Construction Posterior Distribution for Bayesian Mixed ZIP Spatio-Temporal Model

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Abstract:**-** Response variables are scored as counts, for example, dengue hemorrhagic fever (DHF) number cases exposed in densely population of urban areas of Indonesia, for example in Kendari city as the capital of Southeast Sulawesi Province, are often arise in Bayesian analysis. At a certain time is not found (or zero) the DHF cases in the case, but other times appear number of DHF cases. When the number of zeros exceeds the amount expected such as under the Poisson density, the zero inflated Poisson (ZIP) model is more appropriate. In using the ZIP model in DHF studies, it is necessary to accommodate local environmental characters as predictors. This study is proposing a Bayesian mixture ZIP spatio-temporal (BMZIP S-T) model and to construct its posterior distribution.

Keywords:- Bayesian, dengue hemorrhagic fever, posterior distribution, score as count, zero exceeds, ZIP

1.Introduction

Dengue hemorrhagic fever (DHF) cases as scored counts are threatened in densely population areas of Kendari-Indonesia. The Kendari city is capital of Southeast Sulawesi province of Indonesia. Modeling of DHF data that accommodates an environmental character is useful to analyze endemic locations. The endemic locations are source of DHF cases and potential outspread to other locations [1, 2]. The spreading of DHF cases is influenced by mobility of people as random effects [3, 4], spatial heterogeneity [5, 6, 7], and temporal factor [8, 9, 10]. In addition, by [11, 12] outlined that DHF is considerate in two-level hierarchical data.

When the number of zeros exceeds the amount expected under a certain density, as for example, the Poisson density, a possibility for modeling the extra-zeros has been proposed by [13]. In using the Bayesian zero inflated Poisson (BZIP) model in DHF data, it is necessary to accommodate the environmental fluctuates. Model for BZIP count data with random effects accounting for intragroup correlation and dependence of clustered observations either in the logistic regression model of the mixture parameter of the Poisson parameter have been discussed by [14, 15]. In this article, BZIP modeling presents special challenges, in addition of the problem of extra zeros, spatial and temporal dependency, additional of random effects for the correlation within and between clusters, and mixing parameters and distribution. We called Bayesian mixed ZIP Spatio-temporal (or BMZIP S-T) modeling.

The BMZIP S-T model considering the uncertainty factors in space-time term are a complex joint posterior, and then the parameter estimation needs the computational intensive approach [16, 17, 18]. One way to solve the estimation is constructing posterior distribution.

2.Materials and Methods

2.1.Pre-Processing Data

Kendari as the capital of Southeast Sulawesi province of Indonesia, is located geographically in the south of the equator and stretches from west to east (see Figure 2.1). The reason to choose the Kendari is one of the cities in Indonesia (a tropical country) with high DHF cases (2064 cases or 0.66% of population) during period 2013-2015. The population of Kendari was 423.812 in 2015 census with the population density at around 1.094 people per square kilometer (km^2). The city is situated around 3m-30m above sea level with the temperature at 23° C-32° C and the humidity at 81%-85% for the whole year. The wet season usually starts in January and ends in June. The higher rainfall (200mm-300mm) occurs during January-April and the less rainfall is around October-November (below 100 mm). Data reviews were obtained from the Meteorological, Climatological and Geophysics Agency (BMKG) and the Central Bureau of Statistics (BPS) of Kendari.

2.1.1.Distribution Checking

The DHF monthly data, for period 2013-2015, in 10 districts of Kendari, are showing the majority as 90% Poisson distribution and 10% binomial distribution with p-value above 5% (see Table 2.1). Binomial distribution is approached by the Poisson process for large number of population compared to the number of DHF cases.

Figure 2.1: The map of Kendari City

Table 2.1: Adjacency relationships between districts in Kendari and Goodness of fit test

Co-de	District/ location	Adja-cency matrix	K-S (p-value)	Distri-bution
1	Mandonga	3,4,8,10	0,61341	Poisson
2	Baruga	3,5,6,8	0,05417	Poisson
3	Puwatu	1,2,4,5	0,16373	Poisson
4	Kadia	1,3,5,8	0,05119	Poisson
5	Wua-Wua	2,3,4,8	0,0755	Poisson
6	Poasia	2,7,8	0,19575	Poisson
7	Abeli	6	0,30509	Binomial
8	Kambu	1,2,4,5,6	0,14073	Poisson
9	Kendari	10	0,11835	Poisson
10	Kendari Barat	1,9	0,09434	Poisson

In Table 2.1 also outlines adjacency matrix between districts. This compiled based on the code in Figure 2.1. Queen Principle is used to arrange weighting matrix into a spatial contiguity. It was used to test the spatial dependencies.

2.1.2.Spatial Detection

The spread of DHF in a location is affected by other location nearby. If a location is becoming DHF endemic, then the other locations are closed to it are immediately to be high risk. The population of Aedes Aegypti is fluctuating based on the characteristics of the location. Detection is required to determine the spatial dependency of DHF incident, as initial information before used in modeling. Moran index (ρ) is a technique to detect spatial dependency. Moran index value is in the range -1 and 1 [19, 20]. The formula of Moran index,

$$\rho = \frac{S}{\sum_{s=1}^{S}\sum_{j=1}^{S} w_{sj}} \frac{\sum_{s=1}^{S}\sum_{j=1}^{S} w_{sj}\left(y_s - \bar{y}\right)\left(y_j - \bar{y}\right)}{\sum_{s=1}^{S}\left(y_s - \bar{y}\right)^2},$$

where the S is the number of locations, w_{sj} is weighted location, y_s is the number of DHF data at the location s, and $\bar{y}$ is the average of DHF data.

In January, February, March, April, and December, show positive Moran index. This means that DHF cases in adjacent locations have similar patterns. May to November, it is no founding the Moran index, because there is not DHF case (see Table 2.2).

Moran scatter plot is interpreting the relationship of DHF between locations. The spread of DHF cases is divided into four quadrants, there are high-high (HH), low-high (LH), low-low (LL), and high-low (HL). In Table 2.2 also, given a summary of spatial dependency testing of DHF cases in 10 districts of Kendari city, for period 2013-2015.

HH quadrant indicating the location of DHF cases is high case, such as Puwatu, surrounded the location with high DHF cases too, such as Wua-Wua. LH quadrant indicating the location of DHF cases is low case, such as Mandonga, but surrounded the location with high DHF cases, such as Wua-Wua. LL quadrant indicating the location of DHF cases is low case, such as Kendari, surrounded the location with low DHF cases, such as the Kendari Barat. HL quadrant indicating the location is high DHF cases, such as Kadia, surrounded the location with low DHF cases, such as Poasia.

2.1.3.Temporal Detection

To find out the DHF data is temporal dependencies, then it is deemed as time series data. An autocorrelation function (ACF) is a tool to detect temporal dependencies. There are four patterns of time series data, i.e. horizontal, trend, seasonal, and cyclical [18]. Horizontal, mean an incidence of DHF is unpredictable and random. Trend, mean an incidence of DHF is tendency to go up and down. Seasonal is DHF fluctuations occurred periodically at a certain time (quarter, quarterly, monthly, weekly, or daily). Cyclical is DHF fluctuations occurred in a long time.

ACF is a relationship between DHF data, is expressed as a set of all the ACF for various lag, $\rho_k, k=1,2,...$, with $\rho_0 = 1$. The ACF coefficients for the k^{th} lag of time series data, stated:

$$\rho_k = \frac{Cov(\mathbf{y}_t, \mathbf{y}_{t+k})}{\sqrt{Var(\mathbf{y}_t)}\sqrt{Var(\mathbf{y}_{t+k})}}, t = 1,2,...$$

Table 2.2: Moran scatter plot summary of DHF monthly data, period 2012-2014, in 10 districts of Kendari

Time	Quad-rant	District or Location	Moran Index
January	H-H	Puwatu	0,116
	L-H	Mandonga, Baruga, Wua-Wua, Abeli, and Kambu	
	H-L	Kadia and Poasia	
	L-L	Kendari and Kendari Barat	
February	H-H	Puwatu and Wua-Wua	0,359
	L-H	Mandonga, Baruga, Abeli, and Kambu	
	H-L	Kadia and Poasia	
	L-L	Kendari and Kendari barat	
March	H-H	Kendari, Kendari Barat, and Wua-Wua	0,065
	L-H	Mandonga and Puwatu	
	H-L	Kadia	
	L-L	Baruga, Poasia, Abeli, and Kambu	
April	H-H	Poasia	0,104
	L-H	Mandoanga, Baruga, Kadia, Wua-Wua, Abeli, and Kendari	
	H-L	Kambu and Kendari Barat	
	L-L	Puwatu	
May-Nop.		-	Not number
Dec.	H-H	Puwatu, Kadia, and Kambu	0,068
	L-H	Mandonga, Baruga, Wua-Wua, and Kendari	
	H-L	Kendari Barat	
	L-L	Poasia and Abeli	

The DHF data have temporal dependencies if the initial value of the ACF exceeds the boundary line, and then decreases gradually. Detection results show that for DHF data, period 2013-2015, in 10 districts of Kendari, are the initial value of the ACF exceeds the boundary line on the lag-1 then decreases gradually. This means that DHF cases of Kendari is temporal dependencies.

Based on pre-processing of Kendari DHF data for 10 districts, there are spatial and temporal dependencies. Furthermore, the DHF data checking are majority as Poisson distribution. There is not DHF case (or zero case) for May to November.

2.2.Standard ZIP Regression Model

Poisson regression model is starting point of modeling count data and flexible to be parameterize in the form of distribution function [21, 22, 23]. Supposed $y_s, s = 1,...,S$ be a number of DHF cases, where S is number of location, and x_s is a predictor at location *s*. Then, density function is expressed

$$f(y_s|x_s) = \frac{e^{-\lambda_s}\lambda_s^y}{y!}, \lambda_s = \exp(\beta_s x_s^{'}),\ y_s = 0,1,2,...,s = 1,...,S$$

(1)

If count data has excess-zero, then (1) can be modified into ZIP model [2, 24]. The application of the ZIP model using Bayesian approach has been discussed for many subjects, for example, epidemiology [24,25] and health [26].

Observations in the ZIP model are two possible data generating processes [27]. The first process is selected with probability Ω_s (generate always zero count) and second process with probability $1-\Omega_s$ (generate counts from Poisson model). In general, the ZIP model is written

$$P(y_s=0)=(1-\Omega_s)+\Omega_s e^{-\lambda_s},\ 0\le\Omega_s\le 1 \qquad (2)$$

$$P(y_s=k)=\Omega_s\frac{\lambda_s^k e^{-\lambda_s}}{k!},k=1,2,...,\infty,\ 0<\lambda_s<\infty \qquad (3)$$

Some researchers use the Bayesian approach to solve the ZIP model, for example in [28, 29]. They used Markov Chain Monte Carlo (MCMC) method to estimate the parameters of ZIP model. To simplify the computation process, by [30] introduced an alternative model, by mixing Bernoulli distribution.

$$\begin{aligned} &y_s \sim \text{Poisson}(\lambda_s(1-U_s))\ ,U_s \sim \text{Bernoulli}(\Omega_s) \\ &f_{\text{ZIP}}=\Omega_s f_P(y_s;0)+(1-\Omega_s)f_P(y_s;\lambda_s) \end{aligned} \qquad (4)$$

Model (4) is a base for constructing the BMZIP S-T model.

3.Results And Discussion

3.1.BMZIP S-T Regression Model

The BMZIP S-T model is integrating three main components, namely, the spatial heterogeneity as predictor (x_{pst}), two random effects local and global ($u_{st}+v_{st}$), and temporal trend $(\alpha+\delta_s)$. Local random effect is local uncertainty relation, while the global random effect is the relationship between locations [31]. Trend temporal is the temporal occurrence of DHF cases that has same intercept but temporal varying at each location. The GLM concept is also used in the BMZIP S-T structure.

Assumed that the DHF case is count data, y_{st} ,and distributed by i.i.d Poisson distribution with parameter λ_{st} in the district s^{th} at the time t^{th}. Then, BZIP S-T structure is expressed

$$\begin{aligned} &y_{st}\sim \text{Poisson}(\lambda_{st}), y_{st}\in Z^+, t=1,...,T, s=1,...,S \\ &\log(\lambda_{st})=\log(\text{P(Ir)}_{st})+\beta_0+\sum_{p=1}^{P}\beta_p x_{pst}+\Xi_{st}+\Phi_s t_z, \\ &\log\left(\frac{\Omega_{st}}{1-\Omega_{st}}\right)=\log(\text{P(Ir)}_{st})+\beta_0+\sum_{p=1}^{P}\beta_p x_{pst}+\Xi_{st}+\Phi_s t_z. \end{aligned} \qquad (5)$$

where S is the number of districts, T is observation time, P is the number of predictors, P(Ir)_{st} is probability incident risk in district s^{th} at time t^{th}, x_{pst} is p^{th} predictor in district s^{th} at time t^{th}, $\Xi_{st}=u_{st}+v_{st}$ is local and global random effect (CAR model) in district s^{th} at time t^{th}, and $\Phi_s=\alpha+\delta_s$ is trend temporal. The τ_u is precision parameter for u_s, τ_v is precision parameter for v_s, ρ is parameter of spatial dependency which $-1\le\rho\le 1$, D is total neighbor of all locations, and $\varepsilon(s)$ is neighboring number of location of s. The meaning of $\alpha+\delta_s$ is each location has same intercept (α), but each location has different contribution of DHF case (δ_s). Assumed that β_0 is flat distribution, the β_p is normal distribution with zero mean, and τ_β is precision parameter of β_p [29].

3.2.Likelihood, Joint Prior, and Joint Posterior

The parameters of BMZIP S-T are estimated via its FCD respectively. Let $\lambda=\{\beta_0,\beta_p,\alpha,u_s,v_s,\delta_s,\tau_\beta,\tau_u,\tau_v,\tau_\alpha\}$ is parameters vector of BMZIP S-T. The joint posterior as basis for obtaining the FCD is multiplication of likelihood and joint prior. The likelihood of BMZIP S-T, is defined

$$l(y_{1t},...,y_{St}|\lambda)=\prod_{t=1}^{T}\left(\prod_{s=1}^{S}\frac{\left[\mathrm{P(Ir)}_{st}\exp(A)\right]^{y_{st}}}{y_{st}!}\right)\times B, \quad (6)$$

where

$$A=\beta_0+\sum_{p=1}^{P}\beta_p x_{pst}+\Xi_{st}+\Phi_s t_z$$

$$B=\exp\left[-\left(\sum_{t=1}^{T}\sum_{s=1}^{S}\mathrm{P(Ir)}_{st}\exp(A)\right)\right].$$

Meanwhile, a joint prior is

$$J(\lambda)=p(\beta_0)p(\beta_p|\tau_\beta)\,p(\alpha|\tau_\alpha)p(u_s|\tau_u)\,p(v_s|\tau_v)\quad (7)$$
$$p(\delta_s|\tau_\delta)\,p(\tau_\alpha)p(\tau_u)p(\tau_v)p(\tau_\delta)p(\tau_\beta).$$

All priors distribution of (7) are an informative priors because they are obtained from various researchers, for example in [7,32]. Priors distribution ($p(\beta_0)$, $p(\beta_p)$, $p(\alpha)$, $p(u_s)$, $p(v_s)$, $p(\delta_s)$) are normal distribution respectively, whereas the hyper priors ($p(\tau_\beta)$, $p(\tau_\alpha)$, $p(\tau_u)$, $p(\tau_v)$, $p(\tau_\delta)$) are Gamma distribution respectively. The structure of joint Posterior is arranged by Definition 3.1.

Definition 3.1. **(Posterior Distribution)**. *Suppose* $l(\mathbf{y}|\lambda)$ *is likelihood function and* $J(\lambda)$ *is joint prior, then posterior distribution is defined*

$$J_p(\lambda|\mathbf{y})=\frac{l(\mathbf{y}|\lambda)J(\lambda)}{\int_{\Omega_\lambda} l(\mathbf{y}|\lambda)J(\lambda)d\lambda}\propto l(\mathbf{y}|\lambda)J(\lambda),$$ *with*

$\int_{\Omega_\lambda} l(\mathbf{y}|\lambda)J(\lambda)d\lambda$ *is normalize constant.*

Based on the Definition 3.1, likelihood (6), and joint prior (7), then posterior distribution of BMZIP S-T is written as

$$Jp(\lambda|y_{1t},...,y_{St})\propto\prod_{t=1}^{T}\left(\prod_{s=1}^{S}\frac{\left[\mathrm{P(Ir)}_{st}\exp(A)\right]^{y_{st}}}{y_{st}!}\right)\times C\times D, \quad (8)$$

where $C=\exp\left[-\sum_{t=1}^{T}\sum_{s=1}^{S}\mathrm{P(Ir)}_{st}\exp(A)\right]$,

$$D=p(\beta_0)p(\beta_p|\tau_\beta)\,p(\alpha|\tau_\alpha)p(u_s|\tau_u)\,p(v_s|\tau_v)\,p(\delta_s|\tau_\delta)$$
$$p(\tau_\alpha)p(\tau_u)p(\tau_v)p(\tau_\delta)p(\tau_\beta).$$

4.Conclusion and Future Research

This paper has been constructing the posterior distribution of the model BMZIP S-T model (8). Further research is to find a full conditional distribution of the model based on the posterior distribution. The full conditional distribution is used for estimating the parameters of the model in WinBUGS.

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