Nontraditional Approach to Satellite Attitude Estimation

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Abstract: - This paper describes the development of nontraditional attitude determination system that can rely on vector measurements. Vectors coming from the selected sensor data and developed models can be placed in Wahba's problem. The system uses Singular Value Decomposition (SVD) method to minimize the Wahba's loss function and determine the attitude of the satellite. In order to obtain the attitude of the satellite with desired accuracy an extended Kalman filter (EKF) for satellite's angular motion parameter estimation is designed. Different algorithms with or w/o gyro bias estimation are considered and compared in order to achieve better accuracy on the attitude angles and angular rates. Also, a rate gyro is added to the algorithm in addition to the vector measurements and their biases can be estimated.

The SVD and EKF algorithms are combined to estimate the attitude angles and angular velocities. Besides, the proposed algorithm and traditional approach using nonlinear measurements are compared and concluded that SVD/EKF gives more accurate results for most of the time intervals. The algorithm can be used for low-cost small satellites where using high power consuming, expensive, and fragile gyroscopes for determining spacecraft attitude are not reasonable.

Key-Words: - Satellite attitude estimation, magnetometer, sun sensor, extended Kalman filter, angular velocity, nontraditional approach

1 Introduction

Sun sensors and magnetometers are common attitude sensors for small satellites missions; they are cheap, simple, light and available as commercial of-the-shelf equipment. However the overall achievable attitude determination accuracy is limited with these sensors mainly as a result of their inherent limitations and unavailability of the sun sensor data when the satellite is in eclipse. Vectors coming from the selected sensor data and developed models can be placed in Wahba's problem [1, 2]. Coordinate systems used as reference frame and body frame can be transformed to each other with necessary input parameters. The system uses Singular Value Decomposition (SVD) method to minimize the Wahba's loss function and determine the attitude of the satellite. As a reference direction, the unit vectors toward the Sun, and the Earth's magnetic field are used. Cooperating magnetometer sun sensor and rate gyro utilization in small satellite missions is a common method for achieving accurate attitude information. By the use of a Kalman filter algorithm measurement inputs of these sensors can be easily integrated in order to estimate the attitude parameters of the satellite precisely. At this stage, the methods of dynamic filtration (for example Kalman filters) may be useful. In general, two types of Kalman filter algorithms will be taken into consideration:

a) Kalman filter based on linear measurements (nontraditional approach)

b) Kalman filter based on nonlinear measurements

In first case (approach based on linear measurements) attitude angles are found by vector measurements based attitude determination methods at each step. Then these are directly used as measurement input for Kalman filter. Hence measurement model is linear in this case, since the states are measured directly. On the other hand, in the second case, measurement models are based on nonlinear models of reference directions. Therefore there is a nonlinear relation between the measurements the states.

The traditional approaches to design of Kalman filter for satellite attitude and rate estimation use the nonlinear measurements of reference directions (Earth magnetic field, Sun, etc.) [3-5].

Integration of single-frame satellite attitude determination methods with Kalman filter is presented by [6, 7], in which the algebraic method and EKF algorithms are combined to estimate the attitude angles and angular velocities respectively. Attitude determination system use algebraic method (2-vector algorithm). This method is based on the computing any two analytical vectors in the reference frame and measuring these vectors in the body coordinated system [8]. As measuring devices magnetometers, Sun sensors, and horizon scanners/sensors are used. Three different algorithms based on Earth's magnetic field, Sun vector, and nadir vector are used. In order to obtain the attitude of the satellite with desired accuracy an EKF for satellite's angular motion parameter estimation is designed.

After, single frame methods aided KF approach is studied in [9-17].

In this study SVD-aided EKF attitude determination system is presented, in which the SVD and EKF algorithms are combined to estimate the attitude angles. Besides, angular velocities and gyro biases is estimated using presented algorithm. Here, coarse attitude information besides the covariance data coming from SVD is processed in the EKF to estimate much more accurate attitude angles. Also, the robust Kalman filter is compared in the sense of measurement faults with the SVD/EKF algorithm which is also a robust adaptive method naturally.

2 Mathematical Models for Vector Measurements

2.1 Magnetic Field Direction Vector

IGRF model defines the series in nT seen below which depends on 4 input variables (r,θ,ϕ,t), using numerical Gauss coefficients (g, h) - the global variables in the algorithm [18].

$$B(r,\theta,\phi,t) = -\nabla\left\{a\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}[\mathrm{g}_n^m(\mathrm{t})\cos m\phi + \mathrm{h}_n^m(\mathrm{t})\sin m\phi]\times P_n^m(\cos\theta)\right\} \tag{1}$$

Here, r is the distance between center of the Earth and satellite (km), a=6371.2 km (magnetic reference spherical radius), θ colatitude (deg), ϕ longitude (deg). The inputs are coming from the LEO satellite which has an orbit propagation algorithm only regarding to J2 effects for the selected time period. Major axis of the Earth accepted as 6378.137 km. IGRF 11 model makes the calculations at N=13th degree for 5-year intervals. Thus, coefficients of the model are updated at the years of the multiples of five (2010, 2015, etc.). The time dependence of the Gauss coefficients can be denoted as:

$$g_n^m(t) = g_n^m(T_0) + \dot{g}_n^m(T_0)(t-T_0) \tag{2}$$

$$h_n^m(t) = h_n^m(T_0) + \dot{h}_n^m(T_0)(t-T_0) \tag{3}$$

Here, T_0 is the epoch times multiple of five proceeding t and t is in the units of years for the selected time. IGRF-11 model uses predictive secular variation coefficients for 2010-2015 and main field coefficients for 1900-2010.

$$B_o = \begin{bmatrix} B_{x_o} \\ B_{y_o} \\ B_{z_o} \end{bmatrix} = \frac{1}{\sqrt{B_1^2 + B_2^2 + B_3^2}}\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \tag{4}$$

Equation (4) shows the direction cosines for magnetic field model changing between -1 and +1, which only aim to determine the direction of the vector.

Three onboard magnetometers of the satellite measure the components of the magnetic field vector in the body frame. Therefore, for the measurement model, which characterizes the measurements in the body frame, gained magnetic field terms must be transformed by the use of direction cosine matrix, A. Overall measurement model may be given as;

$$B_m(k) = A(k)\mathrm{B}_o(k) + v_H(k), \tag{5}$$

where $B_m(k)$ is the measured Earth magnetic field vector as the direction cosines in body frame, $v_H(k)$ is the magnetometer measurement noise.

2.2 Sun Direction Vector

To determine Sun direction vector in ECI (Earth Centered Inertial) frame, Julian Day (T_{TDB}) should be defined from the satellite's initial data and reference epoch. The first constant is the mean anomaly of the Sun (M_{Sun}) at epoch and the second constant is the change of the mean anomaly during Julian Day that generates. After the calculations, the ecliptic longitude of the Sun ($\lambda_{ecliptic}$) and the obliquity of the ecliptic (ε) can be determined by only the input of date in years, months, days and time in hours, minutes, seconds [19].

$$M_{Sun} = 357.5277233^0 + 35999.05034 T_{TDB} \tag{6}$$

$$\begin{aligned}\lambda_{ecliptic} = \lambda_{M_{Sun}} &+ 1.914666471^0 \sin(M_{Sun}) \\ &+ 0.019994643 \sin(2M_{Sun})\end{aligned} \tag{7}$$

$$\varepsilon = 23.439291^0 - 0.0130042T_{TDB} \quad (8)$$

Finally, the unit Sun vector ($\boldsymbol{S}_{ECI}$) can be found in the inertial frame.

$$\boldsymbol{S}_{ECI} = \begin{bmatrix} \cos\lambda_{ecliptic} \\ \sin\lambda_{ecliptic}\cos\varepsilon \\ \sin\lambda_{ecliptic}\sin\varepsilon \end{bmatrix} \quad (9)$$

The Sun direction vector measurements can be expressed in the following form:

$$S_m(k) = A(k)S_o(k) + v_S(k), \quad (10)$$

where $S_m(k)$ is the measured Sun direction vector as the direction cosines in body frame, $S_0(k)$ represent the Sun direction vector in the orbit frame as a function of time and orbit parameters, $v_S(k)$ is the sun sensor measurement noise.

2.3 Mathematical Model of the Satellite's Rotational Motion

If the kinematics of the small satellite is derived in the base of Euler angles, then the mathematical model can be expressed with a 6 dimensional system vector which is made of attitude Euler angles (φ is the roll angle about x axis; θ is the pitch angle about y axis; ψ is the yaw angle about z axis) vector and the body angular rate vector with respect to the inertial axis frame,

$$x = \begin{bmatrix} \varphi & \theta & \psi & \omega_x & \omega_y & \omega_z \end{bmatrix}^T. \quad (11)$$

Also for consistency with the further explanations, the body angular rate vector with respect to the inertial axis frame should be stated separately as;

$$\omega_{BI} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T, \quad (12)$$

where $\overline{\omega}_{BI}$ is the angular velocity vector of the body frame with respect to the inertial frame. Besides, dynamic equations of the satellite can be derived by the use of the angular momentum conservation law;

$$J_x \frac{d\omega_x}{dt} = N_x + \left(J_y - J_z\right)\omega_y\omega_z, \quad (13)$$

$$J_y \frac{d\omega_y}{dt} = N_y + \left(J_z - J_x\right)\omega_z\omega_x, \quad (14)$$

$$J_z \frac{d\omega_z}{dt} = N_z + \left(J_x - J_y\right)\omega_x\omega_y, \quad (15)$$

where J_x, J_y and J_z are the principal moments of inertia and N_x, N_y and N_z are the terms of the external moment affecting the satellite. For a Low Earth Orbit (LEO) small satellite as in case, gravity gradient torque should be taken into consideration

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = -3\frac{\mu}{r_0^3}\begin{bmatrix} \left(J_y - J_z\right)A_{23}A_{33} \\ \left(J_z - J_x\right)A_{13}A_{33} \\ \left(J_x - J_y\right)A_{13}A_{23} \end{bmatrix}. \quad (16)$$

Here μ is the gravitational constant, r_0 is the distance between the center of mass of the satellite and the Earth and A_{ij} represents the corresponding element of the direction cosine matrix.

Kinematic equations of motion of the picosatellite with the Euler angles can be given as,

$$\begin{bmatrix} \dot\varphi \\ \dot\theta \\ \dot\psi \end{bmatrix} = \begin{bmatrix} 1 & s(\varphi)t(\theta) & c(\varphi)t(\theta) \\ 0 & c(\varphi) & -s(\varphi) \\ 0 & s(\varphi)/c(\theta) & c(\varphi)/c(\theta) \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}. \quad (17)$$

Here $t(\cdot)$ stands for tangent function and p, q and r are the components of $\overline{\omega}_{BR}$ vector which indicates the angular velocity of the body frame with respect to the reference frame. $\overline{\omega}_{BI}$ and $\overline{\omega}_{BR}$ can be related via,

$$\omega_{BR} = \omega_{BI} + A\begin{bmatrix} 0 \\ -\omega_0 \\ 0 \end{bmatrix} \quad (18)$$

where ω_0 denotes the angular velocity of the orbit with respect to the inertial frame, found as $\omega_0 = \left(\mu / r_0^3\right)^{1/2}$.

3 EKF for Satellite Attitude Estimation Based on Nonlinear Measurements-Traditional Approach

Consider the nonlinear mathematical model of the satellite's rotational motion about its mass center driven by white noise with white noise-corrupted measurements defined by

$$x(k+1) = \varphi\left[x(k), k\right] + w(k) \quad (19)$$

$$z(k) = h\left[x(k), k\right] + v(k) \quad (20)$$

where $x(k+1) = \left[\varphi(k+1) \quad \theta(k+1) \quad \psi(k+1) \quad \omega_x(k+1) \quad \omega_y(k+1) \quad \omega_z(k+1)\right]^T$ is the state vector, $z(k) = \left[H_{mx}(k) \quad H_{my}(k) \quad H_{mz}(k) \quad S_{mx}(k) \quad S_{my}(k) \quad S_{mz}(k)\right]^T$ is the measurement at time k, $w(k)$ is the system noise, $v(k) = \left[v_{Hx}(k) \quad v_{Hy}(k) \quad v_{Hz}(k) \quad v_{Sx}(k) \quad v_{Sy}(k) \quad v_{Sz}(k)\right]^T$ is the measurement noise, $\varphi\left[x(k), k\right]$ is the nonlinear state transition function mapping the previous state to the current state, $h\left[x(k), k\right]$ is a nonlinear measurement model mapping current state to measurements.

It is assumed that both noise vectors $v(k)$ and $w(k)$ are linearly additive Gaussian, temporally uncorrelated with zero mean, which means

$$E\left[w(k)\right] = E\left[v(k)\right] = 0, \ \forall k, \quad (21)$$

with the corresponding covariances:

$$E\left[w(i)w^T(j)\right] = Q(i)\delta(ij),$$
$$E\left[v(i)v^T(j)\right] = R(i)\delta(ij), \quad (22)$$

where $\delta(ij)$ is the Kronecker symbol.

It is assumed that process and measurement noises are uncorrelated, i.e.,

$$E\left[w(i)v^T(j)\right] = 0, \ \forall i, j \ . \quad (23)$$

We will consider a real-time linear Taylor approximation of the system function at the previous state estimate and that of the observation function at the corresponding predicted position. The Kalman Filter so obtained will be called the Extended Kalman Filter (EKF). Filter algorithm in this case as is given below [20].

Equation of the estimation value,

$$\hat{x}(k+1) = \hat{x}(k+1/k) + K(k+1) \times \left\{z(k+1) - h\left[\hat{x}(k+1/k), k+1\right]\right\} \quad (24a)$$

Equation of the extrapolation value,

$$\hat{x}(k+1/k) = \varphi\left[\hat{x}(k), k\right] \quad (24b)$$

Filter-gain of EKF

$$K(k+1) = P(k+1/k)H^T(k+1) \times \left[H(k+1)P(k+1/k)H^T(k+1) + R(k)\right]^{-1} \quad (25)$$

where $H(k+1) = \dfrac{\partial h[\hat{x}(k+1/k), k+1]}{\partial \hat{x}(k+1/k)}$ is the measurement matrix constituted of partial derivatives.

The covariance matrix of the extrapolation error is,

$$P(k+1/k) = \frac{\partial \varphi[\hat{x}(k), k]}{\partial \hat{x}(k)} P(k/k) \times \frac{\partial \varphi^T[\hat{x}(k), k]}{\partial \hat{x}(k)} + Q(k) \quad (26)$$

The covariance matrix of the filtering error is,

$$P(k+1/k+1) = \left[I - K(k+1)H(k+1)\right]P(k+1/k) \quad (27)$$

The filter expressed by the formulas (24)-(27) is called the EKF based on traditional approach.

4 SVD Method

After Wahba's optimization problem definition, two or more vectors can be used in statistical methods to minimize the loss [1]. The loss is the difference between the models and the measurements which are found in unit vectors.

$$L(A) = \frac{1}{2}\sum_i a_i \ |\mathrm{b}_i - \mathrm{Ar}_i|^2 \quad (28)$$

$$B = \sum a_i b_i r_i^T \quad (29)$$

$$L(A) = \lambda_0 - tr(\mathrm{AB}^T) \quad (30)$$

where b_i (set of unit vectors in body frame) and r_i (set of unit vectors in reference frame) with their a_i (non-negative weight) are the loss function variables obtained for instant time intervals and λ_0 is the sum of non-negative weights. Also, 'B' matrix is defined to reduce the loss function into the equation (3). Here, maximizing the trace ($tr(\mathrm{AB}^T)$) means minimizing the loss function (L). In this study, Singular Value Decomposition (SVD) Method is chosen to minimize the loss function [21].

$$\mathrm{B{=}USVT{=}U}\mathit{diag}|\mathrm{S}_{11}\ \mathrm{S}_{22}\ \mathrm{S}_{33}|\mathrm{VT} \quad (31)$$

$$A_{opt} = U diag[1 \quad 1 \quad \det(U)\det(V)]V^T \quad (32)$$

The matrices U and V are orthogonal left and right matrices respectively and the primary singular values (S_{11}, S_{22}, S_{33}) can be calculated in the algorithm. To find the rotation angles of the satellite, transformation matrix should be found in the equation (32) first with the determinant of one. "diag" operator returns a square diagonal matrix with elements of the vector on the main diagonal.

Rotation angle error covariance matrix (P) is necessary for determining the instant times which gives higher error results than desired.

$$P_{SVD} = Udiag[(s_2+s_3)^{-1} \quad (s_3+s_1)^{-1} \quad (s_1+s_2)^{-1}]U^T \tag{33}$$

where $s_1 = S_{11}$, $s_2 = S_{22}$, $s_3 = \det(U)\det(V)\, S_{33}$.

The satellite only has two sensors (e.g. sun and magnetic field sensor), thus the SVD-method fails when the satellite is in eclipse period and when the two observations are parallel.

5 SVD Aided EKF for Satellite Attitude Estimation Based on Linear Measurements - Nontraditional Approach

In this study, SVD has been used as the observation model in the EKF framework. The SVD and EKF algorithms are combined to estimate the attitude angles and angular velocities.

5.1 Problem Formulation

In case of EKF design based on linear Euler angle measurements, determination model of the angles that characterizes satellite's attitude, can be given as [7],

$$\begin{aligned} z_\varphi(k) &= \varphi(k) + \mathrm{v}_\varphi(k), \\ z_\theta(k) &= \theta(k) + \mathrm{v}_\theta(k), \\ z_\psi(k) &= \psi(k) + \mathrm{v}_\psi(k) \end{aligned} \tag{34}$$

where $\varphi(k)$, $\theta(k)$ and $\psi(k)$ are the attitude angles determined by SVD method, $\mathrm{v}_{(\cdot)}(k)$ is the measurement noise of the attitude angles. The mathematical expectations and variances of the measurement noises are

$$E\left[\mathrm{v}_{(\cdot)}(k)\right] = 0,\; E\left[\mathrm{v}_\varphi^2(k)\right] = Var\left(\mathrm{v}_\varphi(k)\right),$$
$$E\left[\mathrm{v}_\theta^2(k)\right] = Var\left(\mathrm{v}_\theta(k)\right) \text{ and } E\left[\mathrm{v}_\psi^2(k)\right] = Var\left(\mathrm{v}_\psi(k)\right).$$

It is assumed that both measurement and system noise vectors $\mathrm{v}(k)=\left[\mathrm{v}_\varphi(k) \quad \mathrm{v}_\theta(k) \quad \mathrm{v}_\psi(k)\right]^T$ and $w(k)$ are linearly additive Gaussian, temporally uncorrelated with zero mean and the corresponding covariances:

$$\begin{aligned} E\left[w(i)w^T(j)\right] &= Q(i)\delta(ij), \\ E\left[\mathrm{v}(i)\mathrm{v}^T(j)\right] &= R(i)\delta(ij), \end{aligned} \tag{35}$$

It is assumed that process and measurement noises are uncorrelated, i.e.,

$$E\left[w(i)\mathrm{v}^T(j)\right] = 0,\; \forall i, j\,. \tag{36}$$

It is required to design EKF for satellite attitude and rate estimation.

5.2 EKF Based on Linear Euler Angle Measurements

The mathematical model of the LEO satellite's rotational motion about its center of mass, is linearized using quasi-linearization method. We will consider a real-time linear Taylor approximation of the system function at the previous state estimate. The Kalman Filter which is obtained will be called the Extended Kalman Filter (EKF). Filter algorithm, in this case as, is given below [16]:

Equation of the estimation value,

$$\hat{x}(k+1) = \hat{x}(k+1/k) + K(k+1) \times \left\{z(k+1) - H\hat{x}(k+1/k)\right\} \tag{37}$$

Here $z(k+1) = \left[z_\varphi(k+1) \quad z_\theta(k) \quad z_\psi(k)\right]$ is the measurement vector, H is the measurement matrix. In the investigated case the measurement matrix can be written as

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Equation of the extrapolation value,

$$\hat{x}(k+1/k) = f\left[\hat{x}(k), k\right] \tag{38}$$

Filter-gain of EKF

$$\begin{aligned} &K(k+1) = P(k+1/k)H^T(k+1) \times \\ &\left[H(k+1)P(k+1/k)H^T(k+1) + R(k)\right]^{-1} \end{aligned} \tag{39}$$

The covariance matrix of the extrapolation error is,

$$P(k+1/k) = \frac{\partial f[\hat{x}(k),k]}{\partial \hat{x}(k)} P(k/k) \frac{\partial f^T[\hat{x}(k),k]}{\partial \hat{x}(k)} + Q(k) \tag{40}$$

The covariance matrix of the filtering error is,

$$P(k+1/k+1) = \left[I - K(k+1)H(k+1)\right]P(k+1/k) \tag{41}$$

where $R(k)$ is the covariance matrix of measurement noise, which has diagonal elements built of the variances of angle and angle rate measurement noises and $Q(k)$ is the covariance matrix of the system noises.

Equations given as (37)-(41) represent the EKF, which fulfils recursive estimation of the satellite's rotational motion parameters about its mass centre on the linear attitude measurements.

5.3 Simulation Results

Simulations are realized with a sampling time of $T_s = 1\sec$. As an experimental platform a cube-sat model is used. Nonetheless the orbit of the satellite is a circular orbit with an altitude of $r = 550km$. The time interval 2000-4000s correspond to the eclipse. For the magnetometer measurements, the sensor noise is characterized by zero mean Gaussian white noise with a standard deviation of $\sigma_m = 300\ nT$. The standard deviation for the sun sensor noise is taken as $\sigma_s = 0.002$ (for unit vector measurements). Portion of simulation results are given in Figs.1-4. Absolute errors and variance changes of attitude angles when SVD and SVD+EKF are used are given in Figs.1-3. Here, SVD/EKF attitude estimation results are superior at outside of the eclipse because of the coming covariance knowledge as an adaptation to the filter from SVD method. Measurement model is linear in this case, since the states are measured directly in SVD.

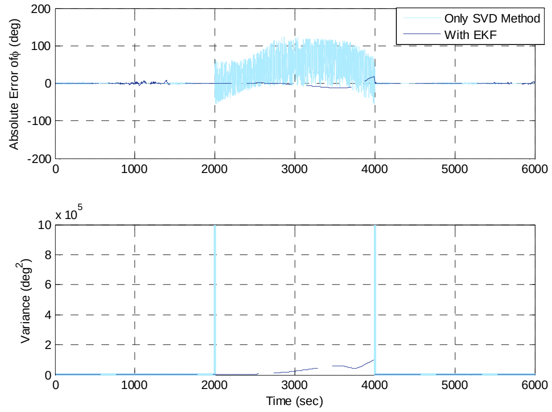

Fig.1. Absolute errors and variance changes of roll angle

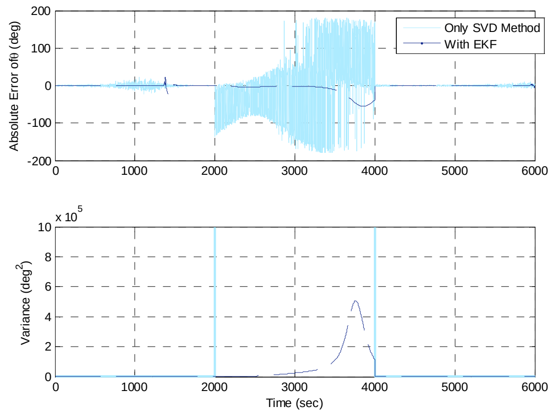

Fig.2. Absolute errors and variance changes of pitch angle

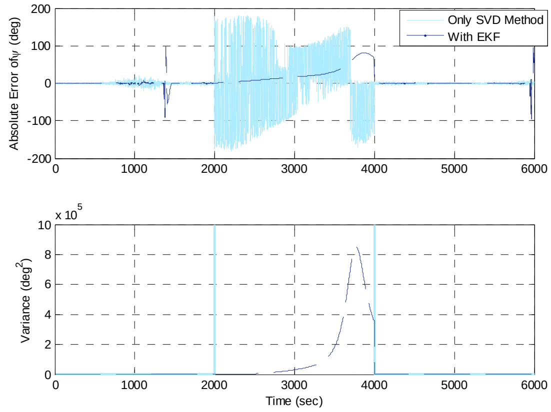

Fig.3. Absolute errors and variance changes of yaw angle

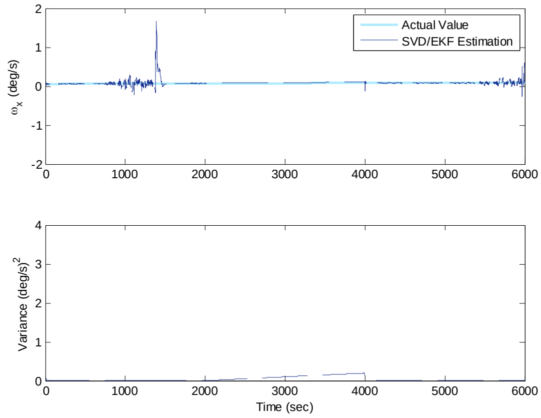

Fig.4. X-axis angular rate estimation results

The X-axes angular rate estimation results are shown in Fig. 4. The angular rates are estimated accurately. The SVD method fails in eclipse since there are no observations coming from the Sun. In this period the EKF gain decreases to very low values, the state update term of the EKF becomes insignificant and the predicted states contribute more to the estimations. In the figures, integrated SVD/EKF attitude estimation error increases through the eclipse. This situation is natural because EKF prediction errors are accumulated in time and after a while, they can get larger errors. In this case, another method can be used for attitude estimation e.g. EKF based on only magnetometer measurements or magnetometer and rate gyro measurements.

6 Nontraditional EKF Approach Based on Gyro and Vector Measurements

6.1 Rate Gyro Measurement Model

Three rate gyros are aligned through three axes, orthogonally to each other and they supply directly the angular rates of the body frame with respect to the inertial frame. Hence the model for rate gyros can be given as;

$$\bar{\omega}_{BI,meas} = \bar{\omega}_{BI} + \bar{b}_g + \eta_2 . \tag{42}$$

where, $\bar{\omega}_{BI,meas}$ is the measured angular rates of the satellite, $\bar{b}_g$ is the gyro bias vector as $\bar{b}_g = \begin{bmatrix} b_{g_x} & b_{g_y} & b_{g_z} \end{bmatrix}^T$ and η_2 is the zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_2(k)\eta_2^T(j)\right] = I_{3x3}\sigma_g^2\delta(kj), \tag{43}$$

Here, σ_g is the standard deviation of each rate gyro random error. Nevertheless, characteristic of gyro bias is given as,

$$\frac{d\bar{b}_g}{dt} = \eta_3 , \tag{44a}$$

where η_3 is also the zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_3(k)\eta_3^T(j)\right] = I_{3x3}\sigma_{gb}^2\delta(kj), \tag{44b}$$

Here, σ_{gb} is the standard deviation of gyro biases.

6.2 EKF Design

The angular velocities $\omega_x, \omega_y, \omega_z$ of satellite are measured through the rate gyroscopes. If the state vector

$$\bar{U}^T = \begin{bmatrix} \varphi & \theta & \psi & \omega_x & \omega_y & \omega_z & b_{g_x} & b_{g_y} & b_{g_z} \end{bmatrix}^T, \tag{45}$$

is arranged and the mathematical model of the LEO satellite's rotational motion about its center of mass, is linearized using quasi-linearization method,

$$\begin{aligned} U(k) &= f\left(\hat{U}(k-1), \bar{\omega}_o(k-1)\right) + F_U(k-1)\left(U(k-1) - \hat{U}(k-1\right) \\ &+ F_o(k-1)\left(\bar{\omega}_o(k-1) - \omega_o^{comp}(k-1)\right) \end{aligned} \tag{46}$$

where $f\left(\hat{U}(k-1), \bar{\omega}_o(k-1)\right)$ is the right hand side of the LEO satellite's rotational motion mathematical model based on estimated values; $\bar{\omega}_o(k-1)$ is the satellite's orbital velocity (system input); $\omega_o^{comp}(k-1)$ is the computational value of the satellite's orbital velocity; F_o is the coefficient matrix of the system input;

$$F_U(k-1) = \left[\frac{\partial f}{\partial U}\right]_{\hat{U}(k-1), \bar{\omega}_o(k-1)},$$

$$F_o(k-1) = \left[\frac{\partial f}{\partial \omega_o}\right]_{\hat{U}(k-1), \bar{\omega}_o(k-1)} \tag{47}$$

Minimum of the error's standard deviation was selected as an optimum criterion. The recursive algorithm for the satellite's attitude estimation is obtained using Bayes' method as bellow,

$$\begin{aligned} \hat{U}(k) &= f\left(\hat{U}(k-1), \bar{\omega}_o(k-1)\right) \\ &+ K(k)\left[z(k) - Hf\left(\hat{U}(k-1), \bar{\omega}_o(k-1)\right)\right] \end{aligned} \tag{49}$$

$$\begin{aligned} P(k) &= M(k) - M(k)H^T \\ &\times \left[R + HM(k)H^T\right]^{-1} HM(k) \end{aligned} \tag{50}$$

$$K(k) = P(k)H^T R^{-1} \tag{51}$$

$$\begin{aligned} M(k) &= F_U(k-1)P(k-1)F_U^T(k-1) \\ &+ F_o(k-1)D_o(k-1)F_o^T(k-1) + Q(k-1) \end{aligned} \tag{52}$$

where $z^T(k) = \left[z_\varphi(k), z_\theta(k), z_\psi(k), z_{\omega_x}(k), z_{\omega_y}(k), z_{\omega_z}(k)\right]$ is the measurement vector; $M(k)$ is the covariance matrix of the extrapolation error, $P(k)$ is the covariance matrix of the estimation error, $K(k)$ is the gain matrix of Kalman filter, $D_o(k-1)$ is the variance which characterizes uncertainty of the calculated values of satellite's orbital velocity, R is the covariance matrix of measurement noise, which is diagonal matrix with diagonal elements built of the variances of angle and angle rate measurement noises and $Q(k-1)$ is the covariance matrix of the system noises.

Equations given as (49)-(52) represent the EKF, which fulfils recursive estimation of the satellite's rotational motion parameters about its mass center and rate gyros biases.

6.3 Simulation Results

In Table 1, RMS Error results can be seen for different time intervals of the satellite's trajectory (including eclipse).

Table 1. RMS Errors for Different Time Intervals in Case of Different Attitude Determination Algorithms (Error1: 0-2000 sec, Error2: 2000-4000 sec, Error3: 4000-6000 sec)

RMS Errors	Gyro Bias for Each Direction (rad/sec)	Error 1 (deg)	Error 2 (Eclipse) (deg)	Error 3 (deg)
Only SVD	-	32	123	16
EKF w/o bias estimation	0	0.08	0.13	0.50
EKF with bias estimation	0	1.12	6.17	0.55
EKF w/o bias estimation	0.005	1.64	17.31	0.97
EKF with bias estimation	0.005	1.01	4.83	0.53

As seen from Table 1, at the beginning, the algorithm using the SVD method only, gives the RMS errors as a coarse attitude of the satellite. Especially in eclipse period, error reaches a very large value because no data from sun for SVD which is depending on the vector measurements only (magnetometer and sun sensor for this case). The rows from 2 to 5 show the SVD aided EKF algorithm with and w/o gyro bias or gyro bias estimation. If gyro biases exist, then the gyro bias estimation in the filter works well even in the eclipse period. If the measurements do not include any gyro biases, bias estimation has a negative effect on the results.

Portion of simulation results are given in Figs.5-10. Absolute errors of attitude angles when SVD and SVD+EKF are used are given in Fig.5. As seen, SVD +EKF attitude estimation results are superior.

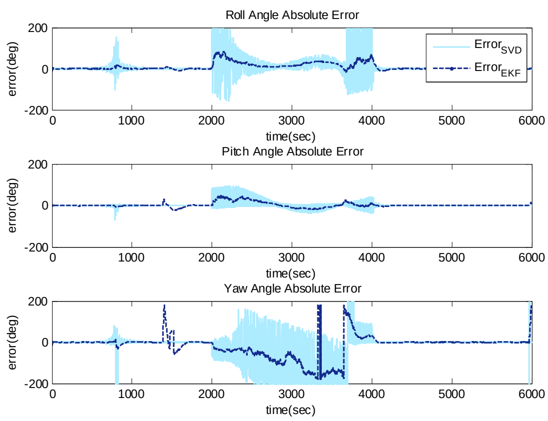

Fig.5. Absolute errors of attitude angles

In Fig.6 the estimated values of Euler angles by SVD and SVD+EKF and the actual values of the angles are shown. As seen from the obtained results, SVD+EKF estimation values are very close to the actual values outside of the eclipse and sufficiently better then the only SVD results even in the eclipse period.

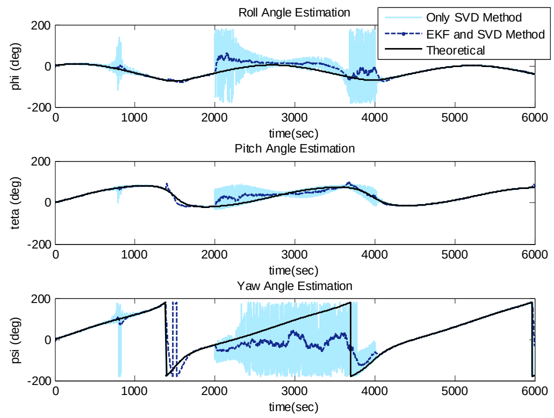

Fig.6. Attitude angles estimation results

In the first panels of Figs.7-10 gray lines indicate the estimation results and black lines actual values of parameters.

In Figs.7-9 the estimated values of biases, the error between the actual values of the biases and their estimated values and variances of the estimation errors in x,y, and z axes respectively are shown. The results show that the gyro biases are estimated accurately and the values of the estimation errors approach to zero for all three cases. The values of estimation error variances decrease with time; therefore variance results support the convergence of the filter.

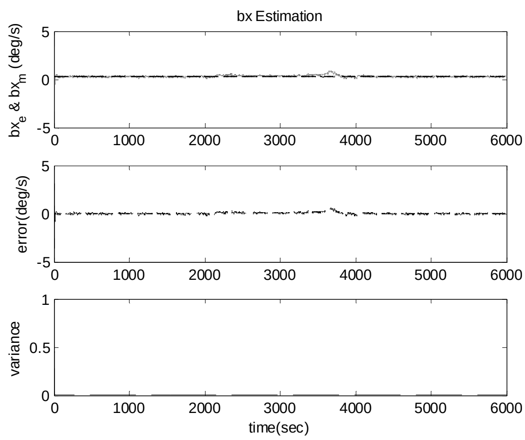

Fig.7. Rate gyro bias estimation results in the X-axis

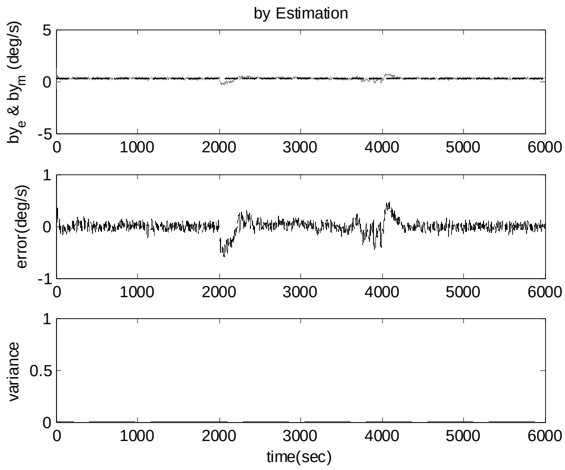

Fig.8. Rate gyro bias estimation results in the Y-axis

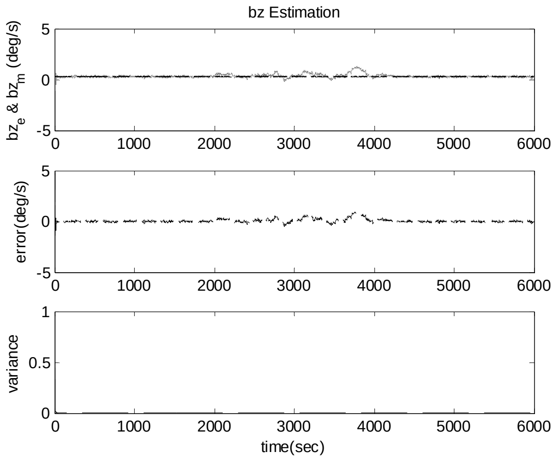

Fig.9. Rate gyro bias estimation results in the Z-axis

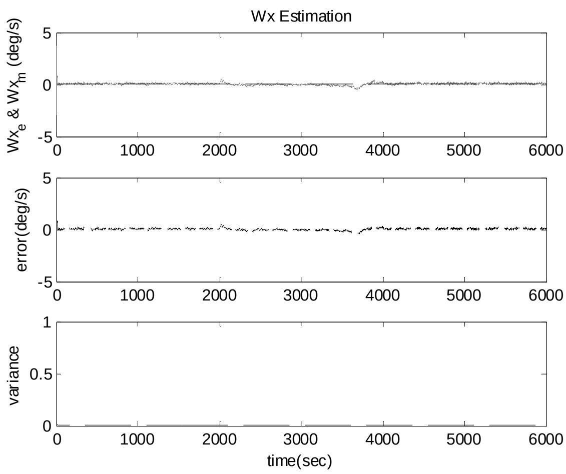

Fig.10. X-axis angular rate estimation results

Fig. 10 characterize the X-axes angular rate estimation results. For outside of the eclipse, the presented filter gives sufficiently good angular velocity estimation results. Also, the estimation results are not deteriorated during the eclipse.

8 Conclusion

In this study the SVD and EKF algorithms are combined as a two-phased estimation algorithm to estimate the attitude angles and gyro biases of small satellite. In the first phase, Wahba's problem, a well-known approach for single frame attitude estimation with vector sun sensor and magnetometer measurements, is solved by the SVD method and Euler angles estimations are obtained for the satellite's attitude. Obtained Euler angles estimations are used as measurement inputs, which forms the second phase of the algorithm and then rate gyros are considered in the filter for EKF. The covariance estimation of the SVD, is used as the part of the measurement noise covariance matrix of the EKF; this is how the filter is tuned specifically in the eclipse period. The EKF provides improved attitude knowledge. The whole algorithm runs recursively.

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