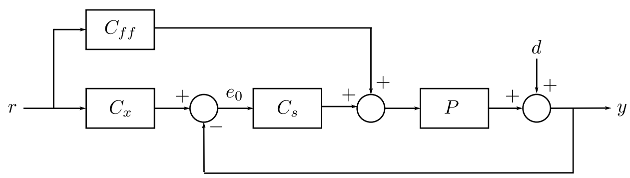


Figure 3: Two degrees of freedom structure, feedforward tracking controller

controller will only act if relation (10) cannot be fulfilled completely or in the case of a disturbance being present. In other words, C_s is responsible for the stability of the closed-loop system and the disturbance rejection performance, which can be treated as a separate sensitivity shaping problem. The separation between the sensitivity shaping problem and the complementary sensitivity shaping problem leads to the idea of using this structure with alternative methods for controller design. The VRFT was found to be suitable for this. In fact, the separation property of this structure is an advantage since two different independent optimizations can be performed to achieve both shaping problems without the drawback of having a compromise between them. However, a reformulation will be needed as the design of C_{ff} and C_x was originally conceived as model based, whereas within VRFT the plant model is not available.
Suppose that the input-to-output target transfer function is given by $M(z)$ and the target Sensitivity Transfer Function is given by $S(z)$, then the input-to-output relationship is given by:

$$y = (C_{ff} + C_x C_s)\frac{P}{1 + C_s P} r + \frac{1}{1 + C_s P} d \quad (11)$$

Note that this relationship could be seen as a general expression for 2DoF controllers, since, from this, other 2DoF cases can be obtained (see [14]). Then, the ideal controllers (that is the controllers that would yield a closed-loop response, exactly given by $M(z)$ and $S(z)$) become (dropping the z terms for the sake of simplicity)

$$C_{so} = \frac{1 - S}{SP} \quad (12)$$

$$\begin{aligned} C_{ffo} &= MP^{-1} + \frac{1 - S}{SP}(M - C_x) \\ &= MP^{-1} + C_{so}(M - C_x) \end{aligned} \quad (13)$$

With C_x a “parametric transfer function”.

4 Alternative 2DoF VRFT Problem Formulation

In order to apply the VRFT’s ideas on the above presented structure, the prefilter is directly set as $C_x(z) = M(z)$. From (13), it is clear that this choice of the prefilter allows the feedforward controller to be independent of the feedback controller, which is the great advantage of this control structure. In what follows, the VRFT problem will be formulated first on this generic 2DoF configuration, followed by considerations on how to find $C_{ff}(z)$ and $C_s(z)$, as well as some considerations with respect to the choice of filters (as is done on the original VRFT). A common practice in data-driven control is to use the disturbance signal at the output of the plant in order to be able to use the sensitivity function $S(z)$ as a design parameter, but it is possible to use the VRFT framework to get a target transfer function from a load disturbance (at the input of the plant) to the closed-loop output. In this paper the disturbance is kept at the output to have a point of comparison with other standard two degrees of freedom VRFT approaches.

4.1 Formulation of the VRFT problem

Using the above presented 2DoF structure, the cost function is as shown in (14) (the z term has been dropped)

$$\begin{aligned} J_{MR}(\theta_{ff}, \theta_s) &= \|(\Psi_M([\theta_{ff}, \theta_s]) - M)\, W_M\|_2^2 \\ &+ \|(\Psi_S([\theta_{ff}, \theta_s]) - S)\, W_S\|_2^2 \end{aligned} \quad (14)$$

with

$$\begin{aligned} \Psi_M([\theta_{ff}, \theta_s]) &= \frac{(C_{ff}(\theta_{ff}) + MC_s(\theta_s))\, P}{1 + PC_s(\theta_s)} \\ \Psi_S([\theta_{ff}, \theta_s]) &= \frac{1}{1 + PC_s(\theta_s)} \end{aligned}$$

Being $\Psi_M([\theta_{ff}, \theta_s])$ and $\Psi_S([\theta_{ff}, \theta_s])$ the achieved reference-to-output transfer function and the achieved sensitivity function, respectively [1].
Using the same procedure as the one originally proposed within the VRFT formulation [1], the ideal controllers are introduced into (14). The ideal controllers, $C_{ff0}(z)$ and C_{s0} for this alternative structure, are given by (12) and (13) with $C_x = M$:

$$\begin{aligned} C_{ff0} &= P^{-1}M \\ C_{s0} &= \frac{1 - S}{SP} \end{aligned} \quad (15)$$

[1] Now $\Psi_M([\theta_{ff}, \theta_s])$ will not be the complementary sensitivity function as $\Psi_M + \Psi_S = 1$ will not necessarily hold

With these ideal controllers, the signal e_0 becomes the reference shaping error, since it reduces to $e_0 = Sd$, and if there is no disturbance, $e_0 = 0$ for any value of r. By introducing (15) into (14), it can be shown that (14) reduces to (16)

$$J_{MR}(\theta_{ff},\theta_s) = \left\| \frac{P\left(C_{ff}-C_{ff0}\right)}{1+PC_s} W_M \right\|_2^2 + \left\| \frac{P\left(C_{s0}-C_s\right)}{(1+PC_s)(1+PC_{s0})} W_S \right\|_2^2 \tag{16}$$

In [1], to find the filter $L_M(z)$, the term $\frac{1}{1+PC_s}$ is approximated $\frac{1}{1+PC_s} \approx S(z)$, which is a good approximation, since the solution of the right part of equation (14) looks for this condition. Since equation (16) is used only for the determination of the filters, as will be shown later, using this approximation, the equation can be written as:

$$J_{MR}(\theta_{ff},\theta_s) = \left\| \left(C_{ff}-C_{ff0}\right) PSW_M \right\|_2^2 + \left\| \frac{P\left(C_{s0}-C_s\right)}{(1+PC_s)(1+PC_{s0})} W_S \right\|_2^2 \tag{17}$$

The first term of 17 is used to find the controller $C_{ff}(z)$, while the second term is used to find $C_s(z)$. This control problem has to be translated into an identification problem to find the parameters of the controllers without the knowledge of any model for the plant. If the ideal controllers were found, the input to the feedback controller should always be zero if a disturbance is not affecting the system. Therefore, using the ideas of the virtual signals of the VRFT, one is able to formulate an identification problem with the controllers $C_{ff}(z)$ and $C_s(z)$ also totally decoupled, and therefore it is possible to optimize each controller for the specific task it is intended to deal with ($C_{ff}(z)$ for tracking and $C_s(z)$ for disturbance rejection). The design algorithm is analogous to that in [2]. Given the reference models $M(z)$ and $S(z)$, and the batch of data $\{u(t), y(t)\}_{t=1,\ldots,N}$:

- Construct the set of "virtual" data $\left(\bar{r}, \bar{d} \text{ and } \bar{y}\right)$, as in [2]:
  - $\bar{r}(t)$ is given by $y(t) = M(z)\bar{r}(t)$
  - $\bar{d}(t)$ is given by $y(t) + \bar{d}(t) = S(z)\bar{d}(t)$
  - $\bar{y}(t)$ is given by $\bar{y}(t) = y(t) + \bar{d}(t)$
- Find the controller parameter vector ($\hat{\theta}_{ff}^N$, $\hat{\theta}_S^N$) that minimizes (18)

$$J_{VR}^N = \frac{1}{N}\sum_{t=1}^{N} \left[L_M(u(t) - C_{ff}(z;\theta_{ff})\bar{r}(t))\right]^2 + \frac{1}{N}\sum_{t=1}^{N} \left[L_S(z)\left(u(t) + C_s(z,\theta_s)\bar{y}(t)\right)\right]^2 \tag{18}$$

Since each part of equation 18 depends only on one of the controller's parameters, it could be solved separately. To check this, let us assume a general function $f(\theta_1, \theta_2)$ which is meant to be minimized. Supposing that $f(\theta_1, \theta_2) \geq 0$ and that it can be written as:

$$f(\theta_1,\theta_2) = f_1(\theta_1) + f_2(\theta_2) \tag{19}$$

with $f_1(\theta_1) \geq 0$ and $f_2(\theta_2) \geq 0$. Minimizing (19) implies

$$\begin{aligned} \frac{\partial f(\theta_1,\theta_2)}{\partial \theta_1} &= 0 \\ \frac{\partial f(\theta_1,\theta_2)}{\partial \theta_2} &= 0 \end{aligned} \tag{20}$$

but given (19), it is evident that

$$\begin{aligned} \frac{\partial f(\theta_1,\theta_2)}{\partial \theta_1} &= \frac{df_1(\theta_1)}{d\theta_1} \\ \frac{\partial f(\theta_1,\theta_2)}{\partial \theta_2} &= \frac{df_2(\theta_2)}{d\theta_2} \end{aligned} \tag{21}$$

therefore, minimizing $f(\theta_1, \theta_2)$ is the same as minimizing each single part individually, given this independence in the variables. For this reason, it is clear that (18) is totally decoupled. It is important to note that the original control criterion is not totally separated, as can be seen in (17). But using the adequate filters L_M and L_S, the decoupled identification criterion (18) can be used to find the controller parameters that approximate the desired control criterion, which is the standard procedure for the VRFT approach (see [1, 2, 4, 10]). The main advantage of applying this alternative two-degrees of freedom controller is that the controllers and the optimization to find each of its parameters are both independent. This independence allows the designer to use different optimization methods, data, or even only find one of the controllers if necessary. Below, the structure of suitable filters $L_M(z)$ and $L_S(z)$ is given.

4.2 Filters Choice

The idea of the filters $L_M(z)$ and $L_S(z)$ is to approximate the identification performance index (18) to the desired model reference control criterion in (14). Applying the Parseval Theorem and (15), it is found that (omitting the $e^{j\omega}$ argument for simplicity)

$$J_{MR} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{|P|^2}{|1+PC_s|}\left|C_{ff}-C_{ff0}\right|^2\left|W_M\right|^2\mathrm{d}\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{|P|^2|S|^2}{|1+PC_s|}\left|C_{s0}-C_s\right|^2\left|W_S\right|^2\mathrm{d}\omega \tag{22}$$

Now, using the results in [1], and considering $J_{VR}(\theta)$ as the asymptotic counterpart of $J_{VR}^N(\theta)$ as $N\to\infty$:

$$J_{VR}(\theta) = E[(u_L(t) - C(z;\theta)e_L(t))^2] \tag{23}$$

Again, applying the Parseval Theorem to J_{VR} (see (18)), it is found that

$$J_{VR}(\theta_{ff},\theta_S) = \frac{1}{2\pi}\int_{-\pi}^{\pi}|C_{ff0} - C_{ff}(\theta_{ff})|^2\psi_M\mathrm{d}\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi}|-C_{s0} + C_s(\theta_s)|^2\psi_S\mathrm{d}\omega \tag{24}$$

with

$$\psi_M = \frac{|P|^2}{|M|^2}|L_M|^2|\Phi_u|^2$$
$$\psi_S = \frac{|S|^2}{|S-1|^2}|P|^2|L_S|^2|\Phi_u|^2$$

To find the filters, (22) and (24) can be compared. Since a plant model is not available, the term $|1 + PC_s(\theta_s)|^{-1}$ can be approximated by $|S|$ (that is, changing $C_s(\theta_s)$ for C_{s0}). If this is done, the filters can be found to be:

$$|L_M|^2 = |M|^2|S|^2|W_M|^2\frac{1}{|\Phi_u|^2} \tag{25}$$

$$|L_S|^2 = |S-1|^2|S|^2|W_S|^2\frac{1}{|\Phi_u|^2} \tag{26}$$

Which turn out to be the same as those found in [2].

4.3 Feedforward Controller Tuning

It is possible to *identify* the $C_{ff}(z)$ controller from the input/output data. If the signal $\bar{r}(t) = M^{-1}(z)y(t)$ is introduced, the output of the feedforward controller should be u. The direct consequence of this fact is that, an identification method can be used to determine $C_{ff}(z)$, using $\bar{r}$ (the filtered version of $y(t)$ through $M^{-1}(z)$) as the input, and $u(t)$ as the output values.
This *identification problem* can also be derived using a VRFT approach: If the output data measured from the experiment performed on the plant (in open-loop) were taken in closed-loop with the ideal $C_{ff}(z)$ controller, then the error e should always be zero. In that case, we should find a *virtual signal* $\bar{r}(t)$ such that $y(t) = M(z)\bar{r}(t)$. Then the controller that should be identified is the one that, with an input signal $\bar{r}(t)$, generates an output $u(t)$ (since the error is always zero). So, the objective is to find $C_{ff}(z)$ as close as possible to $P^{-1}(z)M(z)$. The controller $C_s(z)$ is not involved in the optimization for $C_{ff}(z)$. In (14), the optimization is not totally decoupled, but it is important to note that the filters used in (18) are set in order to approximate both criteria.
In the case of $C_{ff}(z)$, it is very important to have more freedom in the structure, since the advantage of the structure is strongly dependent on how close the controller is to that of the ideal controller. For this reason, using an identification method (such as the OE method: see [16]) to find the feedforward controller parameters can be more useful than using a linear-in-the-parameters structure as was originally proposed in [2]. The use of an identification method to find the parameters of the controller and the idea of using the VRFT in a feedforward controller was first suggested in [4].
In any case, there is no inconvenience in trying to identify a linear-in-the-parameter controller for $C_{ff}(z)$. In such a case, if $C_{ff}(z)$ is defined as $C_{ff}(z,\theta_{ff}) = \beta^T(z)\theta_{ff}$, where β is a vector of transfer functions and θ_{ff} are the parameters of the controller, the performance criterion specifically for the tracking problem becomes

$$J_{VR}^N(\theta_{ff}) = \frac{1}{N}\sum_{t=1}^{N}\left(u_L(t) - C_{ff}(z;\theta_{ff})\bar{r}_L(t)\right)^2 \tag{27}$$

The signals $u_L(t)$ and $\bar{r}_L(t)$ are the filtered versions of $u(t)$ and $\bar{r}(t)$, respectively, filtered by $L_M(z)$. The parameters can be analytically obtained by

$$\hat{\theta}_{ff} = a_N^{-1}f_N \tag{28}$$
$$a_N = \frac{1}{N}\sum_{t=1}^{N}\varphi_L(t)\varphi_L(t)^T$$
$$f_N = -\frac{1}{N}\sum_{t=1}^{N}\varphi_L(t)u_L(t)$$

with

$$\varphi_L(t) = \beta(z)\bar{r}_L(t) \tag{29}$$

4.4 Feedback Controller Tuning

The $C_s(z)$ controller should be optimized to reject the disturbance, since $C_{ff}(z)$ was optimized to solve

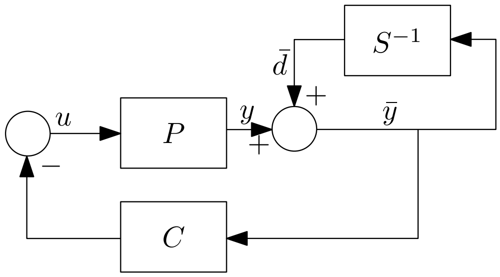


Figure 4: Structure to find the Virtual Signals, for the Sensitivity Shaping problem

the tracking problem. In [3] a method is presented to solve a *Sensitivity Shaping problem* also based on the VRFT formulation. Even though the authors use the controller in the feedback path, the same method can be used with the controller in the direct path, since the problem requires $r(t) = 0$ and the result is independent from $C_{ff}(z)$. According to [3], the structure to find the *virtual signals* is given as in Fig. 4. Once the virtual signals are calculated, the cost function is given by (30).

$$J_{VR}^N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\left(u_L(t) + C_s(z;\theta_s)\bar{y}_L(t)\right)^2 \quad (30)$$

The signals $u_L(t)$ and $\bar{y}_L(t)$ are the filtered versions of $u(t)$ and $\bar{y}(t)$ respectively. According to [3], the filter has to be given by $L_S(t)$ as shown below.
If the controller is linear in the parameters, the solution can be obtained analytically

$$\hat{\theta}_N = a_N^{-1} f_N \quad (31)$$
$$a_N = \frac{1}{N}\sum_{t=1}^{N}\varphi_L(t)\varphi_L(t)^T$$
$$f_N = -\frac{1}{N}\sum_{t=1}^{N}\varphi_L(t)u_L(t)$$

with

$$\varphi_L(t) = \beta(z)\bar{y}_L(t) \quad (32)$$

4.5 Design Guidelines

With data-driven control, choosing the design parameters (the transfer functions $M(z)$ and $S(z)$ and the structure of the controllers $C_s(z)$ and $C_{ff}(z)$) is difficult, since the designer has no model to know exactly what are the limits of the control performance are that he or she can achieve. For this reason some basic knowledge of the plant is needed.
In general terms, $M(z)$ ans $S(z)$ are chosen as first order transfer functions with a settling time between 0.1 and 1 times the settling time of the plant (which can be deduced using, for example, a step change in the plant input during the collection of the data for the VRFT optimization). This is similar to choosing a wider bandwidth for $M(z)$ than the bandwidth of the plant. To find the bandwidth of the plant, the *Empirical Transfer-Function Estimate (ETFE)* [16] has been found to be useful and easy to use.
In the case of the controllers, if its structure is not chosen a priori (for example, restricting the structure to a PID-like controller as in [11]), the designer has to select the structure before the optimization is carried out in order to find the optimal values of the parameters. Again, the choice of structure is an issue that becomes kind of fuzzy, specially when there is no available model of the plant, but only data from it.
The main problem is selecting a set of controller parameterizations that could contain the ideal controllers, but in general, this is not possible. For the feedback controller, a fixed pole could just be chosen at $z = 1$ to guarantee zero stationary error and a certain number of parameters chosen on the numerator. Knowing that the ideal C_{ff} is given in (15), four simple guidelines are presented to choose its structure when minimizing (18):

1. **Finite Impulse Response (FIR) filter**: If the controller is parameterized as

$$C_{ff} = \beta^T(z)\theta_{ff} \quad (33)$$

in order to have a standard least squares problem in (18), the simplest structure will be given by a FIR filter (that is, $\beta^T(z)$ is a vector whose components are ascending powers of z^{-1}). For this configuration, there will have to be many parameters to achieve a good performance, depending on the dynamics of the plant.

2. **Using the denominator of M**: Again, if C_{ff} is linear in the parameters, from (15), it is clear that the denominator of M should also be part of the denominator of the controller, that is, for each element of β and with $M(z) = N(z)/D(z)$

$$\beta_i(z) = D(z)^{-1} \quad (34)$$

The increase in the complexity of M, should decrease the number of parameters needed for a good performance.

3. **Approximation of the plant denominator**: A better approximation to the ideal C_{ff} can be found if controller C_s is already in the closed loop. Supposing that controller C_s is a good controller (that is, the target Sensitivity Function is

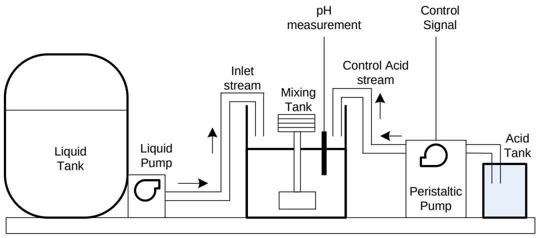


Figure 5: Diagram of the Ph neutralization plant

well approximated using C_s inside the loop), the plant approximation can be given by

$$\tilde{P} = \frac{1-S}{SC_s} \tag{35}$$

So adding the minimum-phase zeros of $\tilde{P}$ along with the denominator of M as the denominator of C_{ff} is an even better approximation.

4. **Free Structure** Also, it is possible to find the parameters of the denominator of C_{ff} if the optimization problem is set as a Least Square problem, with the regressors given by:

$$\varphi^T(t) = [r_v(t)\ r_v(t-1)\ r_v(t-2)\dots r_v(t-n) \\ -u(t-1)\ \ -u(t-2)\dots u(t-m)] \tag{36}$$

which yields a controller of the form

$$C_{ff} = \frac{\alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \cdots + \alpha_n z^{-n}}{1 + \beta_1 z^{-1} + \beta_2 z^{-2} + \cdots + \beta_m z^{-m}} \tag{37}$$

Also, in [4], the use of a standard identification method (for example the O.E. method, see for example [16]) is recommended to get a full parametrized controller. It is possible to obtain an unstable controller using a free structure. Before implementing the controller in the real process, it is necessary to check that the C_{ff} controller does not have poles outside the unit circle.

5 Practical Example

The process under study in this section is the neutralization of an aqueous solution with Hydrochloric Acid (HCl) in a Continuously Stirred Tank Reactor (CSTR). The experimental setup (described in detail in [17]) is shown in Fig. 5. The aim of this experiment is to show how the VRFT methodology, and in particular the alternative 2doF structure, can be applied in a real world plant. It consists of a CSTR where a liquid of variable pH is mixed with a solution of high concentration of HCl. The liquid in the mixing tank overflows (outlet not shown), so the volume of liquid in the tank can be considered constant. The control variable u is the flow rate of the titrating stream. The output variable y is the hydrogen ion concentration in the effluent stream. The control was implemented using the OPC toolbox in MATLAB/Simulink, using discrete-time filters.

Table 1: Steady-state operating point of the nonlinear pH model

Parameter	Value
F_A	1.8×10^{-4} l/s
C_A	0.0708 mol/l
F_S	7.73×10^{-3} l/s
C_S	0.0308 mol/l
V	0.63 l
τ	0.0127 s
x_A	1.63×10^{-3} mol/l
x_S	30.1×10^{-3} mol/l

Due to the nonlinear dependence of the pH value on the amount of titrated agent, the process will be inherently nonlinear. Moreover, variations of the buffering effects could make the process time-varying. Both effects make the process difficult to control with classical process control techniques [6].

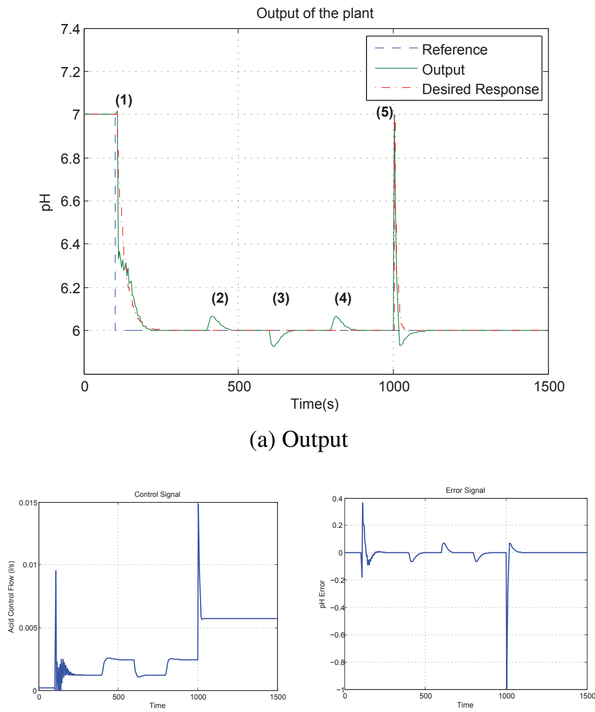
The methodology was firstly tested in simulation using the following model [18], where the acid is hydrochloric acid (HCl) and the inlet is an aqueous solution of sodium acetate (CH_3COONa):

$$\begin{aligned} -x_A + 10^{-pH} - 10^{pH-14} + \tfrac{x_S}{1+10^{pKS+pH-14}} &= 0 \\ V\frac{dX_A}{dt} &= F_A C_A - (F_A + F_S)x_A \\ V\frac{dx_S}{dt} &= F_S C_S - (F_A + F_S)x_S \\ \tau\frac{dpH^*}{dt} &= pH - pH^* \end{aligned} \tag{38}$$

where $x_A = \left[\mathrm{Cl}^-\right]$ and $x_S = \left[\mathrm{Na}^+\right]$ are the negative and positive ion concentration within the tank respectively. F_A is the control acid stream flowrate, F_S is the inlet stream flowrate, C_S is the concentration of sodium acetate in the inlet stream, C_A the acid concentration in the control acid stream, the measured pH is pH^* which is supposed to have a constant time τ, $pKS = -10\log_{10} k_S$, with dissociation constants $k_w = 10^{-14}\mathrm{mol}^2$

The purpose of the simulated model is to test the control methodology before implementing it in the real plant. The nonlinear model was excited with a pseudo random binary signal in open-loop, and the controller was calculated as pointed out in section 4. The target closed-loop response is set as a first order transfer function with time constant equal to 25s (which represents a settling time of approximately 150s). The settling time of the sensitivity function was chosen as 40s. The filters W_M and W_S were set to 1.

For a linear-in-the-parameter structure for C_s, with 2 parameters in the numerator (θ_s =[-0.01237, 0.01012]), denominator $1 - z^{-1}$; and a fully parameterized C_{ff} with 3 parameters in the numerator (θ_{ffnum} = [0.0001073, -0.007215, 0.006206]) and 3 in the denominator (θ_{ffden}= [1, 0.1178, -0.6]); using a sampling time of $T_s = 1.5s$ and a saturation at the input of the controller to avoid negative values of the acid stream flowrate, the result of the controlled system is as shown in Fig. 6. The control signal is the sum of both the C_{ff} and the C_s outputs in Fig. 6b. Fig. 6c shows the e_0 signal.
As can be seen in Fig. 6a, the controller achieved a response that is close to the target one (1), but the reference tracking controller presents a non-minimum phase zero which produces an undesired oscillatory response. On the other hand, the C_s controller has a good response to (2) a step disturbance in the inlet concentration, (3) a step disturbance in the acid concentration, (4) a step disturbance in the inlet flowrate and (5) a unit step disturbance at the output of the plant. In cases (1) to (4), the disturbance step is equal to 100% of the value of the steady-state operating point given in Table 1. The data used for this experiment is presented in Fig. 7a.
Taking advantage of the separation between the controllers, new data can be found that is specifically addressed to finding the reference tracking controller without changing the feedback controller. This new data takes into account the fact that the pH cannot rise as as fast as it decreases, since it is impossible to actively withdraw acid from the tank. For this reason, the new data was made to change more slowly, and is presented in Fig. 7b. With this new data, the C_{ff} controller's parameters became (θ_{ffnum} = [-0.001424, 0.000263, 0.0009215]) and (θ_{ffden}= [1, -0.6911, -0.07566]). With this controller, the results are as given in Fig 8. As can be seen, the response is better without altering the disturbance rejection. The reference tracking was set faster than the open-loop response, since being able to take the system to a new operating point faster than in open loop is an important control task, which can be achieved independently from the disturbance rejection thanks to the extra degree of freedom of the controller. Another test where only changes in the reference signal are applied is presented in Fig. 9. The error signal depicted in Fig. 9b



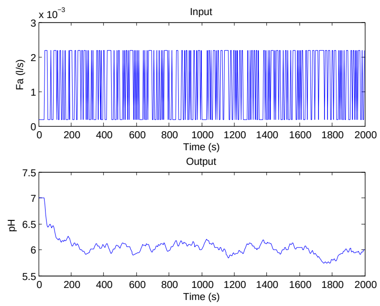
(a) Output

(b) Control Signal
(c) e_0 signal

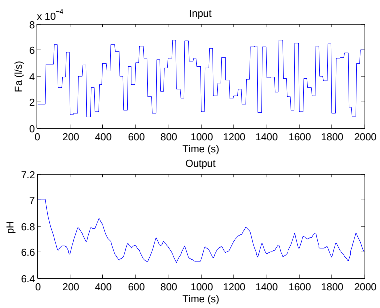
Figure 6: Results from simulation of the nonlinear model, controlled with the VRFT controllers with several disturbances.

shows that there is an error in the transient part of the response due to imperfections in the structure of the C_{ff} controller, but the C_s controller is able to cope with these errors. The saturation of the input signal is responsible for the overshoot at the end of the test, when a higher pH is required.

For the real plant in Fig. 10, instead of sodium acetate, the inlet is just liquid water with variable pH around the value of 7. In order to have the data with the correct magnitude, a new batch of data was taken using the OPC server connected to the system [19]. This input to the real plant cannot be a pseudo random signal as in the simulation example, because the peristaltic pump cannot act as quickly as the data changes. The data collected from the experiment in open-loop is shown in Fig. 11: a series of step changes were performed in order to excite the plant at several operation points. It is well known that, from a system identification point of view, the data has to be persistently exciting [16], which is why, under the physical restriction of the plant, this input signal was selected. When performing the optimization, the data was filtered with a third order Butterworth filter with cut frequency equal to 0.25 times the sampling frequency. It was decided to have a settling time of 150s in the response between the reference and the output. The C_s controller is a PI-like controller with two parameters in the numerator.

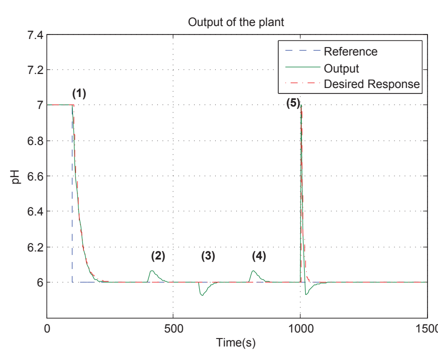


(a) First data

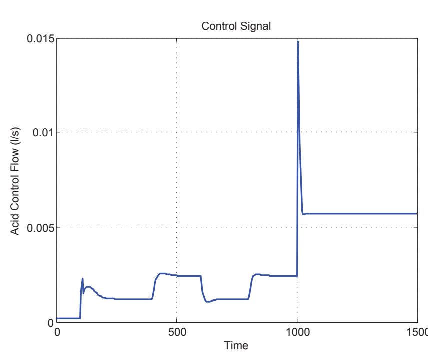


(b) Second data

Figure 7: Open-loop data used to find the VRFT controllers for the simulated case

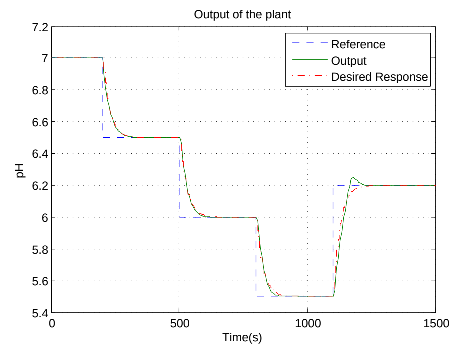


(a) Output

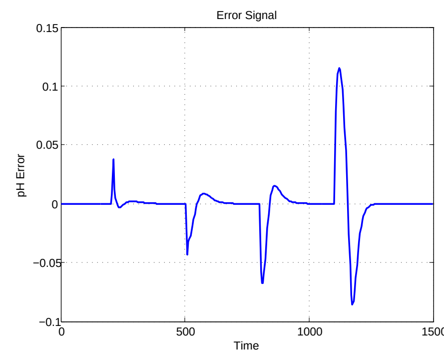


(b) Control signal

Figure 8: Change in the C_{ff} controller to avoid oscillation in the value of pH



(a) Response to step changes in the reference



(b) Error signal

Figure 9: Test of the C_{ff} controller for different changes in the reference signal

The resulting closed-loop, using the same sampling period of the simulation, is shown in Fig. 12. In Fig. 12 b), $u_{OpPoint}$ is the output of the controller at the operation point, $u_{C_{ff}}$ is the output of the feed-forward controller, u_{C_s} is the output of the feedback controller and u is the sum of all the control signals. It was decided not to go beyond a pH of 6.24 to ensure good functioning of the peristaltic pump. It was found that for higher values of the pH, the response is not as desired, but the constant time is very similar. For lower values of pH, the response is very close to the desired one (neglecting the noise). The controllers

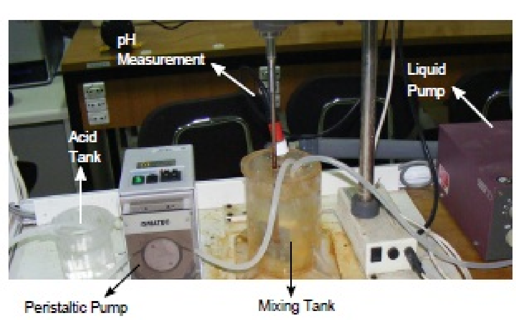


Figure 10: Photograph of the real pH neutralization process

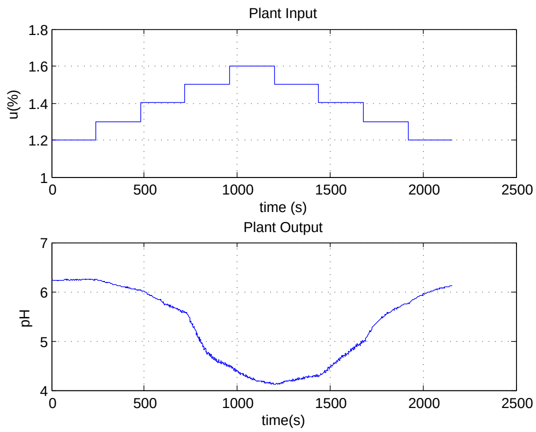


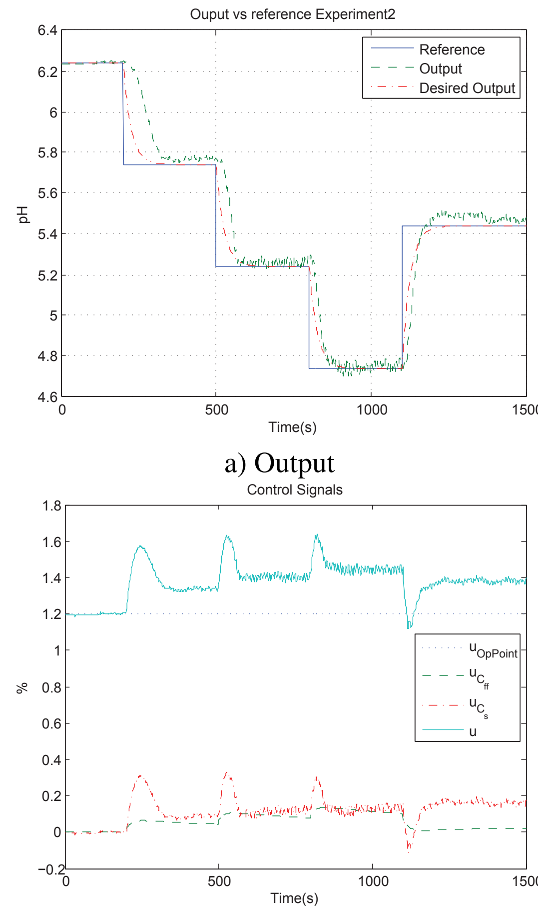
Figure 11: Batch of data used for computing the controllers in the real plant

and the design transfer functions are as follows:

$$
\begin{aligned}
M(z) &= \frac{0.05824z^{-1}}{1-0.9418z^{-1}}\\
S(z) &= \frac{1-z^{-1}}{1-0.9418z^{-1}}\\
C_s(z) &= \frac{-0.8235+0.8201z^{-1}}{1-z^{-1}}\\
C_{ff}(z) &= \frac{\begin{pmatrix} -0.04827+0.1322z^{-1}\\ -0.1203z^{-2}+0.03634z^{-3}\end{pmatrix}}{\begin{pmatrix} 1-2.871z^{-1}+2.748z^{-2}\\ -0.8762z^{-3}-0.0001912z^{-4}\end{pmatrix}}
\end{aligned}
\tag{39}
$$

It was found that the sampling time for this application was too high: the controller had poles very near to the unit circle and therefore, having the exact values of the parameters became critical. Because of this, it was decided to use a larger sampling time ($T_s = 4.5s$). The same batch of data was used, but decimated by 3. In this case, the response is as given in Fig. 13. As expected, the system degrades its performance, but the controller poles are farther from the unit circle. With this sampling time, an oscillatory behavior affects the response, and overshoot is found as the reference changes. A lower sampling frequency made the entire system oscillate. The response of the system with $T_s = 7.5s$ is shown in Fig. 14.

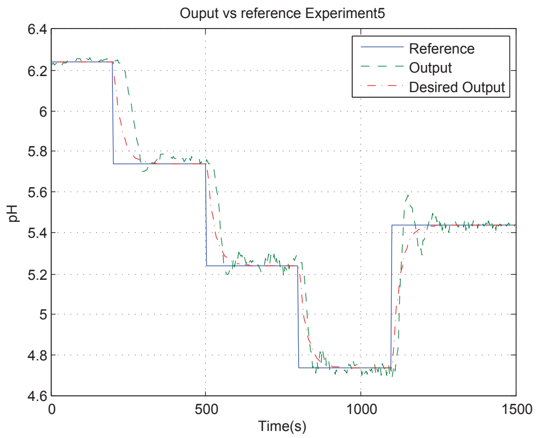
It is interesting to note that both the C_s and the C_{ff} controllers have the same output when a stationary point is reached. This is because the C_{ff} controller is not equal to the ideal C_{ff0}, as expected, given the non-linearity of the plant. That is why the C_s controller has to act when the reference is changed. Otherwise, the output of the feedback controller should only act when a disturbance is present in the plant.



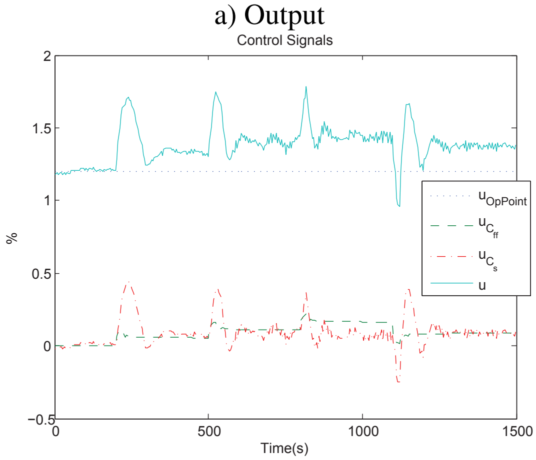
a) Output

b) Control Signals

Figure 12: Response of the closed-loop system in the real plant



a) Output



b) Control Signals

Figure 13: Response of the closed-loop system in the real plant for a sampling time of $T_s = 4.5s$

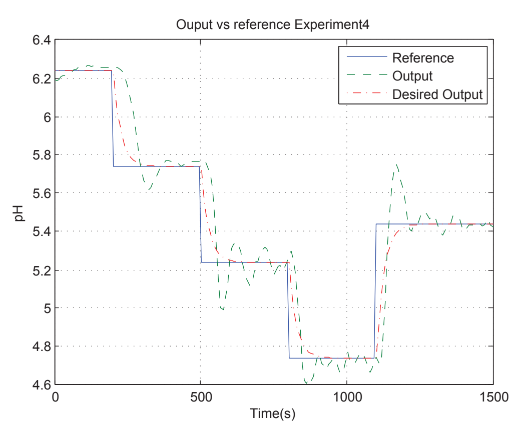


Figure 14: Response of the system with a sampling period of $T_s = 7.5$. The response became oscillatory

Also, it is interesting to note that the controller is able to cope with the noise in the measurements and keep the system near the reference points.

6 Conclusion

In this paper, the Virtual Reference Feedback Tuning was applied to an alternative Two Degrees of freedom controller aimed at totally decoupling the reference tracking response to the disturbance rejection. The tuning process of both controllers was also decoupled, since this topology allows the parameters of the feed-forward and feedback controller to be independently computed. Also, the methodology was tested successfully in the control of a pH neutralization bench plant. More research has to be done, aimed at finding a way of determining the structure and the number of parameters needed for certain applications, based only on data from the plant, in order to use restricted order data-driven controllers.

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