Optimal PMU Placement Considering Fault Tolerant Enhancement

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Abstract— Phasor measurement unit (PMU) has been becoming an important device to implement wide area measurement system (WAMS). Practical implementation of WAMS needs to an economic study due to the high cost of PMUs, the technical benefits should also be studied to enhance the fault tolerance of the monitoring system. In this paper, PMU placement from fault tolerant point of view is proposed. The proposed method provides the optimal PMUs placement (OPP) under observability constraint. The objective function of proposed method get priority to install PMUs at the critical buses in the entire system. Additionally, a bi-level optimization framework is proposed to select the optimal PMUs placement which increases the measurements reliability, that to enhances the network fault tolerance. In this paper graph theory visualization is used to visualize the PMU placement results. The proposed method has been tested and validated on IEEE 6 bus, 30 bus, 39 bus, 57 bus, and 118 bus test systems.

Keywords: Phasor measurement unit, optimal PMU placement, wide area measurement system, reliability, fault tolerance.

1. Introduction

Phasor measurement unit (PMU) one of the most important devices in of the wide area monitoring system(WAMS) [1]. PMU provides an accurate synchronized phasor measurement of voltage and current in as electric power system. The sampling time of measured quantity (voltage and current) synchronized with global positioning system (GPS). PMU provides high refresh rate compared with conventional measuring unit.

However, due to the high cost of PMU and its installation cost, not economical to install PMU at each bus in the power system. To solve this, many researchers have tackled the question of minimum number of PMUs needed to achieve full observability of the power system, which is named as optimal PMU placement (OPP) problem. Several research papers have been proposed for minimizing the number of PMUs with a complete system observability. In [2] an integer linear programming (ILP) is used to solve the OPP problem. The authors formulated the OPP as linear optimization problem considering the line and PMU outage contingency. In [3] the OPP problem has solved using practical swarm optimization. However, many researchers have been solved the OPP problem with various mathematical and heuristic optimization techniques include, Tabu search [4], simulated annealing [5], simulated annealing combined with Tabu search [6], exhaustive binary search [7], Spanning Tree [8].

The previous research works solve the OPP problem by minimize the number component reliability; they assume network components with high reliability decrease the contingency probability of the network. An ILP optimization with stability consideration to solve the OPP problem was proposed in [16], the authors used stability criteria to rank the buses that to ensures a priori observability of most vulnerable buses. In [13] the authors proposed solved OPP problem to enhances the fault tolerance of the monitoring system, they used a bi-level optimization framework with two objective function. The primary objective function was the PMU cost, and the second objective function was vulnerability analysis.

In this paper, the OPP problem have formulated to enhance the monitoring fault tolerant and prevent any interruption in system monitoring due to PMU outage. The proposed method characterized as a generical formulation of and more realistic. The main contributions of this paper are:

- Introduce the aspect hybrid observability to enhance the fault tolerance of monitoring system and to tradeoff between the system reliability and number of PMUs.
- Propose bi-level optimization method to minimize the number of PMUs as a first objective and maximize the system reliability as a second objective.
- Propose a method to solve the OPP problem allows the designer to increase the redundant observability for predefined critical buses.
- Propose a new method to present the OPP result by using graph theory rather than the tables, which is enhances designer engineer to choose the proper solution for large power system.

The organization of the rest of this paper as, section II presents the general formulation of OPP problem. Section

III presents the proposed method of OPP problem formulation. Section IV presents the results and discussion of the test systems. Finally, the conclusion presented in section V.

2. General Formulation of OPP Problems

The general formulation of the OPP problem founds the minimum number of PMUs as will as their locations in the entire system. the objective function minimizes the number of PMUs as presnted in equation (1), which is subjected to the complete observability constraints as in equation (2),

Subjected to,

$$min \sum_{i=1}^{n} x_i \quad (1)$$

$$a_{ij}.x_i^T \geq b_i^T \quad (2)$$

Where, a_{ij} is a binary connectivity matrix of the entire system which is defined as in equation (3), xi is a vector of length n indicates to PMU location as in equation (4), and bi is unit vector of length n

$$a_{ij} = \begin{cases} 1 & if\ bus\ i = bus\ j \\ 1 & if\ bus\ i\ and\ j\ are\ connected \\ 0 & otherwise \end{cases} \quad (3)$$

$$x_i = \begin{cases} 1 & if\ PMU\ is\ installed\ at\ bus\ i \\ 0 & otherwise \end{cases} \quad (4)$$

If all buses are observable, the observability constraint (2) will be equal or greater than 1. In some cases, the vector x_i replaced by $C_i x_i$. where, the variable C_i represents the PMU installation cost at bus *i*. Anyway, that additional variable dose not effects on the linearity of the objective function.

2.1 Effect of Zero Injection Buses

The buses which are not connected with loads or generators are called zero injection buses (ZIBs). Those buses enhance in reducing of needed PMUs to achieve the full observability of the system.
The effect of ZIB on the system observability can be summarized as:

1. In case bus *i* is ZIB, bus *i* can be considered observable if all adjacent buses to bus i are observable.
2. An unobservable bus that connected with ZIB, it can be considered observable if the ZIB and other adjacent buses to the ZIB are observable.

ZIBs are modeled in this paper by adding additional inequality constraint for each ZIB and modified the inequality constraint in (2). But, firstly, the adjacent buses for each ZIB must be selected as:

$$B_i = |A_i| \cup \{i\}\ \forall i \in Z \quad (5)$$

Where, A_i is set of busses adjacent to zero injection bus *i*. *Z* is set of ZIB. The additional constraint for each ZIB can be given as:

$$\sum_{k \in B_i} u_k \geq |A_i|\ \forall i \in Z \quad (6)$$

Where, u_k is the observability of bus k, B_i given in equation (5). Additionally, the vector b_i in the inequality constraint in equation (2) need to be modified as:

$$b_i = \begin{cases} 0 & for\ i \in B_i \\ 1 & otherwise \end{cases} \quad (7)$$

3. Fault Tolerance Enhancement

Fault tolerance is an ability that allows the system to continue functioning properly in an event of failure of any of its components. The OPP problem can help the fault tolerance of the wide area measurement system (WAMS). Where, the rest of system components or at least the critical components of the power system must be remained observable during the failure event in WAMS components.

3.1 Single PMU outage

The PMU failure effects on the complete observability of the power system. PMU failure include the failure in PMU itself, the communication link between the PMU and Phasor Data Concentrator (PDC), and measurement instruments failure. The PMU failure can be modeled by modified b_i in the complete observability constraint in equation (2) to be as:

$$a_{ij}.x_i^T \geq [2, \quad 2, \quad \ldots \quad 2]_i^T \quad (8)$$

Where, a_{ij} is a binary connectivity matrix of the entire system which is defined as in equation (3), x_i is a vector of length n indicates to PMU location. The modified constraint in equation (8), which is called in this paper as redundant observability constraint, means that all buses in the system observable by two PMUs. In case any single PMU outage all buses remain observable. Certainly, this will increase the number installed PMUs and thus the cost will increase.

3.2 Hybrid observability

Due to the high cost of PMUs, the hybrid observability constraint is proposed in this paper, which is merge between the complete observability constraint and the redundant observability constraint. In the hybrid observability constraint, only the critical buses have redundant observability, and the others have complete observability as:

$$x_i = \begin{cases} 2 & for\ i \in C_i \\ 0 & otherwise \end{cases} \tag{9}$$

Where, C_i is set of critical buses in the system. the modification in equation (9) ensures that the critical buses must be observable by at least by two PMUs.

To apply the proposed hybrid observability, decision makers or utilities owners must select the critical buses in the power system, which are buses should be observed with at least two PMUs. The critical buses selection could be based on power system stability, system topologies, load importance, and others.

3.3 Maximize the measurement redundancy

Maximum measurement redundancy can be added as 2nd level objective function in OPP to increase the fault tolerance of the monitoring system. Where the OPP may own more than one solution with same number of PMUs. The best solution can be selected by maximize the measurement redundancy as

$$max \sum_{i=1}^{n} b_i \tag{10}$$

Where,

$$b_i = \left(\sum_{J \in i} a_{ij}\right). x_i \qquad \forall i \in I \tag{11}$$

Where, a_{ij} is a binary connectivity matrix of the entire system, x_i is a vector of length n indicates to PMU location. The 2nd objective function (10) is restricted to the optimum number of PMUs, which produced by the 1st objective function (1).

In this paper to maximize the measurement redundancy for specific critical buses the vector S_i has been added to the 2nd objective function (10) to be as:

$$max \sum_{i=1}^{n} s_i . b_i \tag{12}$$

Where,

$$S_i = \begin{cases} 1 & for\ i \in C_i \\ 0 & otherwise \end{cases} \tag{13}$$

Where, C_i is set of critical buses in the system. S_i is a vector that related to the critical buses.

3.4 Network Visualization by Graph Theory

In the literature the OPP solution presented only by tables. The designers and planners may require visualizing the PMUs placement in graph for better planning process. In the present work, a graph theory is used to visualize the network and the OPP solution. Graph theory is a method uses to model the pair-wise relations between objects where, the graph *G (V, E)* mainly composes from set of vertices *V* and set of edges *E*. In power system the vertices represent the nodes or buses, and the edges represent the transmission lines between to buses. The nodes in graph can be colored according to the PMUs, ZIB, and critical buses locations.

4. Results and Discussion

The proposed OPP formulation has been applied in IEEE test system, to demonstrate the ability the proposed OPP to solve small and large power system. the test systems include IEEE 14 -bus, 30-bus, 39-bus, 57-bus, and 118-bus systems. The lines data of the test systems are obtained from [17], and it is used to construct the connectivity matrixes using MALAB program. Integer linear programming ILP is used to solve the optimization problem using MATLAB optimization toolbox. The test systems information included number of lines, number of ZIB, and ZIB location are presented in Table I.

TABEL I: TEST SYSTEMS INFORMATION

Test System	No. of Lines	No. of ZIBs	ZIBs Locations
IEEE 14-Bus	20	1	7
IEEE 30-bus	41	6	6, 9, 22, 25, 27, 28
IEEE 39-bus	46	12	1, 2, 5, 6, 9, 10, 11, 13, 14, 17, 19, 22
IEEE 57-bus	80	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48
IEEE 118-bus	186	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81

4.1 Base case scenario

The base case represents the OPP solutions based on generic formulation of OPP problem with considering the ZIBs and without. The base case results for test systems are presented in Table II. The OPP results show that ZIBs enhance to decrease the total number of required PMUs to achieve full observability of the entire system.

As in Table II, the minimum number of needed PMUs to achieve full observability of 13, 30, 39, 57, and118 bus system are 3, 7, 9, 13, and 28 PMUS, respectively. The effect of ZIBs present clearly in large system. The number of needed PMUs without ZIBs was 32 PMUs for 118 bus system, but with ZIB considering the number of needed PMUs decreased to 28 PMUS.

4.2 Single PMU outage scenario

To enhance fault tolerance of WAMS a single PMU outage is considered and the OPP problem is solved for all test systems using the modified constraint as in equation (8). Table III presents the OPP results with considering a single PMU outage.

TABEL II : PMUS PLACEMENT BASED ON BASE CASE

Test System	Without ZIBs		With ZIBs	
	NO. of PMUs	PMUs Placement	NO. of PMUs	PMUs Placement
14-Bus	4	2, 6, 7, 9	3	2, 6, 9
30-bus	10	1, 7, 8, 9, 10, 12, 15, 19, 25, 27	7	2, 4, 10, 12, 18, 24, 27
39-bus	13	2 ,6, 9, 13,14, 17, 22, 23, 25, 29, 32, 33, 34	9	3, 8, 11, 13, 16, 20, 23, 25, 29
57-bus	17	1, 4, 9, 14, 19, 22, 25, 26, 29, 32, 36, 39, 41, 45, 48, 50, 53	13	1, 4, 9, 14, 19, 22, 25, 28, 32, 37, 50, 53, 56
118-bus	32	2, 5, 10, 11, 12, 17, 21, 25, 28, 34, 37, 42, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 84, 87, 89, 92, 96, 100, 105, 110, 114	28	3, 8, 12, 15, 19, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 64, 70, 75, 77, 80, 85, 87, 90, 94, 102, 105, 110

TABEL III: PMUS PLACEMENT CONSIDERING SINGLE PMU OUTAGE

Test System	Without ZIBs		With ZIBs	
	NO. of PMUs	PMUs Placement	NO. of PMUs	PMUs Placement
14-Bus	9	2, 4, 5, 6, 7, 8, 9, 10, 13	7	2, 4, 5, 6, 9, 10, 13
30-bus	21	2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 15, 16, 19, 20, 22, 23, 25, 26, 27, 28, 30	15	1, 3, 5, 7, 10, 12, 13, 15, 16, 19, 20, 24, 25, 27, 29
39-bus	28	1, 2, 3, 6, 8, 9, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38	18	3, 6, 8, 9, 12, 14, 15, 18, 20, 21, 23, 25, 26, 29, 34, 36, 37, 38
57-bus	33	1, 2, 4, 6, 9, 11, 12, 15, 19, 20, 22, 24, 25, 26, 28, 29, 31, 32, 33, 34, 36, 37, 38, 41, 44, 46, 47, 50, 51, 53, 54, 56, 57	26	1, 2, 4, 6, 9, 12, 14, 19, 20, 24, 25, 27, 29, 30, 32, 33, 38, 39, 41, 44, 46, 50, 51, 53, 54, 56
118-bus	68	2, 3, 5, 7, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 30, 31, 32, 34, 36, 37, 40, 42, 43, 45, 46, 49, 51, 52, 53, 56, 57, 59, 62, 64, 65, 67, 68, 70, 71, 73, 75, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 102, 105, 106, 109, 110, 111, 112, 115, 116, 117, 118	63	1, 3, 5, 7, 9, 10, 11, 12, 15, 17, 19, 21, 22, 26, 27, 28, 29, 32, 34, 35, 37, 40, 41, 43, 45, 46, 49, 50, 51, 52, 54, 56, 59, 62, 66, 68, 70, 71, 72, 75, 76, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 117

It is observed that the minimum number of PMUs are approximately doubled when single PMU outage is considered, that to maintain the system observability during single PMU loss.
Additionally, considering ZIBs can be enhanced to decrease the minimum number of required PMUs even with single PMU outage scenario. For example, as in Table III, the minimum number of PMUs required for single PMU outage scenario in 57 bus system was 33 PMUs. On the other hand, this number was decreased to 26 PMUs when ZIBs was considered.

4.3 Hybrid observability scenario

Since the single PMU outage scenario afford an additional observable source for each bus in the system, the minimum number of required PMUs is increased, which increases the total insulation cost. To decrease the total installation cost and improve the fault tolerance of the monitoring system the hybrid observability is proposed in this paper using the modified constraint in equation (9), that to tradeoff between the system reliability and the total installation cost.

To apply the proposed hybrid observability, the decision makers or utilities owners must select the critical buses in the entire system, which are buses should be observed with at least two PMUs. The critical buses selection could be based on power system stability, system topologies, load importance, and others. In this paper the critical buses are randomly selected.

TABEL IV: OPTIMAL PMUs PLACEMENT FOR PROPOSED HYPRID OBSERVABILITY

Test System	Critical Buses	NO. of PMUs	PMUs Placement
14-Bus	5, 9, 11	5	2, 6, 7, 10, 13
30-bus	10, 12, 13, 22, 26	12	2, 3, 6, 9, 10, 12, 13, 19, 24, 25, 26, 30
39-bus	5, 12, 14, 21, 33, 37	17	2, 4, 6, 9, 10, 11, 13, 16, 17, 19, 20, 22, 23, 25, 29, 33, 37
57-bus	7, 12, 13, 26, 33, 37, 50, 53, 56, 57	22	2, 6, 12, 13, 19, 22, 24, 26, 29, 31, 32, 33, 36, 39, 41, 44, 47, 49, 50, 52, 54, 56
118-bus	8, 12, 15, 26, 32, 35, 50, 52, 55, 57, 98, 100, 110	39	2, 5, 10, 12, 15, 17, 21, 23, 25, 28, 30, 36, 37, 42, 44, 46, 50, 51, 53, 55, 57, 59, 63, 67, 68, 71, 75, 77, 80, 84, 87, 89, 92, 94, 100, 103, 105, 110, 114

Table IV presents the selected critical busses and the OPP results for the test systems based on the proposed hybrid observability. It is observed that the number of needed PMUs to achieve hybrid observability less than the redundant observability, which are presented in Table III. As a comparison the number of needed PMUs for 57 bus system in single PMU outage scenario (redundant observability) was 33 PMUs. On the other hand, the number of needed PMUs was 22 PMUs for the proposed method. For more investigation Table V summarizes the minimum number of PMUs for single PMUs outage scenario and for hybrid observability.

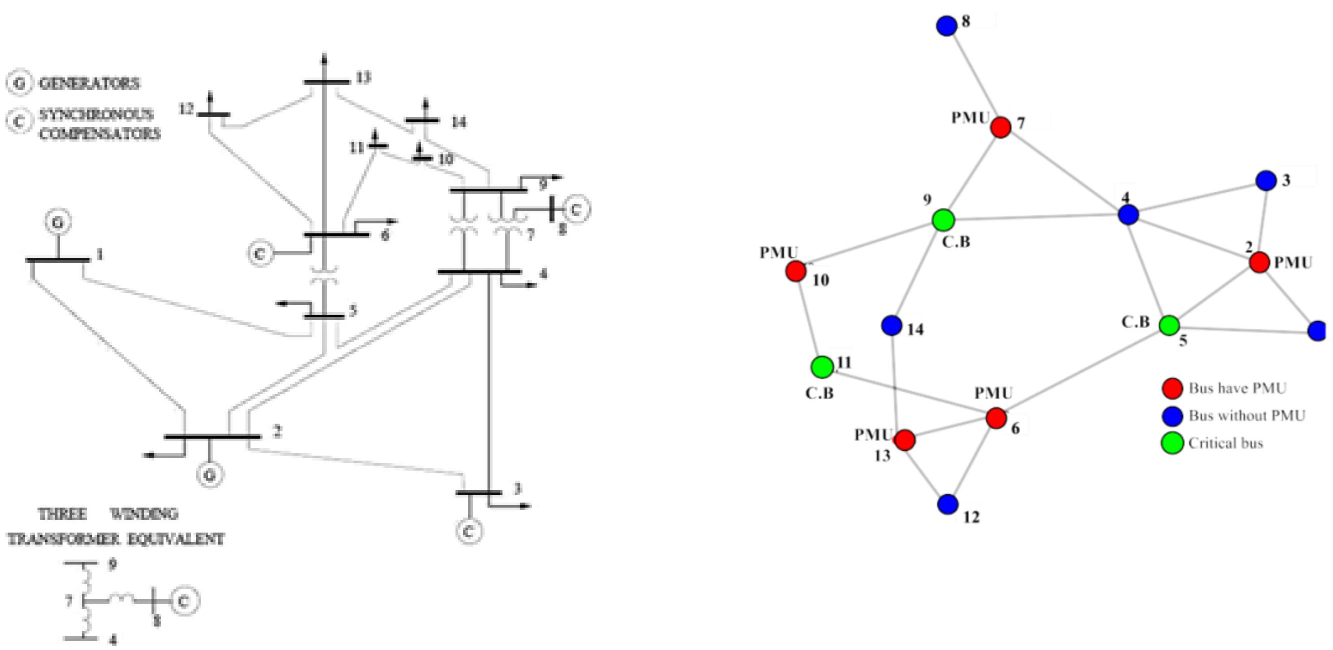

Fig. 1. Real IEEE 14 Bus system and visualization using graph theory with proposed hybrid observability.

TABEL V: COMPARISON BETWEEN REDUNDANT AND HYBRID OBSERVABILITY

System	Redundant Observability NO. of PMUs	Hybrid Observability List of Critical Buses C_i	Hybrid Observability NO. of PMUs
14-Bus	9	5, 9, 11	5
30-bus	21	10, 12, 13, 22, 26	12
39-bus	28	5, 12, 14, 21, 33, 37	17
57-bus	33	7, 12, 13, 26, 33, 37, 50, 53, 56, 57	22
118-bus	68	8, 12, 15, 26, 32, 35, 50, 52, 55, 57, 98, 100, 110	39

Fig. 1 presents graph theory visualization for IEEE 14 bus system. where the tables dose not present clear visualization of the OPP results. It observed when PMUs installed at bus 2, 6, 7, 10, and 13 the system is observable and the critical buses, which are 5, 9, and 11 have redundant observability. Fig.2 presents the graph theory visualization for IEEE 30 bus system It observed when PMUs installed at bus 2, 3, 6, 9, 10, 12, 13, 19, 24, 25, 26, and 30 the system is observable and the critical buses, which are 10, 12, 13, 22, and 26 have redundant observability. For example, bus number 10 is a critical bus in IEEE 30 bus system, as in Table IV, and it observable for two

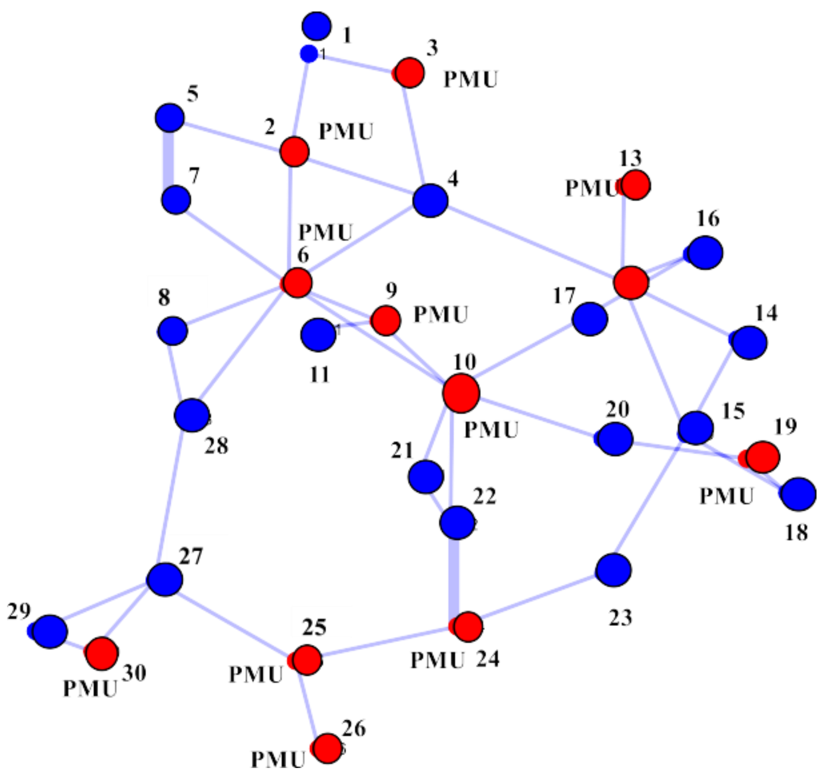

Fig. 2. IEEE 30 Bus system Visualization using graph theory with proposed hybrid observability.

PMUs located at bus 9 and 10. Similarly, for bus 12, 13 it is observable from two PMUs located at bus 12 and 13, bus 22 observable from two PMUs located at bus 24 and 10.

Fig.3 presents the graph theory visualization for IEEE 39 bus system It observed when PMUs installed at 2, 4, 6, 9, 10, 11, 13, 16, 17, 19, 20, 22, 23, 25, 29, 33, and 37 the system is observable and the critical buses, which are 5, 12, 14, 21, 33, and 37, have redundant observability.

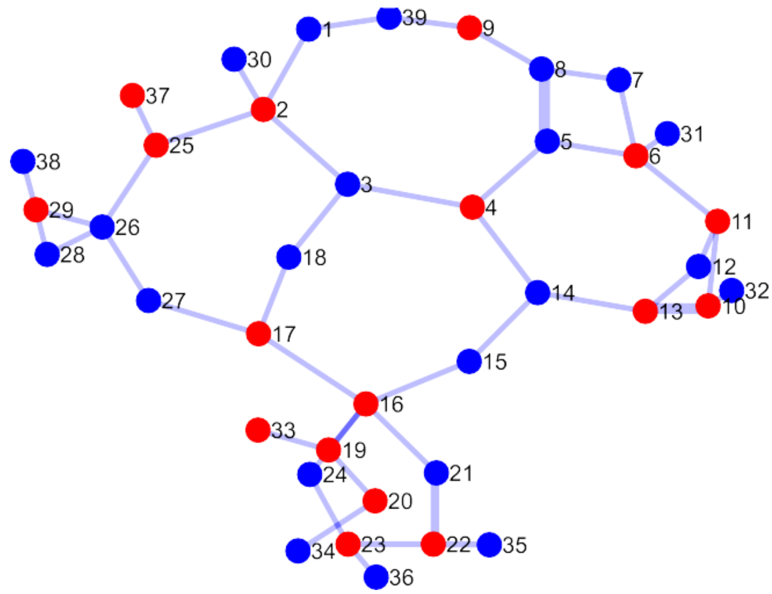

Fig. 3. IEEE 39 Bus system visualization using graph theory with proposed hybrid observability.

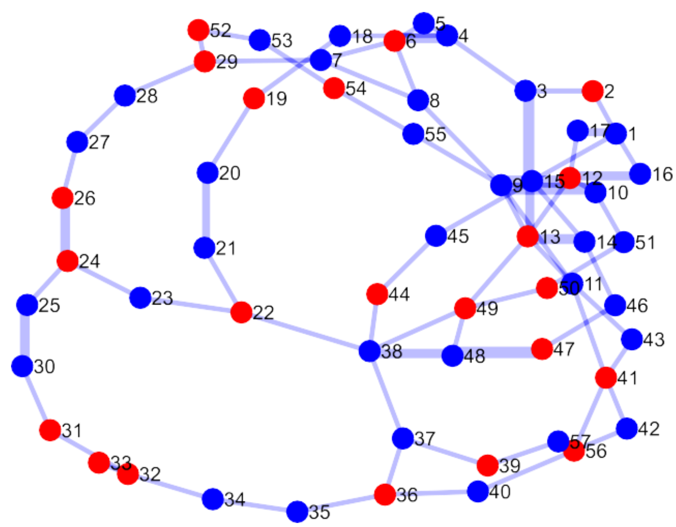

Fig. 4. IEEE 57 Bus system Visualization using graph theory with proposed hybrid observability.

Similarly, the graph theory presentation for 57, and 118 bus system are presented in Fig. 4, and 5, respectively. The IEEE 57 bus system has 22 PMUs and the critical buses, which are 7, 12, 13, 26, 33, 37, 50, 53, 56, and 57 have redundant observability. The IEEE 118 bus system has 39 PMUs and the critical buses, which are 8, 12, 15, 26, 32, 35, 50, 52, 55, 57, 98, 100, and 110 have redundant observability.

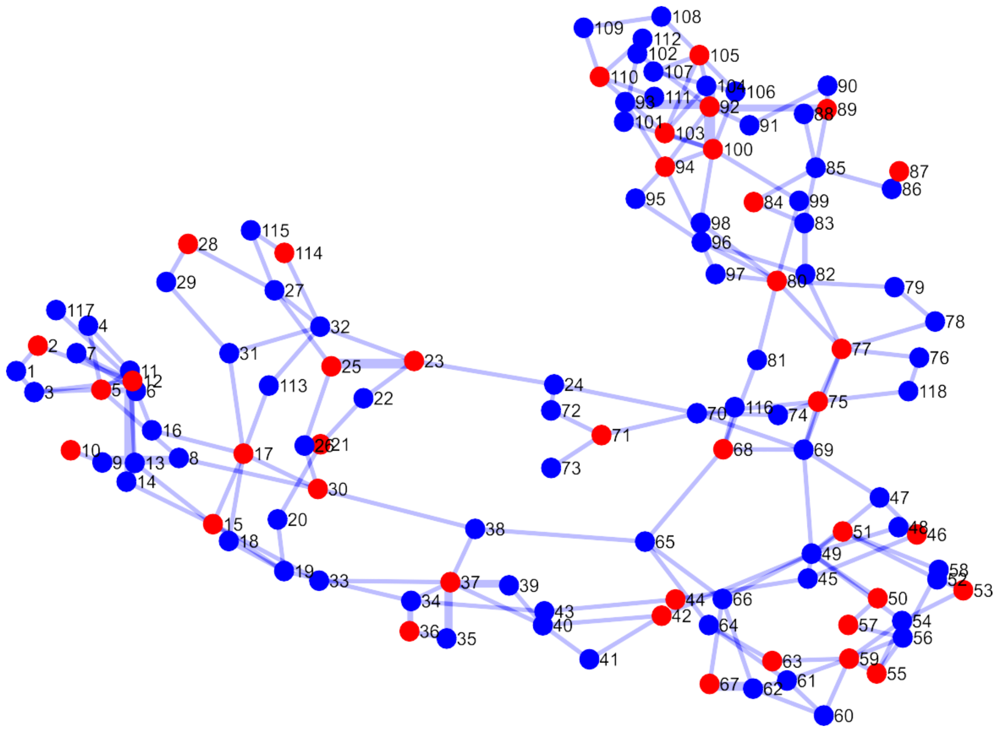

Fig. 5. IEEE 118 Bus system Visualization using graph theory with proposed hybrid observability.

TABEL VI: OPTIMAL PMUs PLACEMENT FOR ONE- AND TWO-LEVEL OPTIMIZATION WITH ZIB CONSEDERING

Test System	**NO. of PMUs**	**One Level Optimization**		**Two Level Optimization**
		PMUs Placement	M.R*	PMUs Placement
14-Bus	3	2, 6, 9	15	2, 6, 9
30-bus	7	2, 4, 10, 12, 18, 24, 27	35	2, 4, 10, 12, 18, 24, 27
39-bus	9	3, 8, 11, 13, 16, 20, 23, 25, 29	37	3, 6, 8, 13, 16, 20, 23, 25, 29
57-bus	13	1, 4, 9, 14, 19, 22, 25, 28, 32, 37, 50, 53, 56	53	1, 6, 9, 15, 20, 25, 27, 32, 37, 48, 50, 53, 56
118-bus	28	3, 8, 12, 15, 19, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 64, 70, 75, 77, 80, 85, 87, 90, 94, 102, 105, 110	145	3, 8, 12, 15, 17, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 59, 62, 70, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110

M.R*: Sum of measurement redundancy for all system, which is the objective function as in equation (10)

TABEL VII
OPTIMAL PMUs PLACEMENT FOR PROPOSED HYPRID OBSERVABILITY WITHOUT ZIB

Test System	Critical Buses	NO. of PMUs	M.R.C*	PMUs Placement
14-Bus	5, 9, 11	3	15	2, 6, 9
30-bus	10, 12, 13, 22, 26	7	13	2, 4, 10, 12, 19, 24, 30
39-bus	5, 12, 14, 21, 33, 37	9	4	3, 8, 11, 14, 16, 20, 23, 25, 29
57-bus	7, 12, 13, 26, 33, 37, 50, 53, 56, 57	13	19	1, 4, 13, 20, 25, 26, 29, 32, 37, 48, 51, 54, 56
118-bus	8, 12, 15, 26, 32, 35, 50, 52, 55, 57, 98, 100, 110	28	32	3, 8, 12, 15, 19, 21, 27, 28, 32, 34, 41, 45, 49, 52, 56, 62, 65, 70, 75, 77, 80, 85, 87, 91, 94, 101, 105, 110

M.R.C*: Sum of measurement redundancy for critical buses, which is the objective function as in equation (12)

4.4 Maximize The Measurement Redundancy

When the OPP result has more than one solution, the 2nd objective function can be used to maximize the measurement redundancy. Table VI shows the OPP results for one- and two- level optimizations. In two-level optimization, the first level minimizes the required number PMUs for full observability and the second level maximizes the measurement redundancy as in (10) – (11). In small test systems such 14 and 30 bus systems there are no reality different between the one-level and two-level OPP results.

In IEEE 39 bus test system the sum of measurement redundancy increased from 37 to 38. In IEEE 57 and 118 bus system the sum of measurement redundancy increased from 53 and 145 to 55 and 151, respectively. As stated early this method increases the measurement redundancy in global manner. In other word this method unable to increase the measurement redundancy for specific critical buses.

To maximize the measurement redundancy for specific critical buses the proposed objective function (12) is used. The critical buses, OPP result, and sum of measurement redundancy for critical buses are presented in Table VII. It is observed that for

14 bus test system the measurement redundancy for the selected critical buses, which are 5, 9, and 11, was 15. By using the proposed method, the measurement redundancy increased only for predefined critical buses. This helps the monitoring system fault tolerance since some of the buses in power system have characterized with high priority and importance.

4. Conclusions

This Paper proposed a hybrid observability method to enhance the fault tolerance of the monitoring system in electrical power system. the proposed hybrid observability combines the redundant observability and the full observability method, that to tradeoff between the measurement redundancy and the installation cost. Additionally, this paper proposed a new method to maximize the measurement redundancy for specific predefined critical buses in the entire system.

A new method to visualize the optimal PMUs placement results is proposed in this paper, which is based on graph theory Visualization. The proposed Visualization helps designers to choose the proper PMUs placement in graph or map manner.

The proposed methods are tested and investigated using IEEE 14, 30, 39, 57, and 118 bus test system. Integer Linear Programming ILP is used to solve the proposed optimization using optimization toolbox in MATLAB program.

References

[1] Kyriakides and S. Chakrabarti, "Optimal placement of phasor measurement units for power system observability," IEEE Transation on Power System, vol. 23, no. 1, p. 1433–1440, 2008.

[2] F. Aminifar, A. Khodaei, M. Fotuhi-Firuzabad and a. M. Shahidehpour, "Contingency-constrained PMU placement in power networks," Transation on Power System, vol. 25, no. 1, p. 516–523, 2010.

[3] M. Hajian, A. M. Ranjbar, T. Amraee and a. B. Mozafari, "Optimal placement of PMUs to maintain network observability using a modified BPSO algorithm," International Journal of Electrical Power & Energy System, vol. 33, no. 1, p. 28–34, 2011.

[4] J. Peng, Y. Sun and a. H. F. Wang, "Optimal PMU placement for full network observability using Tabu search algorithm," Elect. Power Syst. Res, vol. 28, no. 4, p. 223–231, 2006.

[5] T. L. Baldwin, L. Mili, M. B. Boisen and a. R. Adapa, "Power system observability with minimal phasor measurement placement," IEEE Transaction on Power System, vol. 8, no. 2, p. 707–715, 1993.

[6] K. S. Cho, J. R. Shin and a. S. H. Hyun, "Optimal placement of phasor measurement units with GPS receiver," in Proc. IEEE Power Eng. Soc. Winter Meeting, Columbus, 2001.

[7] E. Kyriakides and S. Chakrabarti, "“Optimal placement of phasor measurement units for power system observability," IEEE Transaction on Power System, vol. 23, no. 3, p. 1433– 1440, 2008.

[8] R. F. N. a. A. G. Phadke, "Phasor measurement unit placement techniques for complete and incomplete observability," IEEE Transaction on Power System, vol. 20, no. 4, p. 2381–2388, 2005.

[9] M. B. Mohammadi, R. A. Hooshmand and a. F. H. Fesharaki, "A new approach for optimal placement of PMUs and their required communication infrastructure in order to minimize the cost of the wams," IEEE Trans. Smart Grid, vol. 7, no. 1, p. 84– 93, 2016.

[10] Z. H. Rather, Z. Chen, P. Thgersen, P. Lund and a. B. Kirby, " Realistic approach for phasor measurement unit placement: Consideration of practical hidden costs," IEEE Trans. Power Del., vol. 30, p. 3–15, 3–15.

[11] Y. Shi, H. D. Tuan, T. Q. Duong, H. V. Poor and a. A. V. Savkin, "PMU Placement Optimization for Efficient State Estimation in Smart Grid," IEEE Journal on Selected Areas in Communications, vol. 38, no. 1, pp. 71-83, 2020.

[12] K. Al-Maitah, B. Al-Khriesat and A. Al-Odienat, "The Improvement of Weighted Least Square State Estimation Accuracy," WSEAS TRANSACTIONS on POWER SYSTEMS, vol. 15, no. 1, pp. 1-7, 2020.

[13] S. Almasabi and a. J. Mitra, "A Fault-Tolerance Based Approach to Optimal PMU Placement," IEEE Transactions on Smart Grid, vol. 10, no. 6, pp. 6070-6079, 2019.

[14] F. Aminifar, M. Fotuhi-Firuzabad, M. Shahidehpour and a. A. Khodaei, "Probabilistic Multistage PMU Placement in Electric Power Systems," IEEE Transactions on Power Delivery, vol. 26, no. 2, pp. 841-849, 2011.

[15] S. Kumar, B. Tyagi, Chohan, S. and V. Kumar, "Optimization of Phasor Measurement Units Placement under Contingency using Reliability of Network Components," IEEE Transactions on Instrumentation and Measurement, pp. 1-13, 2020.

[16] V. S. S. Kumar and a. D. Thukaram, "Approach for Multistage Placement of Phasor Measurement Units Based on Stability Criteria," IEEE Transactions on Power Systems, vol. 31, no. 4, pp. 2714-2725, 2016.

[17] R. D. Christie, "Power Systems Test Case Archive," 1999. [Online] https://lamarr.ece.uw.edu/research/pstca/index.html