On the topological connection of inertial systems on space-time manifolds

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Abstract: - St. Hawking proved that the conformal diffeomorphisms of the space-time manifold can be represented by autohomeomorphisms of a Zeeman topology [17]. The corresponding physical invariants to this symmetry group are well known [17]. For accelerated reference frames, there are contradictions in measurements of observables, since the locally used Lorentz transformations require inertial frames that are compatible [13]. I use a finer Zeeman topolo y generated by piecewise timelike geodesics representing inertial motions. The corresponding autohomeomorpism group preserves the Nonregularity of this topology . The non-regularity was proven by the Bulgarian topologist Strassimir Popvassilev [12], [9]. We construct the possible relationship for inertial frames by the aid of this proof .

Key-Words: - Space time manifold, Zeeman topologies, conformal mapping

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1 Introduction

Symmetry groups in connection with physical laws form an important component in mathematical physics. In electrodynamics, for example, the group of conformal diffeomorphisms for space-time manifolds plays an important role in energy transport in the electrovacuum.

Hawling King and McCarthy proved a representation of this group by the autohomeomorpism group of a Zeeman topology induced by continuous time-like worldlines in the space-time manifold.

Heinzmann and Mittelstaedt [13] showed for accelerated reference frames that the tensor character is locally preserved via Lorentz transformations, but the measurement of the observables can lead to physical nonsense. (Radiating or non-radiating electron).

2 Problem Formulation

If an initial inertial frame can be transformed suitably, this contradiction would not arise. In order to be able to get a mathematical extension for inertial systems in the space-time manifold, the following way could be successful.

I. Definition of a strictly finer Zeeman topology by reduction to piecewise time-like geodesics Zpg.
II. Use of the topological property that Zpg is not regular, and use of the construction in Popvassilev's proof.
III. Transport of the construction with the help of the autohomeomorpism group H(Zpg).
IV. Interpretation of the time-like geodesics as force-free motions and the representation of unertial frames bytheis assumption.

The local relationship of inertial frames is in discussion.

2.1 Zeemantopologies and group representations

On a topological space X be given a family S of subsets ş carring the subspace topology. The finest topology Z(X,S) which coincids on every subspace ş with the topology X is called Zeemantopology on X induced by S.

For our application it is important the uniqueness of this topology and representing some information about the subspace family.

E.C. Zeeman was able to show that the Lorentz group agrees with the autohomeomorpism group H (Z) by choosing X as the Minkowski space and the family S of the time-like world lines [3].

Many generalities on space-time manifolds were investigated [4], [5], [7]) using different global conditions of causality to get easier mathematical proofs.

Especially we remember the result of Hawking, King and McCarthy [7]. X be the space-time manifold topology and S be the family of continuous time-like paths. This Zeeman topology was called path topology Po. Their main result has been

Proposition: The group of autohomeomorhisms H(Po) of Po is equal to the group of conformal diffeomorphisms ConDiff in the space-time manifold M. In their proof the important relation H(Po) part of H(M) was guaranteed by a global causality condition. Malament [8] proved his result without global causal condition.

2.2 Nonstandard topological extension

The topology Po does not distinguish geodesics from arbitrary continuous time-like paths. For our purpose, the difference between force-free and accelerated movements is essential.

The Zeeman topology Zg, which is defined by the family S of time like geodesics is equal to the Zeeman topology Zpg which is defined by the family S of piecewise time like geodesics.

Therefore will a Zpg-homeomorphism only transform invariantly time-like geodesics.

From many investigated properties of Zpg [9], [10], [11], [12] we need in this paper the negative separation property Non-regular of Zpg.

Popvassilev [12] proved the Non regularity of Zpg . Therefore is Zpg not completely regular.

The continuality of continuous real-valued functions is only supported on completely regular spaces [15]. The topology Zpg supports the continuation of geodesic pieces from border points into the interior of open charts of the manifolds with the help of non-regularity. In Popvassilev's proof, it is essentially used that the geodesists are of second category.

We remark that most proofs where developed in Minkowski-space using timelike lines and generalized by exponential maps to space-time manifolds.

2.3 Autohomeomorpism group H

The additional group of auto homeomorphisms H(Zpg) to the group lf conformal diffeomorphisms represented by H(Po) has to be carefully interpreted. Regarding auto bijections which leave invariant piecewise geodesics there are interesting geometric examples using the dimension greater or equal 3. [16]. There exist deformations of hyperplanes in space-time, which can have physical interpretations concerning symmetry and invariance. We use only the transformation of the Nonregularit by the Zpg-homeomophism.

2.4 The principle of relativity

In the work of Heintzmann and Mittelstaedt [13] it is shown that various physical laws apply to accelerated reference frames only to a limited extent.

A physcal law wellformulated by tensoanalysis can be transformed locally by Lorentzian transformations in an accelerated frame. The character of tensors will remain but the measurements of observables is difficult. An example is the radiation of an accelerated electron. The transformation into an accelerated reference frame, where the electron does not move would lead to non radiation.

3 Solution

An inertial system in the space-time manifold is defined by all locally existing time-like geodesics, which we assume represent force-free motions.

In Minkowski space, we define analogously with time-like straight lines. The Zeeman topologies Zpg uniquely creates a structure whose invariance, expressed by the autohomeomorpism group H(Zpg), is essential for the appropriate definition of an inertial frame. Locally occurring changes due to symmetry transformations can be explained with the help of the nonregularity of Zpg. Consider a Zpg-closed densely lying subset A located in the open interior of a local time cone, which is not Zpg-separable from the starting point Q of the time cone. The use of a time-like geodesic piece through Q allows the Interpretation ofpermissible changes due to Zpg-autohomeomorphism.

The Zpg-autohomeomorpisms allow unsteady transformations, but preserve the property that the piecewise timelike geodesics consist of finitely many tricks.

The contradiction in the example of a radiating electron is resolved by the use of permissible transformations of an inertial frame.

4. Conclusion

In this work, we have investigated the topological structure of inertial systems on space-time manifolds through the lens of a strictly finer Zeeman topology. This topology not only reflects the physical distinction between inertial (force-free) and accelerated motion but also incorporates essential topological features—most notably its nonregularity, as established by Popvassilev—which enable meaningful interpretations of transformations between inertial frames. By employing the group of autohomeomorphisms, we extend the representation of symmetries beyond the conformal group traditionally associated with the path topology. These homeomorphisms preserve the structure of inertial motion while admitting certain discontinuities that are still physically permissible, offering a refined framework for analyzing transformations in space-time. This approach resolves long-standing contradictions in the treatment of accelerated frames—such as the paradox of a radiating versus non-radiating electron—by allowing for more nuanced transformations within a topologically consistent model. Our construction bridges the gap between mathematical rigor and physical intuition, suggesting that the topology of space-time plays a foundational role in the formulation and interpretation of the principle of relativity. Ultimately, this study contributes to a deeper understanding of the interplay between geometry, topology, and physics, offering a potential pathway toward a more complete theory of reference frames within the general framework of space-time manifolds.

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