SID (Strain Integral Damping) Static Load Effects

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Abstract: - The theory of SID (Strain Integral Damping) is thoroughly explained in references [1] through [6]. Reference [1] has been included because of the very important concept that forces cannot be functions of accelerations. The present paper deals with the critical case of strongly non linear elastic elements (elastomers) with heavy static load. The ability of SID to simulate hysteresis cycles that are practically independent from the frequency (as Natural Hysteretic Damping is found to be in nature), is confirmed by two examples at 30 and 70 Hz and a SCILAB script (in appendix) is provided to the user to perform the calculations and better understand what is done. Instructions about how to set the important "ω1" parameter are also given to the user as well as cautions about how to set the zero of the displacements used for strain integral calculation.

Key-Words: - vibration; damping; secant stiffness; static load; SID; convergence; bushings; hysteresis; cycles;

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1 Introduction

Nomenclature:

csil	Tg(φ) hysteretic damping coefficient
csilfact	csil multiplying factor
dd	displacement
dt	time step
factdetr	precompression prior to constraint
factmult	sinusoid multiplying factor
force	force
force1	force
freqq	frequency of sinusoidal constraint
f1	constant to define parabolic spring
h1	SID "ω1" parameter
ll	2π
m1	constant to define parabolic spring
k1	constant to define parabolic spring
secstiff	secant stiffness
ss1	SID strain integral "y"
tt	time
va	velocity
zh1	SID "ω2" parameter
z1	SID integral "z"
z1av	SID subtractive weighted average

We have above in the Nomenclature a list of the SciLab script variables and constants. All displacements are in meters and all forces in Newtons. The computed transient has 2 seconds time length.

The SciLab script of reference [5] was used to perform all calculations once modified to take static load into account. The user should carefully read what follows.

It must be clear that the zero of the displacements used to calculate the strain integral must correspond to the precompression "factdetr" parameter which ensures that such a zero is close to the average displacement during the transient. Such a provision helps guarantee against strain integral divergence described in the theory: see the references.

Parameter "ω1" is of the utmost importance in the static load case considered. The user will adopt a suitable estimate of such parameter. A value of 20 Hz is used here which is fair enough when interested in the 30 through 70 Hz band of mechanical frequency.

The "csil" parameter can be used to swell the cycle if wanted. In any case experience will be the best guide.

It is important to remark that the secant stiffness "secstiff" instead is computed using displacement from zero load: both for the blue and red curves. That is both in presence and in absence of static load: see the theory discussions in the references.

2 Materials and Methods

In figures 1 and 2 only the last 0.25 seconds of the transients are displayed. That corresponds to about 7 cycles in the 30 Hz case and about 17 cycles in the 70 Hz case. As we are at the end of the transient the cycles are stabilized and superimpose looking like just one cycle. The beginnings of the transients were not included because the interest is in the stabilized cycles and their frequency invariance. Such frequency invariance is clearly verified by the cycles in the two figures.

Including the whole transients would cause difficult reading of the interesting result.

The displacements of Fig 3 and 4 instead are limited for clarity to the first 0.25 seconds. It is recommended that the user uses both the script of reference [5] and the script in the present paper to study the effect that all parameters have on SID behaviour. That will help effectively using SID to simulate the behaviour of different elastomers which can be various. In the references it is explained that the two parameters “ω1” and “ω2” of SID are generally assigned the same value. The user may discover that assigning different values as is done in this paper’s example, might better fit reality instead. It all depends on measured data that are available like the ones displayed in reference [6] for example. It must be stressed that a certain change of the cycle is in reality possible when the frequency varies. SID constitutes in any case a good starting point to add other features like a bit of viscous damping or more SID spring dampers to make models similar to those made of Maxwell elements for example. Fitting to reality might be more perfect then although in my experience, complication seldom leads to real improvement.

3 Figures

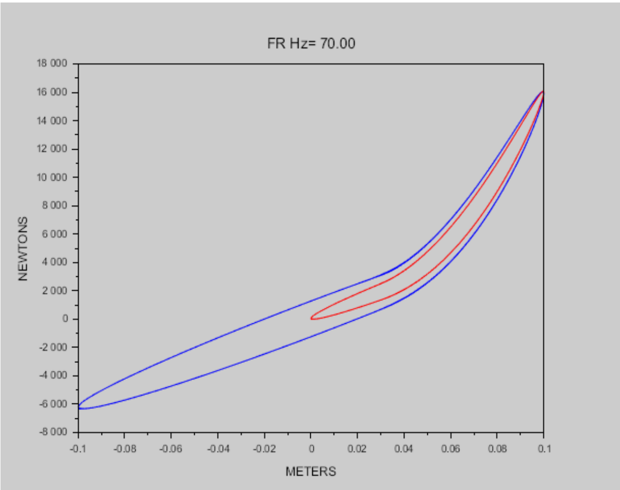

Fig. 1. No static load blue cycle and static load red

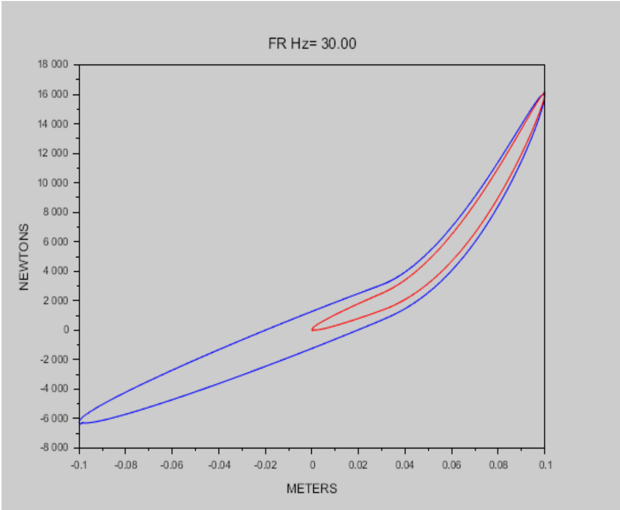

Fig. 2. No static load blue cycle and static load red

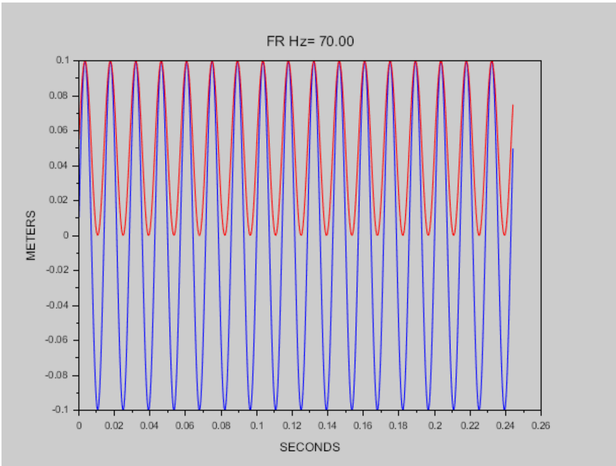

Fig. 3. Displacements

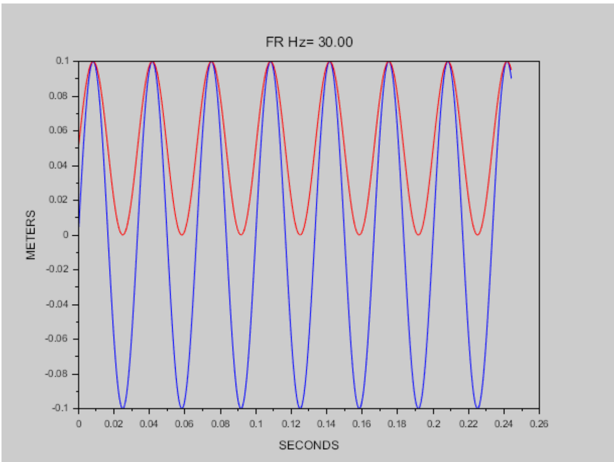

Fig. 4. Displacements

4 Scilab script

```
clear all;
freqq = 30; fig1 = 1; fig2 = 2;
figure(fig2); title('FR Hz= ' + msprintf('%.2f',freqq));
xlabel('METERS'); ylabel('NEWTONS');
csi1 = 0.2; h1 = 20;
for ncalc = 1 : 2;
if ncalc == 1 then; factmult = 1; factsum = 0.0;
 csilfact = 1;factdetr = 0.0; end;
if ncalc == 2 then; factmult = 0.5; factsum = 0.05; csilfact = 1; factdetr = 0.05;end;
tt=(1:8192)/4196; ll = 2* %pi; dd = factsum + factmult * sin(ll*tt*freqq)/10;
f1 = 2; m1 = 400; k1 = (ll*f1)*(ll*f1)*m1; dt = 1/4096;
ss1 = 0; z1 = 0; zh1 = 0.0001; force1 = 0.;
for kk = 1 : 8192 - 1;
va = (dd(kk+1) - dd(kk))/dt;
z1 = z1 + (-z1 * zh1 + dd(kk)-factdetr) * dt;
z1av = z1 * zh1 / max(0.0001,1-exp(-zh1*tt(kk)));
ss1 = ss1 + (-ss1 * h1 + dd(kk)-factdetr - z1av) * dt;
force1 = dd(kk)*k1 + 2.*(dd(kk) > 0.03)*(dd(kk)-0.03)**2*1000000;
secstif = abs(force1/ dd(kk));
force1 = force1 + (-0.*0.15+sign(va))*(abs(ss1 * va))**0.5 * secstif * csi1*csilfact;
force(kk) = force1;
end
dd = dd(7168:length(dd)-1); force =force(7168:length(force));
figure(fig2);
if ncalc == 1 then; plot((dd),force,'b'); end;
if ncalc == 2 then; plot((dd),force,'r'); end;
end
```

5 Results

As an engineer I can say that *as a main result* SID simplified my profession and *should result in* simplify*ing* also the task of other engineers. SID was made to be simple: remark how short is the script we use here.

6 Conclusion

I couldn't claim that there is a real link between the strain integral and physical reality. I am a nuclear engineer but I am not a physicist. Assuming that forces can also be functions of variables other than displacement and velocity as is the case of SID, would revolutionize physics and open enormous theoretical problems. Is the "strain integral" one of the "hidden variables" physicists write and discuss so much about today? I couldn't answer such a fascinating question.

References:

[1] Pars, L.A.; A treatise on analytical dynamics. *Ox Bow Press* Woodbridge Connecticut **1979**, *Volume 1*, pp 11-12.

[2] Tornar, U.; Modelizing Structural Hysteretic Damping by means of MDI ADAMS. Proceedings of the 1997 ADAMS users conference, Marburg, Germany.

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[5] Tornar, U.; *Non linear Strain Integral Damping*, Proceedings of the ICCM2016 Conference, Berkeley, California, USA.

[6] Tornar, U.; *SID (Strain Integral Damping) comparisons with experiment and imaginary stiffness damping*, Proceedings of the ISMA 2022 Conference, Leuven, Belgium.