# Robust $H_{\infty}$ State Feedback plus State-Derivative Feedback Controller Design for Uncertain T-S Fuzzy Systems

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Abstract: - This paper examines the problem of designing a robust  $H_{\infty}$  state feedback plus state-derivative feedback control for a class of uncertain nonlinear systems which is described by a Takagi-Sugeno (T-S) fuzzy model. A linear matrix inequalities (LMIs) approach is employed to obtain the robust controller for such a system. Simultaneously, the illustrative example is given to show the effectiveness of the proposed methodology. The results show that the proposed approach guarantees the fulfilments on both the asymptotic stability and the performance index. the page.

*Key-Words:* - Robust  $H_{\infty}$  control, State feedback, State-derivative feedback, Linear matrix inequalities (LMIs), Takagi-Sugeno (T-S) fuzzy model.

#### **1** Introduction

Over the past two decades, there has also been a rapidly growing interest in application of the topic of  $H_{\infty}$  control problem [1-2]. To achieve the stabilization with the prescribed performance index, the controllers are widely synthesized using methods. In spite of these successes, there are many basic issues that still remain to be addressed [3-4]. As occurring so often in reality, the strong nonlinearities of the plants may lead to severe difficulties for the stability analysis and the controller synthesis of nonlinear systems.

Recently, fuzzy systems in the T-S model can well solve for those  $H_{\infty}$  problems [5-7]. The nonlinear system models can be explained by the T-S fuzzy model construction procedures. The T-S fuzzy control design is derived by utilizing the concept of the parallel distributed compensation (PDC); i.e., a fuzzy system is represented by each plant rule model. Thus, the T-S fuzzy design procedure can simply describe the global behaviour of a nonlinear system. In addition, T-S fuzzy model based LMI technique can be used to solve that the stability analysis and the control design problems. During the past decades, the issues of uncertainty nonlinear control via  $H_{\infty}$  fuzzy approach have been reported in several literatures [8-10].

Notwithstanding the  $H_{\infty}$  fuzzy approach, the controlled method for many practical applications cannot easily meet the prescribed performance index of system in uncertain nonlinear systems. The nonlinearity and uncertainty always exist in real-life

applications such as industrial applications, telecommunication networks and chemical systems. The high nonlinearities and external disturbances noises are considered as a source of poor control performances and instabilities. In the last few years, increasing attentions have been investigated the problem of uncertainty nonlinear system. Recently, the uncertainty systems control designs using the  $H_{\infty}$  fuzzy approaches have been discussed [11-12].

However, some problems are occurred on the real mechanical control systems where the obtained measurable signals are the state feedback and the state-derivative feedback signals. For instance, the real physical nonlinear system can be seen in the control of suppression systems where the accelerometers presented principal sensors of vibration. Therefore, the state variables are defined by the velocities and displacement as the statederivative feedback and the state feedback, respectively [13]. Some state-derivative feedback approach results have been reported in [14-16]. Furthermore, the novel results are acquired in [17] and [18] by developing the design of  $H_{\infty}$  fuzzy control applied with the LMIs technique. Unfortunately, those results are not applied for the nonlinear system which includes uncertainties. As reported in several literatures, those designing approaches have not yet been adequately researched and those design problems have still been challenged.

As motivated by the facts abovementioned, in this paper, we examine the problem of designing a

robust  $H_{\infty}$  fuzzy state feedback plus state-derivative feedback controller for a class of uncertain nonlinear systems. First, T-S fuzzy model is applied to approximate the uncertain nonlinear systems. Then, LMIs methodology will be used to generalize a strategy for designing a robust  $H_{\infty}$  fuzzy state feedback plus state-derivative feedback controller with ensuring the fulfilments on both the performance and the robustness. This paper is organized as follows. Section 2 presents the preliminaries. Section 3, the proposed control strategy is demonstrated. Section 4, the result of this methodology is illustrated. Finally, the paper is concluded in Section 5.

### 2 Preliminaries

The T-S fuzzy model is demonstrated toward IF-THEN rules which might have the ability to estimate the nonlinear system toward joining together the linear models via nonlinear membership functions. A T-S fuzzy model will be analyzed in the *i*-th rule as follows:

*Plant rule* : IF  $x_{k1}(t)$  is  $M_{1i}(t)$  and...and  $x_{kj}(t)$  is  $M_{ii}(t)$  THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t) + B_w w(t),$$
 (1)

$$z(t) = C_i x(t) \tag{2}$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$  and  $z(t) \in \mathbb{R}^s$  is the state vector, the input and the controlled output, respectively.  $w(t) \in \mathbb{R}^p$  is the input disturbance which belongs to  $L_2[0, \infty)$ ,  $M_{ji}(t)$  is the fuzzy sets, r is the number of IF-THEN rules,  $x_{k_j}(t)$  is the premise variables, the matrices  $A_i, B_i, B_w$  and  $C_i$  are suitable matrices of the system. In this paper, it is assumed that  $x_k(t)$  is the vector containing all the individual elements  $x_{k_1}(t), \dots, x_{k_j}(t)$ . For any specified state vector and the control input, the T-S fuzzy model is inferred as follows.

Let

$$\varpi_i(x(t)) = \prod_{j=1}^{\nu} M_{ji}(x_k(t)),$$

and

$$\mu_i\bigl(x(t)\bigr) = \frac{\varpi_i(x(t))}{\sum_{i=1}^r \varpi_i(x(t))}$$

where  $M_{ji}(x_k(t))$  is the grade of membership of  $x_k(t)$  in  $M_{ji}$ .

It is assumed in this paper that

$$\varpi_i(x(t)) \ge 0, \qquad i = 1, 2, \dots, \nu, \tag{3}$$

$$\sum_{i=1}^{r} \varpi_i \big( x(t) \big) > 0 , \qquad i = 1, 2, \dots, \nu, \tag{4}$$

where v are the number of premise variables, for all t. For he simplicity of the notations, we use  $\varpi_i = \varpi_i(x(t))$  and  $\mu_i = \mu_i(x(t))$ . Thus, we can generalize that the T-S fuzzy models as the weighted average of the following forms:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i \left[ A_i x(t) + B_i u(t) + B_w w(t) \right], (5)$$
  
$$z(t) = \sum_{i=1}^{r} \mu_i \left( C_i x(t) \right), \quad i = 1, 2, 3, ..., r. \quad (6)$$

However, the phenomena of uncertain parameters and disturbances are frequently encountered in most real dynamical systems. Those problems are found within the complexity of the designing problem. Robust control methods aim to achieve the robust performance and the stability in the presence of bounded modelling errors. Thus, the T-S fuzzy system can be considered with parametric uncertainties as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i \left[ (A_i + \Delta A_i) x(t) + (B_i + \Delta B_i) u(t) + (B_w + \Delta B_w) w(t) \right],$$
(7)

$$z(t) = \sum_{i=1}^{r} \mu_i \left[ (C_i + \Delta C_i) x(t) \right], i = 1, 2, ., r \quad (8)$$

where the matrices  $A_i, B_i, B_w$  and  $C_i$  are defined same as the previous section and the matrices  $\Delta A_i$ ,  $\Delta B_i, \Delta B_w$  and  $\Delta C_i$  represent the uncertainties in the system and satisfy the following assumption.

Assumption 1 :

$$\Delta A_i = F(x(t), t)H_{1i}, \Delta B_w = F(x(t), t)H_{2i},$$
  
$$\Delta B_i = F(x(t), t)H_{3i}, \Delta C_i = F(x(t), t)H_{4i},$$

where  $H_{j_i}$ , j = 1, 2, 3, 4 are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\|F(x(t),t)\| \leq \rho ,$$

for any known positive constant  $\rho$ . Next, let us recall the following definitions.

*Definition 1:* Suppose  $\gamma$  is a given positive real number: A system of the form (7) is said to  $L_2$ -gain less than or equal to  $\gamma$  if

$$\int_0^{T_f} z^T(t) z(t) dt \leq \gamma^2 \left[ \int_0^{T_f} w^T(t) w(t) dt \right], (9)$$

for all  $T_f \ge 0$  and  $w(t) \in L_2[0, T_f]$ .

*Definition 2*: (Asymptotic stability) Let  $x_e = 0$  be an equilibrium for  $\dot{x} = f(x)$ . Let  $V : \mathbb{R}^n \to \mathbb{R}$  be a continuously differentiable function such that

- V(0) = 0 and V(x) > 0 for all  $x \neq 0$ ,
- $\dot{V}(x) < 0$  for all  $x \neq 0$ ,  $\dot{V}(0) = 0$ .

Then,  $x_e$  is asymptotically stable and is the unique equilibrium point.

Note that for the symmetric block matrices, we use (\*) as an ellipsis for terms that are induced by symmetry. So the following results deal with the system (7) - (8).

### 3 Main Result

This section begins with considering the problem of designing a robust  $H_{\infty}$  state feedback plus statederivative feedback controller which guarantees the  $L_2$  gain from the exogenous input noise to the regulated output being less than or equal to some prescribed values. An LMIs methodology will be used to infer a fuzzy controller which stabilizes the system (7). Suppose there exists a fuzzy state feedback plus state-derivative controller of the term:

Controller rule j: IF  $x_{k1}(t)$  is  $M_{1i}(t)$  and...and  $x_{kj}(t)$  is  $M_{ji}(t)$  THEN

$$u(t) = K_{s_i} x(t) - K_{d_i} \dot{x}(t), \forall j = 1, 2, ..., r (10)$$

where x(t) is state vector  $K_{s_j}$  and  $K_{d_j}$  are the controller gain of a state feedback controller and state-derivative feedback controller respectively. Finally, the fuzzy controller as shown in Fig.1 can be inferred as

$$u(t) = \sum_{j=1}^{r} \mu_j \left( K_{s_j} x(t) - K_{d_j} \dot{x}(t) \right), \forall_j = 1, 2, ., r.$$
(11)

Considering the system (7)-(8) with the controller (11) as shown in Fig.2 yields

$$[I + \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i} \mu_{j} B_{i} K_{d_{j}}] \dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i} \mu_{j}$$
$$[A_{i} x(t) + B_{i} K_{s_{j}} x(t) + \tilde{B}_{w} \tilde{w}(t), (12)$$

where  $\tilde{B}_{w_i} = [\delta I \ I \ \delta I \ B_w]$  and the disturbance is

$$\widetilde{w}(t) = \begin{pmatrix} \frac{1}{\delta} F(x(t), t) H_{1_i} E_{ij} x(t) \\ F(x(t), t) H_{2_i} w(t) \\ \frac{1}{\delta} F(x(t), t) H_{3_i} E_{ij} K_{s_j} x(t) \\ w(t) \end{pmatrix}.$$

The issue is to obtain state-feedback gains and state derivative feedback gains  $K_{s_j}$ ,  $K_{d_j}$ , j = 1, 2, 3, ..., r, respectively, such that the following conditions hold:

- 1) Matrices  $(I + B_i K_{d_j})$ ,  $\forall i, j = 1, 2, 3, ..., r$ , have a full rank.
- 2) The system (7)-(8) with the fuzzy controller (11) is asymptotically stable and the  $H_{\infty}$  performance is satisfied for all admissible values based on the sufficient condition for a prescribed scalary > 0.

To establish the proposed results and without loss of generality, we assume that there exists the following assumption: rank  $[I|B_i] = n$  exists. Thus, it is easy to known that if rank  $[I|B_i] = n$  holds, then there exists  $K_{d_j}$  such that rank  $[I + B_iK_{d_j}] = n$ , that is, matrices  $(I + B_iK_{d_j}), \forall i, j = 1, 2, 3, ..., r$ , have full rank. From the above conditions and assumption, we define

$$E_{ij} = \left(I + B_i K_{d_j}\right)^{-1}, \qquad (13)$$

then (12) can be written as

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i} \mu_{j} \left[ E_{ij} (A_{i} + B_{i} K_{s_{j}}) x(t) + E_{ij} \tilde{B}_{w} \widetilde{w}(t) \right],$$
(14)

An LMIs approach will be applied to derive a fuzzy controller which stabilizes the system (14) and guarantees the disturbance rejection of level  $\gamma > 0$ , immediately. Firstly, to design the state feedback plus state-derivative feedback controller, the following design objectives are satisfied;

- (a) The closed loop system is asymptotically stable when w(t) = 0,
- (b) Under zero initial condition, the system (14) satisfies ||z||<sub>2</sub> ≤ γ ||w||<sub>2</sub> for any non-zero w(t) ∈ L<sub>2</sub> [0, +∞) where γ > 0 is a prescribed constant.



Fig. 1 The weighted average of fuzzy controller model.



Fig. 2 The closed-loop fuzzy system.

*Theorem 1* : Consider the system (7)-(8). Given a prescribed  $H_{\infty}$  performance  $\gamma > 0$  and a positive constant  $\delta$ , if there exist symmetric matrices P > 0 and matrices  $Y_{s_j}, Y_{d_j}, j = 1, 2, ..., r$ , satisfying the following linear matrix inequalities:

$$\Psi_{ii} < 0, \quad i = 1, 2, \dots, r, \quad (15)$$

$$\Psi_{ij} + \Psi_{ji} < 0, \quad i < j \le r,$$
 (16)

where

 $\Psi_{ii} =$ 

			-						
$\Theta_{1_{ij}}$	$(*)^T$	$(*)^T$							
δΙ	$-\gamma I$	$(*)^{T}$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^{T}$
Ι	0	$-\gamma I$	$(*)^{T}$	$(*)^{T}$	$(*)^{T}$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^{T}$
δΙ	0	0	$-\gamma I$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^{T}$
$B_w$	0	0	0	$-\gamma I$	$(*)^{T}$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^{T}$
$\Theta_{2ii}$	0	0	0	0	$-\gamma I$	$(*)^T$	$(*)^T$	$(*)^T$	$(*)^{T}$
0	0	0	0	0	0	$-\gamma I$	$(*)^T$	$(*)^T$	$(*)^{T}$
$\Theta_{3_{ij}}$	0	0	0	0	0	0	$-\gamma I$	$(*)^T$	$(*)^T$
$\Theta_{4_{ii}}$	0	0	0	0	0	0	0	$-\gamma I$	$(*)^T$
$\Theta_{5_{ij}}$	0	0	0	0	0	0	0	0	-P /
									(17)

with

$$\Theta_{1ij} = PA_i^T + A_i P + Y_{sj}^T B_i^T + B_i Y_{sj} + B_i Y_{dj} A_i^T + A_i Y_{dj}^T B_i^T,$$
(18)

$$\Theta_{2ij} = \frac{\gamma \rho}{\delta} H_{1i}^T P + \frac{\gamma \rho}{\delta} H_{1i}^T Y_{dj}^T B_i^T, \qquad (19)$$

$$\Theta_{3ij} = \sqrt{2\lambda\rho}H_{3i}^{I}P + \sqrt{2\lambda\rho}H_{3i}^{I}Y_{dj}^{I}B_{i}^{I}, \qquad (20)$$

$$\Theta_{4_{ij}} = \sqrt{2\lambda C_i^I P} + \sqrt{2\lambda C_i^I Y_{d_j}^I B_i^I}, \qquad (21)$$

$$\Theta_{5_{ij}} = (Y_{s_j} + Y_{d_j})^T B_i^T , \qquad (22)$$

and

$$\lambda = \left(1 + \rho^2 \sum_{i=1}^r \sum_{j=1}^r \left[ \left\| H_{2_i}^T H_{2_i} \right\| \right] \right)^{1/2}.$$
 (23)

Furthermore, the suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{r} \mu_j \left( K_{s_j} x(t) - K_{d_j} \dot{x}(t) \right), \forall j = 1, 2, ..., r, \quad (24)$$

where

$$K_{s_i} = Y_{s_i} P^{-1} (25)$$

and

$$K_{d_j} = Y_{d_j} P^{-1}. (26)$$

*Proof.* For brevity, the full proof have been omitted in this paper

## **4** Illustrative example

Consider the following uncertain nonlinear system which is described by the equation follows [19]:

$$\dot{x}_{1}(t) = -\sigma x_{1}(t) + \sigma x_{2}(t) + u(t) + 0.1w_{1}(t)$$
  

$$\dot{x}_{2}(t) = rx_{1}(t) - x_{2}(t) - (x_{1}(t))x_{3}(t)$$
  

$$+0.1w_{2}(t) \qquad (27)$$
  

$$\dot{x}_{3}(t) = (x_{1}(t))x_{2}(t) - bx_{3}(t) + 0.1w_{3}(t)$$
  

$$z(t) = [x_{1}^{T}(t) \quad x_{2}^{T}(t) \quad x_{3}^{T}(t)]^{T}$$

where  $x_1(t)$ ,  $x_2(t)$ ,  $x_3(t)$  denote the state vectors, u(t) is the control input, z(t) is the regulated output,  $w_1(t)$ ,  $w_2(t)$ ,  $w_3(t)$  are the disturbance noise inputs and the bounded uncertain parameters  $\sigma$ , r and b are given by  $10 \pm 30\%$ ,  $28 \pm 30\%$  and  $8/3 \pm 30\%$ , respectively. Note that the variables  $x_1(t)$ ,  $x_2(t)$  and  $x_3(t)$  are treated as the deviation variables (variables deviate from the desired trajectories). The control objective is to control the state variable  $x_1(t)$  for the range  $x_1(t) \in [N_1 N_2]$ . The nonlinear system plant can be approximated by two T-S fuzzy rules. Let choose the membership functions of the fuzzy sets as:

$$M_1(x_1(t)) = \frac{-x_1(t) + N_2}{N_2 - N_1}$$

and

$$M_2(x_1(t)) = \frac{x_1(t) - N_1}{N_2 - N_1}$$

Note that  $M_1(x_1(t))$  and  $M_2(x_1(t))$  can be interpreted as the membership functions of fuzzy sets.

Knowing that  $x_1(t) \in [N_1 \ N_2]$ , the nonlinear system (27) can be approximated by the following two rules T-S model:

Plant rule 1: IF 
$$x_1(t)$$
 is  $M_1(x_1(t))$  THEN  
 $\dot{x}(t) = [A_1 + \Delta A_1]x(t) + B_{1_1}w(t)$   
 $+B_{2_1}u(t), x(0) = 0,$   
 $z(t) = C_1x(t),$ 

Plant rule 2: IF  $x_1(t)$  is  $M_2(x_1(t))$  THEN  $\dot{x}(t) = [A_2 + \Delta A_2]x(t) + B_{1_2}w(t)$   $+B_{2_2}u(t), x(0) = 0,$  $z(t) = C_2x(t),$  where

$$A_{1} = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -N_{1} \\ 0 & N_{1} & -8/3 \end{bmatrix},$$

$$A_{2} = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -N_{2} \\ 0 & N_{2} & -8/3 \end{bmatrix},$$

$$B_{1_{1}}, B_{1_{2}} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$B_{2_{1}}, B_{2_{2}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, C_{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\Delta A_{1} = F(x(t), t)H_{1_{1}}, \Delta A_{2} = F(x(t), t)H_{1_{2}}$$

$$x(t) = [x_{1}^{T}(t) \quad x_{2}^{T}(t) \quad x_{3}^{T}(t)]^{T} \text{ and}$$

$$w(t) = [w_{1}^{T}(t) \quad w_{2}^{T}(t) \quad w_{3}^{T}(t)]^{T}.$$

Now by assuming that  $||F(x(t), t)|| \le \rho = 1$ , we have

$$H_{1_1}, \ H_{1_2} = \begin{bmatrix} -0.3\sigma & 0.3 & 0\\ 0.3r & 0 & 0\\ 0 & 0 & -0.3b \end{bmatrix}.$$

Using the LMIs optimization algorithm and Theorem 1 with  $\gamma = 1$  and  $\delta = 1$ , with the fuzzy controller

$$u(t) = \sum_{j=1}^{2} \mu_j \left( K_{s_j} x(t) - K_{d_j} \dot{x}(t) \right)$$
(28)

where

 $\mu_1 = M_1(x_1(t))$  and  $\mu_2 = M_2(x_1(t))$ .

*Remark 1:* The fuzzy controller (28) ensures that the inequality (9) holds. As depicted in Fig. 3, after 1.3 seconds, the ratio of the regulated output energy to the disturbance input noise energy tends to a constant value which is less than the prescribed value 1.

#### **5** Conclusion

This paper has investigated a robust  $H_{\infty}$  fuzzy state feedback plus state-derivative feedback controller design procedure for a class of uncertain nonlinear systems that guarantees the  $L_2$ -gain from an exogenous input to a regulated output to be less or equal to a prescribed value. Based on a linear matrix inequalities (LMIs) approach, LMIs based sufficient conditions for the uncertain Takagi-Sugeno (T-S) fuzzy model to have an  $H_{\infty}$  performance are established. The effectiveness of the proposed de-



sign methodology is demonstrated through the illustrative examples. However, the time-varying delay system can be easily found in many real physical control problems. Thus, the robust  $H_{\infty}$  fuzzy state feedback plus state-derivative feedback controller for the uncertain nonlinear with time-varying delay system can be studied in the future research work.

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