

# Modeling of the Galvanic Anode Cathodic Protection System with Dynamic Polarization Characteristics

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*Abstract:* - Cathodic protection systems are used for protection of underground or underwater metallic infrastructure against corrosion. Protection by cathodic protection system is achieved by polarizing protected object i.e. by shifting equilibrium potential to more negative value. Value of the electric potential and current density on the surface of protected object are essential data for evaluation of efficiency of the cathodic protection system. Value of these parameters can be determined by using numerical techniques. This paper deals with the mathematical modeling of cathodic protection system when taking into account dynamic nonlinear polarization characteristics on the electrode surface. Firstly, mathematical model is described in detail. Numerical procedure presented in this paper is divided in the two parts. First part is the calculation of distribution of electric potential and current density in spatial domain using direct boundary element method, and calculation of the time changes of these parameters by finite difference time domain method. Finally, presented mathematical model was used for calculation of parameters of one geometrically simple cathodic protection system example.

*Key-Words:* - Cathodic Protection System, Galvanic Anode, Dynamic Nonlinear Polarization Characteristics, Boundary Element Method, Finite Difference Time Domain Method, Newton-Raphson Technique.

## 1 Introduction

Cathodic protection is most widely used technique for protection of the underground and underwater metallic structures from corrosion [1]. This technique is based on the shifting the equilibrium potential of protected structure to more negative value. This can be done by connecting the protected object with additional electrode(s) placed in same electrolyte (ground or water), which equilibrium potential is more negative than equilibrium potential of the protected object [2]. After installation of the

cathodic protection system, electric potential value on the entire surface of protected structure must be lower than minimum protection potential value defined by standards [3]. Also, current density distribution on the surface of protected structure should be uniform as possible [4]. Therefore, for evaluation of efficiency of the cathodic protection system, electric potential and protection current density distribution on the protected metallic structure surface need to be known [5]. These parameters can be determined by solving Laplace

partial differential equation along with the adequate boundary conditions [1]. This type of the problems can be easily solved by using numerical techniques. For modeling of the cathodic protection systems, boundary element method is mostly used [6-8]. The main advantage of this method, in comparison to the other numerical methods, is that it requires discretization only of boundaries of considered domain and there is no need for discretization of infinite boundaries [9]. Additionally, iterative techniques for solution of nonlinear equations (such as Newton – Raphson technique) must be included because nonlinear boundary conditions (nonlinear polarization characteristics) are given on electrode surfaces of cathodic protection system.

In some situations, these nonlinear boundary conditions can change over the time (dynamic nonlinear polarization characteristics) [10-12]. These time changes of boundary conditions affect the electric potential and current density distribution in the system. Therefore, for modeling of the cathodic protection system with dynamic nonlinear polarization characteristics, some modifications in mathematical model must be done. In this paper, this problem is solved by using combined boundary element method and finite difference time domain method.

## 2 Mathematical Model

As previously mentioned, the purpose of modeling of the cathodic protection system is to determine the distribution of protection current density and electric potential on the surface of the protected structure in order to verify the effectiveness of the system. Electric potential distribution of cathodic protection system can be determined by solving Laplace partial differential equation for static current field:

$$\nabla^2 \varphi = 0 \quad (1)$$

where  $\varphi$  is electric potential and  $\nabla$  is Nabla operator.

To obtain a unique solution it is necessary to add the appropriate boundary conditions on domain boundaries to a Laplace partial differential equation. Boundaries that are taken into account when modeling cathodic protection system are anode/electrolyte interface and cathode/electrolyte interface. Boundary conditions applied to these boundaries are called polarization characteristics and they are influenced by electrochemical reaction that takes place on electrode surfaces. Polarization characteristics represent the correlation between

current density and electric potential on electrode surfaces and they are nonlinear. When polarization characteristics change over the time then they are dynamic polarization characteristics. These changes of polarization characteristics over the time can be caused by calcareous and magnesium deposition on the electrode surface. This phenomenon is characteristic for the cathode surface [11,13].

Dynamic polarization characteristic on the cathodic surface is defined by the following mathematical relation:

$$j_C = j_{O_2}^* f_{O_2} + j_{H_2} \left( 10^{\left( \frac{\varphi - \varphi_{H_2}^*}{\beta_{H_2}} \right)} f_{H_2} \right) - j_{Fe} \left( 10^{\left( \frac{\varphi - \varphi_{Fe}^*}{\beta_{Fe}} \right)} f_{Fe} \right) \quad (2)$$

where  $j_{O_2}^*$  is threshold current density of oxygen reduction,  $j_{H_2}$  is current density of the hydrogen separation,  $j_{Fe}$  is current density of the metal dissolution,  $\varphi_{H_2}^*$  and  $\varphi_{Fe}^*$  are equilibrium potentials for corresponding electrochemical reactions,  $\beta_{H_2}$  and  $\beta_{Fe}$  are Tafel's coefficients,  $\varphi$  is potential difference of interface metal/electrolyte and  $f_{O_2}$ ,  $f_{H_2}$  and  $f_{Fe}$  are time-dependent factors. For all three partial electrochemical reactions, the time-dependent factors have a following form:

$$f_R = e^{-\frac{t}{\tau_R}} + f_{\infty,R} \left( 1 - e^{-\frac{t}{\tau_R}} \right) \quad (3)$$

where index  $R$  represents a corresponding electrochemical reaction.

On the anode surface, polarizing characteristic is defined by the following mathematical relation:

$$j_a = j_{O_2} \left( 10^{\left( \frac{\varphi - \varphi_a}{\beta_a} \right)} - 1 \right) \quad (4)$$

where  $j_{O_2}$  is current density corresponding to the oxygen reduction,  $\varphi_a$  is equilibrium potential of galvanic anode and  $\beta_a$  is Tafel slope of galvanic anode.

Graphical representation of the polarization characteristics for anode and cathode surface is given on the Figure1. Polarization characteristics are

constructed using relations (2) and (4) and data from literature [13].

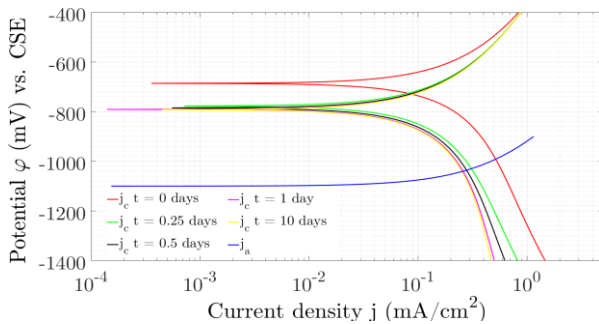


Figure 1. Polarization characteristics of the electrode surfaces

For calculation of the electric potential and current density distribution of the cathodic protection system with dynamic polarization characteristic, numerical procedure based on boundary element method and finite difference time domain method was used. This numerical procedure can be divided into two parts. First part is calculation of electric potential and current density distribution in spatial domain at  $t = 0$  using boundary element method and second part for calculation of electric potential and current density change over the time using finite difference time domain method.

## 2.1 BEM formulation in spatial domain

Boundary element method for calculation of the electric potential and current density distribution in spatial domain is based on solving the integral field equation. Integral field equation can be obtained by applying Green's symmetric identity on Laplace partial differential equation (1). Then integral field equation have the following form:

$$\begin{aligned} c(q)\varphi(q) + \int_{\Gamma} \varphi(p) \cdot (\vec{n} \cdot \nabla G(p, q)) d\Gamma = \\ = -\rho \cdot \int_{\Gamma} G(p, q) \cdot j(p) d\Gamma + \varphi_{\infty} \end{aligned} \quad (5)$$

where  $p$  is position of the source point,  $q$  is position of the field point,  $\Gamma$  is boundary of the domain,  $\varphi(p)$  is electric potential of source point  $p$ ,  $\varphi(q)$  is electric potential of field point  $q$ ,  $c(q)$  is a constant,  $\vec{n}$  is normal unit vector,  $\rho$  is resistivity of surrounding electrolyte,  $j(p)$  is current density at source point  $p$ ,  $\varphi_{\infty}$  is constant potential at infinite boundary and  $G(p, q)$  is Green's function whose form for 2D plan parallel problems is as follows:

$$G(p, q) = \frac{1}{2\pi} \ln \frac{1}{|p - q|} \quad (6)$$

For problems with infinite boundaries, Gauss boundary condition must be added [14]:

$$\int_{\Gamma} j(p) d\Gamma = 0 \quad (7)$$

In this way it is ensured that there is no flow of current on infinite boundary, and by using Gauss boundary condition there is no need for discretization of infinite boundary.

In the process of calculation of electric potential and current density distribution using boundary element method, it is necessary to discretize domain boundary on appropriate number of boundary elements. After discretization of domain boundaries, field integral equation (5) is solved over every element. After application of the Collocation method at the point and numerical integration, solution of the electric potential and current density can be obtained for each node of all boundary elements by solving following matrix equation [15]:

$$[H] \cdot \{\varphi\} = [G] \cdot \{j\} \quad (8)$$

where  $\{\varphi\}$  is vector of unknown electric potentials in collocation points,  $\{j\}$  is vector of current densities in collocation points,  $[H]$  and  $[G]$  are matrix of influence coefficients of system geometry.

Since nonlinear boundary conditions are specified, to solve the matrix equation (8) it is necessary to apply an iterative technique. In this paper, matrix equation (8) is solved by using Newton – Raphson iterative technique proposed in [11,12]. The first step in solving a matrix equation (8) by Newton - Raphson technique is expansion of the vector current density in the Taylor series, as follows:

$$\{j\}_k = \{j\}_{k-1} + \left[ \frac{\partial j}{\partial \varphi} \right]_{k-1} \cdot \{\Delta\varphi\}_k \quad (9)$$

where  $k$  is number of the current iteration,  $[\partial j / \partial \varphi]$  is Jacobian matrix, and  $\{\Delta\varphi\}$  is vector of electric potential increments between the two adjacent iteration, and can be written as:

$$\{\Delta\varphi\}_k = \{\varphi\}_k - \{\varphi\}_{k-1} \quad (10)$$

By including matrix equations (9) and (10) in the equation (8), following equation can be written [11,12]:

$$\left( [H] - [G] \left[ \frac{\partial j}{\partial \varphi} \right] \right) \cdot \{\Delta \varphi\}_k = [G] \cdot \{j\}_{k-1} - [H] \cdot \{\varphi\}_{k-1} \quad (11)$$

From previous matrix equation vector of electric potential increments in  $k$  - th iteration can be calculated. Further, this vector is used in matrix equations (9) and (10) for calculation of current densities and electric potentials in same iteration. Iterative cycle is repeated until a convergence is achieved.

Since, this calculation can be time consuming some modifications are done in this paper in order to accelerate calculation procedure. In matrix equations (9) and (11) Jacobian matrix is used. This matrix is rectangular matrix whose all elements outside the main diagonal equal to zero. Therefore, instead of memorizing a square matrix, non - zero elements of Jacobian matrix are stored in form of vector. Also, GMRES iterative technique was used for calculation of matrix equation (11) in each iteration of Newton - Raphson method.

## 2.2 FDTD formulation

On cathode surface dynamic (time - varying) polarization characteristics are assumed. Therefore, to calculate change of the electric potential and current density over the time previously presented mathematical model needs to be expanded.

$$\{j\}_n = \{j\}_{n-1} + \left[ \frac{\partial j}{\partial \varphi} \right]_{n-1} \cdot \{\Delta \varphi\}_n + \left[ \frac{\partial j}{\partial t} \right]_{n-1} \cdot \{\Delta t\}_n \quad (12)$$

where  $n$  is time step at which current density is calculating,  $[\partial j / \partial \varphi]$  is matrix of first derivatives of current density function in the time,  $\{\Delta t\}$  is vector of time steps and  $\{\Delta \varphi\}_n$  is vector of electric potentials increments in  $n$ -th time step and can be written as:

$$\{\Delta \varphi\}_n = \{\varphi\}_n - \{\varphi\}_{n-1} \quad (13)$$

From equation (4) it is noticeable that polarization characteristic of anode is time independent, therefore in matrix equation (12) elements of matrix  $[\partial j / \partial \varphi]$  that corresponds to colocation points placed on anode surface are equal to zero.

In matrix equation (12) two vectors are unknown. By including matrix equations (12) and (13) in the matrix equation (8), matrix equation with one unknown vector can be obtained, as follows:

$$\left( [H] - [G] \left[ \frac{\partial j}{\partial \varphi} \right]_{n-1} \right) \cdot \{\Delta \varphi\}_n = [G] \cdot \{j\}_{n-1} + [G] \cdot \left[ \frac{\partial j}{\partial t} \right]_{n-1} \cdot \{\Delta t\}_n - [H] \cdot \{\varphi\}_{n-1} \quad (14)$$

By solving previous matrix equation vector of electric potential increments in new time step is obtained. Further, this vector is used in matrix equations (12) and (13) for calculation of current densities and electric potentials in same time step.

## 3 Case Study

Application of the previously presented mathematical model is demonstrated on example given on the Figure 2. Radius of anode is 1 m while radius of cathode is 5 m. Both, anode and cathode are placed in the electrolyte with resistivity 25  $\Omega\text{cm}$ .

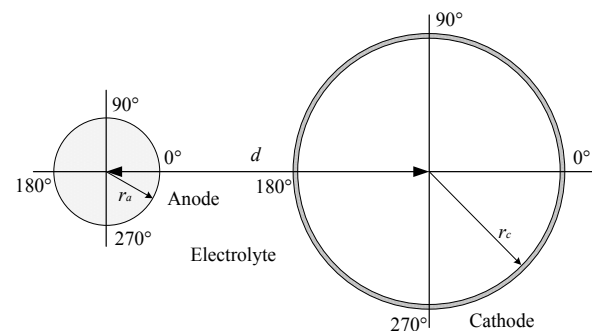


Figure 2. Geometry of analyzed galvanic anode cathodic protection system

Below are presented results of the calculation of electric potential and current density on the cathode and anode surface of example given in Figure 2. Distribution of electric potential on the cathode surface is given on the Figure 3.

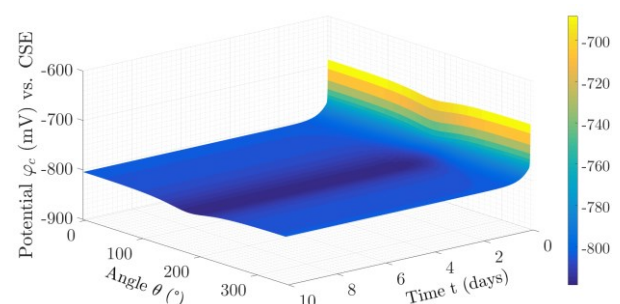


Figure 3. Electric potential distribution on the cathode surface

Results given on the Figure 3 indicate that areas that are closer to the anode (the nearest point of the cathode to the anode is located at 180 degrees) have a

lower value of electric potential. This further means that this area is more protected than the other parts of the cathode. Also, it can be noticed that there is a change of the electric potential to negative side in all points of the cathode, which supports the cathodic protection of surfaces against corrosion.

Change of the current density distribution on the cathode surface in the discrete time steps is given on the Figure 4.

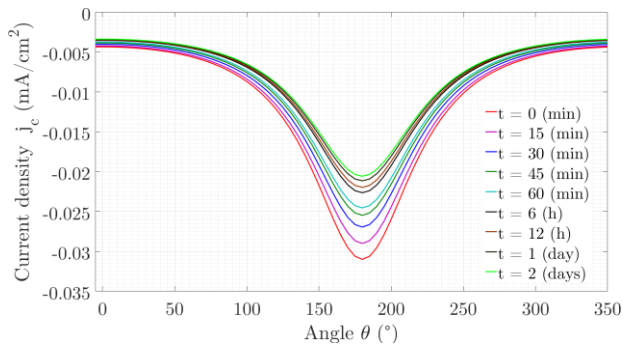


Figure 4. Current density distribution on the cathode surface over the time

From results given on the Figure 4 it is noticeable that current density is higher (in absolute sense) at areas that are closer to the anode. Also, increase of the current density over the time (also in absolute sense) is higher in areas that are closer to the anode.

Electric potential and current density distribution on the anode surface over the time is given on the Figures 5 and 6, respectively.

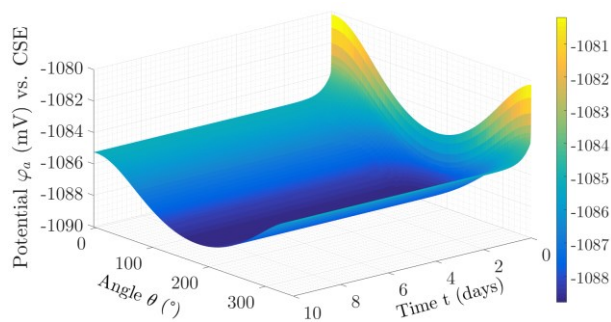


Figure 5. Electric potential distribution on the anode surface over the time

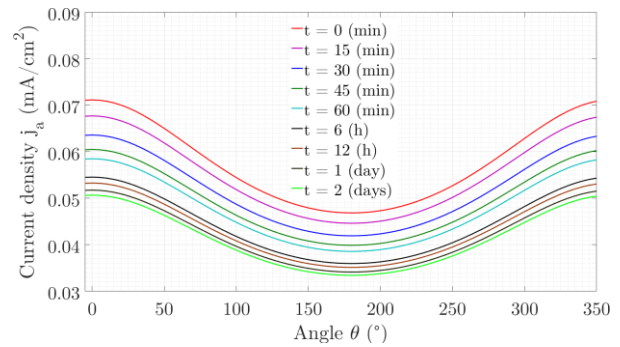


Figure 6. Current density distribution on the anode surface over the time

On anode surface it can be noticed that the electrical potential is more positive on area where the anode surface is closer to the cathode (the nearest point of the anode to the cathode is located at  $0^\circ$ ). Also, on this area current density is the highest.

Change of the current intensity in analyzed cathodic protection system, caused by dynamic polarization characteristics are given on the Figure 7.

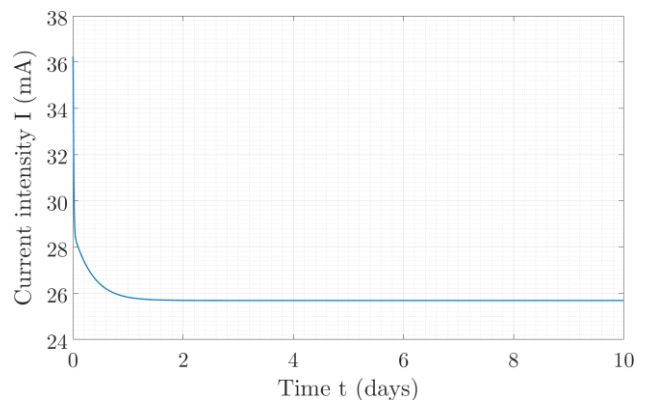


Figure 7. Change of the current intensity in system over the time

From results given on the Figure 7 it can be noted that current intensity decreases over the time. Also, highest change of the current intensity is at first day, while after two days current intensity is constant.

## 4 Conclusion

Design of modern cathodic protection system requires application of very accurate and precise methods for calculating of relevant parameters. The reason for this is the fact that the boundary conditions on the electrode surface of the system are nonlinear and in some situations, such as deposition of calcareous and magnesium on the cathode surface, and may be time-varying (dynamic). In this



paper, mathematical model for calculation of the electric potential and current density distribution in galvanic anode cathodic protection system with dynamic nonlinear polarization characteristics is presented. Presented mathematical model is based on combination of boundary element method and finite difference time domain method. Application of presented mathematical model was demonstrated on one 2D simple geometry problem.

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