Paradoxes or Contradictions? Exploring the Riemann-Zeta Function and Riemann Hypothesis by Euler's Identity and Category Theory

YANG I. CAO
Independent
Bundesasyizentrum
Freiburgerstrasse 50, 4057 Basel
SWITZERLAND

Abstract: - The research studies the Riemann Zeta Function (RZF) and the Riemann hypothesis (RH) with the Harmonic Series by Euler's identity and category theory. I attempt to simplify the RZF by a metric space with geometric analysis. I further explore the nondiscrete mathematical relations of Euler's identity and the basic trigonometric functions in the analytic geometric space, and some morphisms are constructed. The study demonstrated the viability of the three-dimensional coordinate construction with its topological relations to be further explored and justified. The current form of the solutions corroborates with the nontrivial zero solutions, and further tests on the RH will need a paradigm shift on the preliminary results.

Key-Words: - non-algebraic numbers; morphism; injection; zero morphism; discrete and nondiscrete mathematics; unit sphere; powers and exponents; structuralism

Received: March 29, 2025. Revised: April 19, 2025. Accepted: May 14, 2025. Published: June 2, 2025.

1 Introduction

There is a common scientific consensus that Gödel's Incompleteness Theorems (GITs) falsify any possible theory of everything in physics. It is believed that for the descriptive and causally deterministic theories in physics, the GITs imply that they are consistent but incomplete, inconsistent but complete, or inconsistent and incomplete [1]. ANDRÉKA, MADARÁSZ [2] proved, with first-order logic, that the relativity theories can be complete, i.e., determinable, but inconsistent. An example that falsifies the Minkowski spacetime prerequisite of the proof can be seen in the non-perturbative solution with the mathematical method element of integration [3]. Therefore, the case from the unquestionable laws in contemporary physics and astrophysics supports the corollary from Myers and Hadi Madjid [4] that

> "There is no logical ground to exclude any of the uncountable set of potential explanations of given evidence prior to additional evidence not yet on hand."

Or in the words of Stephen Hawking [5]:

> "The theories we have so far, are ~both inconsistent, and incomplete."

2 Problem Formulation

For me, the problems of inconsistency and incompleteness always come back to the numbers, -1, 1, 0, and i. For example, between

$$X = \sum_{n=0}^{\infty} n \tag{1}$$

and

$$X' = \sum_{n=0}^{\infty} 2n + 1, \tag{2}$$

is it comparable for the two terms of infinity between equation (1) and equation (2)? If not, we only have two different patterns of infinity series, and if yes, both commutative and noncommutative operators may apply to the variations of infinity series as if they were simple calculus and algebra.

There seems to be a common consensus though in infinite series that the matter of discussions only applies to the cases where $n \geq 1$, with one possible reason that infinity does not have a unidirectional property. Nonetheless, the logical premise paves an interesting connection between the Harmonic Series (HS) and Riemann-Zeta Function.

2.1 The Harmonic Series in Euler's Identity

It is notable that by the adaptation of commutative and noncommutative operators on infinite series, we're only applying the abstract notion of static representations to a dynamic one that assumes consistency at the very least. It is not that consistent, however, when I reformulate the HS by Euler's identity that

$$HS = 1 + \sum_{n=2}^{\infty} \frac{1}{n}, n \in N = -ne^{i\pi} - \sum_{n=2}^{\infty} \frac{n-1}{n} \quad (3).$$

Now to find the Riemann-Zeta zeros one only needs to solve the equation

$$\sum_{n=1}^{\infty} \frac{1}{n} + \frac{1}{n^s}, n \in N, s \in C = -ne^{i\pi} - \sum_{n=2}^{\infty} \frac{n-1}{n} \quad (4),$$

i.e.,

$$\sum_{n=1}^{\infty} \frac{n^{s-1}+1}{n^s}, n \in N, s \in C = -ne^{i\pi} - \sum_{n=1}^{\infty} \frac{n-1}{n} \quad (5),$$

$$\sum_{n=1}^{\infty} \frac{n^s+1}{n^s}, n \in N, s \in C = -ne^{i\pi} \quad (6),$$

$$\sum_{n=1}^{\infty} \frac{1}{n^s}, n \in N, s \in C = -n(e^{i\pi}+1) \quad (7).$$

Equation (7) implies that there must be a one-to-one correspondence between the summation set of the Riemann-Zeta Function (RZF) and the infinite set of the Euler's identity, by the value of $s \in C$. Or in the other form set forth in equation (8), the axis of the summation term, depending on the value of s, ought to be self-complimentary to 0 for all Riemann zeros.

$$1 + \sum_{n=1}^{\infty} \frac{1}{n^{s-1}}, n \in N, s \in C = -e^{i\pi} \quad (8).$$

Essentially the Riemann Hypothesis (RH) asserts that $\sum_{n=1}^{\infty} \sqrt{n} \times n^{-p}, n \in N, p = s - \frac{1}{2} \in$ imaginary parts oscillate to zero, namely,

$$\ln \sum_{n=1}^{\infty} \frac{1}{n^{s-1}} = 2mi\pi, n \in N, S \in C, m \in Z$$

[the proof is trivial] (9);

or if we dissect the topological space of $\ln(1^{s-1} + 2^{s-1} + 3^{s-1} \ldots)$ to a metric set of $\ln 1^{s-1} + \ln 2^{s-1} + \ln 3^{s-1} \ldots$,we obtain

$$\ln \lim_{n\to\infty} n!^{1-s} = 2mi\pi, n \in N, S \in C, m \in Z \quad (10),$$

then,

$$\lim_{n\to\infty} n!^{1-s} = 1 = i^{4m} = e^{2mi\pi}, \quad n \in N, S \in C, m \in Z \quad (11).$$

2.2 Traditional and Alternative Approaches to Riemann Hypothesis

Traditional approaches to RH have been largely constructivist and positivist (see [6-8] as some examples diverse in methodological approaches but uniformly takes the theoretical premise as given), and some deconstructivist and postpositivist stances can also be found in Alan Turing's later works [8]. The skeptics from Turing can be interpreted as fundamentally the skeptics on the foundations of Calculus in general with integration and differentiation, two of which are heavily involved in the methodological approaches to the proof of the RH. Or more specifically, it might be the doubt on the consistency of the RZF.

Indeed, differential calculus with the undefined tangent problems of $\frac{\pi}{2} + m\pi, m \in Z$ does not necessarily capture the essence of the critical stripe of nontrivial zeros in question. Rather, in my understanding, the tangent problem is the question to the imaginary parts of the nontrivial zero solutions other than the theoretical premise of a hypothesis, as we derive equations (9) and (10) into:

$$\frac{2}{3} \ln \sum_{n=1}^{\infty} \frac{1}{n^{s-1}} = -\frac{4}{3}\left(i\sqrt[3]{m}\right)^3 \pi, n \in N, S \in C, m \in Z \quad (12),$$

and

$$\frac{2}{3}\ln\lim_{n\to\infty} n!^{1-s} = -\frac{4}{3}\left(i\sqrt[3]{m}\right)^3\pi,$$
$$n \in N, S \in C, m \in Z \quad (13).$$

Even though Rolle's theorem seems to apply if the RH were true, equations (12) and (13) suggest it is not that case, unless the value of s can make the natural number line's value cyclic to a sum of zero to the infinite distance. Therefore, I infer that the RZF is a non-continuous function.

2.2.1 The Question of Geometry

Even though it is not clear for the visual representation of an i-metric unit circle in the spherical form, the deviations of the infinite sum of the RZF are injective if not bijective to the i-metric sphere. Moreover, a contradiction or paradox can be seen between equations (10) and (11) with the special case of e, where

$$\ln\lim_{n\to\infty} n!^{1-s} = \ln 1 = \ln i^{4m} = 2mi\pi = 0,$$
$$n \in N, S \in C, m \in Z \quad (14).$$

The contradiction or paradox arises from the natural logarithm applied to equation (8) with the element of 1 present. Therefore, by applying natural logarithm to make an i-metric-based radius sphere, an axial shift must have formed that is not in the logical rules of calculus and algebra. Hereby, the GITs seem to imply that the RZF is both inconsistent in its number theoretical foundation and incomplete in its geometrical morphology. From the geometric representation, it looks that the metric space of the perimeter of the sphere's plane is net zero, and so is its dimensional projection to the spherical surface area.

2.2.2 The Origin of Negative Numbers

In my previous justification approach with discrete number theory to the RH, I formed the corollary that, "there is no natural extension to the dimension of negative numbers, and its dimension originates from the negative number -1."[9] Thereby, the dimensionality of the i-metric-radius sphere can be considered as the dimensional extension in bridging the numbers between 0 and -1 that mirror the ones between 0 and 1. Therefore, the determination of the value of s can also be considered as the determination on the center of the i-metric-radius sphere in the number lines.

The connotations of the nontrivial zeros in the RZF with multiple solutions then imply that the axial shift for the i-metric-radius sphere consists of a set of zeros from the real number lines to the complex number lines, i.e., $\ln i^{4m} = 0, m \in Z$, contrary to the normative definitions that 0 is merely a real number.

So, with Euler's Trigonometric Identities that

$$\begin{cases} \cos x = \frac{e^{ix}+e^{-ix}}{2} \\ \sin x = \frac{e^{ix}-e^{-ix}}{2i} \end{cases} \quad (15),$$

we have

$$\tan\left(\frac{\pi}{2}+m\pi\right) = \frac{e^{\frac{2m+1}{2}i\pi} - e^{-\frac{2m+1}{2}i\pi}}{i\left(e^{\frac{2m+1}{2}i\pi} + e^{-\frac{2m+1}{2}i\pi}\right)}$$
$$= \frac{-e^{\frac{2m+1}{2}} + e^{\frac{2m+1}{2}}}{-2ie^{\frac{2m+1}{2}}} = \frac{\ln i^{4m}}{-2ie^{\frac{2m+1}{2}}}, m \in Z$$
$$(16).$$

And the reciprocal tangent can also be derived

$$e^{-i\pi\frac{2m+1}{2}} = \cos\frac{2m+1}{2}\pi - i\sin\frac{2m+1}{2}\pi$$
$$\cot\left(\frac{\pi}{2}+m\pi\right) = i \pm e^{\frac{2m+1}{2}} = 0, m \in Z$$
$$(17)$$

or in the tangent form

$$\ln i \tan\left(\frac{\pi}{2}+m\pi\right) = m + \frac{1}{2}$$
$$\tan\left(\frac{\pi}{2}+m\pi\right) = \frac{e^{\frac{2m+1}{2}}}{i}, m \in Z$$
$$(18).$$

If we assume the indications of equations (16) and (18) are true, it is then derived that

$$\frac{\ln i^{4m}}{-2ie^{\frac{2m+1}{2}}} = \frac{e^{\frac{2m+1}{2}}}{i}, m \in Z$$
$$\ln i^{4m} = -2e^{2m+1} = 0, m \in Z \quad (19).$$

With equations (17) and (19), it becomes that

$$i = e^{\frac{2m+1}{2}} \pm (2)e^{2m+1}, m \in Z \quad (20),$$

which implies that the value of i projected on the real number line can be both discrete and divergent. This is in accordance to the possible geometry of the i-metric-radius sphere question.

Another contradiction or paradox arises from equation (17) by another solution also exists

$$-i\pi\frac{2m+1}{2}=\ln\frac{\cot\frac{2m+1}{2}\pi}{i}=\ln\frac{0}{i},m\in Z$$
$$e^{\frac{2m+1}{2}}=-\frac{0}{i},m\in Z \quad (21).$$

By equations (20) and (21), we get

$$i=e^{\frac{2m+1}{2}}\pm(2)e^{2m+1}=-\frac{0}{e^{\frac{2m+1}{2}}},m\in Z$$
$$e^{2m+1}\pm(2)e^{3m+\frac{3}{2}}=-0,m\in Z \quad (22),$$

which are equivalent to equation (19).
From the tangent analyses, it is inferred that the center of the i-metric-radius sphere originates from the zero point of the complex number lines.

2.2.3 Paradoxes in the Numerical Values

It is seen that the application of logarithms on the trigonometric numbers created the new paradoxes in discrete real number lines with the set of undefined negative numbers' logarithm values in section 2.2.2. From the body of analyses from the above section, however, equations (20) and (22) is similar in form to the surface area of a cone in complex morphologies

$$\begin{aligned}\ln -1 &= i\pi \\ &= \pi e^{2\left(\frac{m}{2}+\frac{1}{4}\right)}\pm(2)\pi e^{\frac{m}{2}+\frac{1}{4}}\times e^{\frac{3}{2}m+\frac{3}{4}} \\ &= \pi e^{\frac{m}{2}+\frac{1}{4}}[e^{\frac{m}{2}+\frac{1}{4}}(1\pm(2)e^{m+\frac{1}{2}})],m\in Z\end{aligned} \quad (23),$$

and

$$\begin{aligned}&\pi e^{2\left(m+\frac{1}{2}\right)}\pm(2)\pi e^{m+\frac{1}{2}}\times e^{2m+1} \\ &= -0 = -\ln i^{4m},m\in Z\end{aligned} \quad (24).$$

Therefore, to match the projection of the i-radius sphere volumes' points of values on the real number line requires the solutions for the complex number representations of m's apparent value on the integer number line.

2.3 Current Applications of the RH and RZF

Roger Penrose has applied the concept of the extended space from the RH and RZF for the mathematical basis of Conformal Cyclic Cosmology (CCC). Daly [10] solutions on active galactic nuclei and galactic black holes also found that a dimensionless efficiency factor correlational to the dimensionless mass accretion rate can be a constant with the solutions $A\simeq\frac{1}{2}$ and $a=\frac{1}{2}$. Both tendencies seem to imply that the RZF with RH may lead to new spatial morphism in astrophysical and cosmological research. The research seeks to expand the conceptual possibilities of the topological and metric spaces derived by the RZF, whereby I found that the real part of $\frac{1}{2}$ in the nontrivial zero solutions to the RZF is a dimensional description of the dimensional parity between 0s and -1s, hence the $-\frac{1}{2}$ value of the real part was proposed for the RH [9].

3 Problem Solution

From the geometric analyses in section 2.2.2, some metric relations between the imaginary i-radius sphere and the real number coordinate have been derived:

$$e^{2m+1}: -\frac{\ln i^{4m}}{2}\mapsto 0,m\in Z \quad \text{[from equation (19)]} \quad (25)$$

$$\cot(\frac{\pi}{2}+m\pi)-i:\pm\sqrt{e}^{2m+1}\mapsto i,m\in Z \quad \text{[from equation (17)]} \quad (26)$$

$$\ln\cot(\frac{\pi}{2}+m\pi): \frac{2m+1}{4}\pi^2\mapsto \ln 0,m\in Z \quad \text{[from equation (21)]} \quad (27)$$

with the identity morphism

$$\ln\lim_{n\to\infty}n!^{1-s}:2mi\pi\mapsto 1,$$
$n\in N,S\in C,m\in Z$ [from propositional logic in equation (14) to predicate logic with the projective plane [11]] (28).

The morphisms are in accordance with my previous lemma that

> the convergence of the whole numbers line gives 1's dimensional property equal to $1+(\infty-1)$ with $m\in W\wedge m>0\wedge\lim_{m\to\infty}\frac{m}{m}=1$. If and only if the dimension of m sequentially increases, 0

and 1 on the whole number line belong in the same dimension of 1 [9, 12].

The origin of the *i*-metric-radius sphere's center's dimensionality is then established from the 0-centered 1-mirrored metric space, together forming a zero morphism illustrated as a derivative work from Gunther and Wereon on Wikipedia in fig. 1.

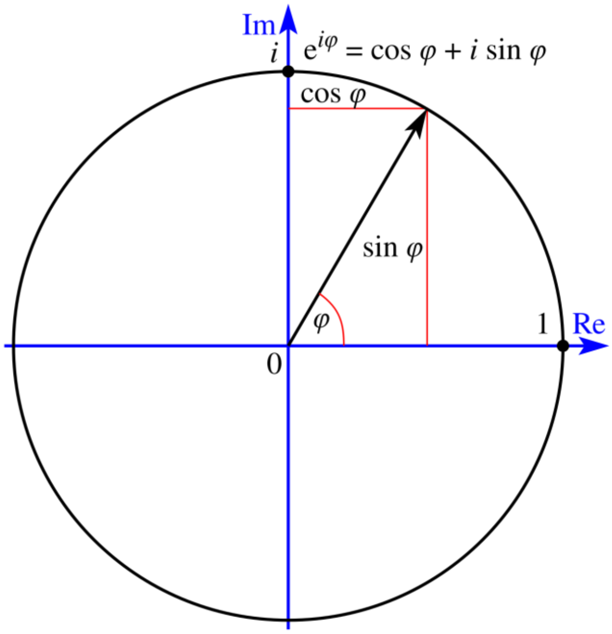

Fig. 1 Euler's formula illustrated in the complex plane.

The three-dimensional plane for the *i*-metric-radius sphere's center coordinate can then be derived from Euler's identity that

$$
\begin{gathered}
e^{ix} = \cos x + i \sin x \\
-\sec x = 1 + i \tan x \\
\tan x = \frac{e^{i\pi} - \sec x}{i} = i + i \sec x \\
e^{i\pi} = \sec x + i \tan x = -1 \\
i\pi = \ln(\sec x + i \tan x) = \ln -1 \qquad (29).
\end{gathered}
$$

The coordinate is illustrated in fig. 2 as a derivative work from fig. 1.

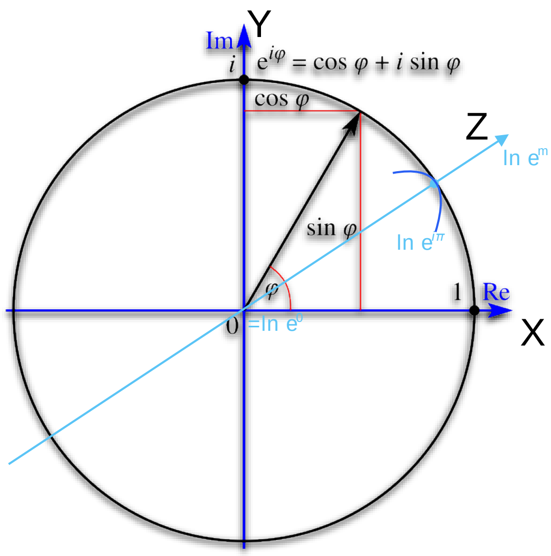

Fig. 2 The three-dimensional coordinate extended from the real number line for the geometry of the *i*-metric-radius sphere.

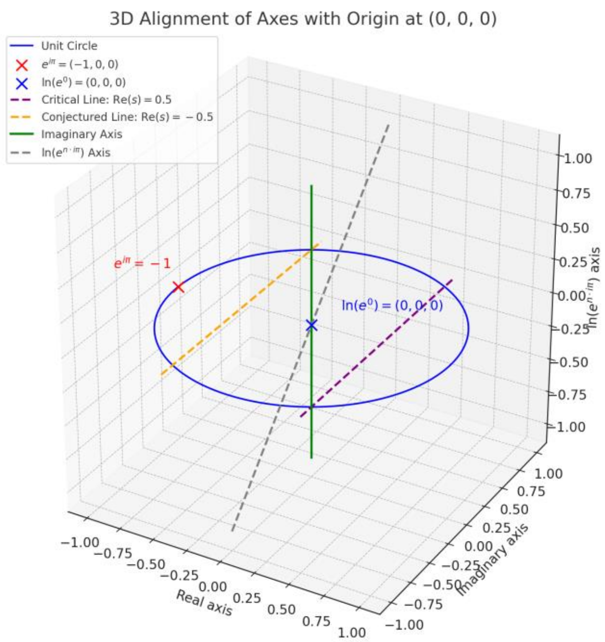

Fig. 2A An enhanced version of the visualization of Fig. 2 generated by ChatGTP.

Combined with fig. 2, equation (11), and the identity morphism equation (28), it is seen that the trivial zeros for RZF are the recurrent mapping of $\ln e^{2mi\pi}$ to the value of 0 from the Z axis, namely, the negative even integer values of *s* in the RZF. Therefore, the nontrivial zeros belong to another zero-morphism group in the same zero-morphism category, which does not yield even power to the basic element (identity) of $e^{i\pi}$. The clue can be seen with the metric space transformation from equation (9) to equation (10), which created another zero point when we take $s = 1$, hence the morphisms in equations (26) and (27) for the Z axis, with a possible closure solution for the simple pole of the RZF.

To reconstruct from the metric space expanded from equations (10) and (11) to the illustration in fig. 2, I use p to denote the pure imaginary part of the nontrivial s solutions, and q to denote the pure real part of s, $s = q + p, q \in R, p \in imaginary\ numbers$. Then

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \sum_{n=1}^{\infty} n^{-(q+p)} = \sum_{n=1}^{\infty} n^{-q+pi^2}$$
$$s \in C, n \in N, q \in R, p \in imaginary\ numbers \quad (30)$$

and by the term pi^2, it is implied that the imaginary part of s naturally belongs to the i-metric-radius sphere with the radius $\sqrt[3]{\frac{3p}{4i\pi}}i$. Therefore, the RZF with the RH can be redefined as

$$\zeta(p,q) = \sum_{n=1}^{\infty} \frac{n^{\frac{4}{3}\left(\sqrt[3]{\frac{3p}{4i\pi}}i\right)^3 \pi}}{n^q} = e^{i\pi} + 1,$$
$$n \in N, q \in R, p \in imaginary\ numbers \quad (31).$$

The redefinition separates the metric space and the topological space of the RZF coherent to the above solutions with the Z axis in fig. 2. From equation (23) and the morphism established in equation (26), the tangent value morphism can be defined with the i-metric-radius bases on the X-Z plane

$$\tan\left(\frac{\pi}{2} + m\pi\right) =$$
$$\begin{cases} \dfrac{e^{\frac{m}{2}+\frac{1}{4}}}{\sqrt{\left[\pm(2)e^{m+\frac{1}{2}}\right]^2 - e^{m+\frac{1}{2}}}} = \begin{cases} -\frac{\sqrt{i}}{8} - \frac{1}{8\sqrt{i}} \\ -\frac{\sqrt{i}}{2} - \frac{1}{2\sqrt{i}} \end{cases} \\ \dfrac{e^{m+\frac{1}{2}}}{\sqrt{\left[\left(1\pm(2)e^{m+\frac{1}{2}}\right)\right]^2 - e^{2m+1}}} = \\ \begin{cases} -\frac{1}{3}\ \{when - 1\} \\ 1\ \{when\ 1\} \\ \frac{1}{3i-5} = \frac{5}{34} + \frac{3i}{34}\ \{when - 2\} \\ \frac{1}{3i+3} = \frac{1}{6} + \frac{i}{6}\ \{when\ 2\} \end{cases} \end{cases}, m \in Z \quad (32).$$

The polymorphism exhibited by equation (32) with the coefficient implies that the vertex to the Y axis of the cone with the base on the X-Z plane is dynamic depending on the coefficient which does not affect the mapping from the X axis with integer, and equation (29) applies. The incomplete discrete verification from equation (32) suggests the two possible cones with different bases and varied slant lengths may yield infinite identities of $\sqrt{i}$, which are algebraically metric to the number set of i (Y axis), and injective to the Z axis, indicaing that they belong to the Y-Z plane.

The topological space we need to construct from the metric coordinate is then

$$\zeta(p,q) = \begin{bmatrix} q & p \\ n & 0 \end{bmatrix} \quad (33).$$

Even though the logarithm functions, geometric analysis, and some other few methods may bridge the non-algebraic transcendental numbers with the real number equivalences in algebraic operations, the eigenvalue and eigenvector solutions to bridge the potential topological space with the metric space established within the coordinate framework set in this paper may require another theoretical framework in pure mathematics, such as structuralism and ring theory, for more detailed analyses in order to define the differentiation of classes in the metric space's domain [13]. Then the deduction and simplification of the vector parts of the quaternions can more thoroughly evaluate the values of the scalar parts, i.e., the real part of the nontrivial zero solutions to the RZF. For example, if it can be justified that

$$\ln e^{ix} = \ln\cos x + \ln i \sin x$$
$$ix = \ln i \cos x \sin x = \frac{i\pi}{2} + \ln\cos x \sin x$$
$$\frac{e^{ix}}{i} = e^{\left(x-\frac{\pi}{2}\right)i} = \cos x \sin x \in [-\tfrac{1}{2}, \tfrac{1}{2}] \quad (34),$$

further analyses would have been made much simpler

4 Conclusion

The research has demonstrated that by combining the RZF with the HS can simplify the solution-finding process of RH with Euler's identity. It further demonstrated that by expanding the i-metric unit circle to an i-metric unit sphere is possible with category theory for the $\zeta(s)$ morphisms to the value of zero, with a third dimension (Z axis) by the metric of $\ln e^m, m \in C$.

By a nondiscrete mathematical approach, some properties of the undefined trigonometrical functions are derived and mapped, which corroborate with the trivial zero solutions to RZF with the constructed three-dimensional coordinate [14]. The topological space for the coordinate regarding the RZF remains to be further explored.

A trigonometric hypothesis regarding the Euler's identity in the topological space devised is proposed for further work in line with the train of thought.

Acknowledgement:
I thank the conference organizers' invitation that incentivized my further inquiries into the subject.

References:

[1] Ben-Ya'acov, U., *Gödel's incompleteness theorem and Universal physical theories.* Journal of Physics: Conference Series, 2019. **1391**(1): p. 012067.

[2] ANDRÉKA, H., J. MADARÁSZ, and I. NÉMETI, *DECIDABILITY, UNDECIDABILITY, AND GÖDEL'S INCOMPLETENESS IN RELATIVITY THEORIES.* Parallel Processing Letters, 2012. **22**(03): p. 1240011.

[3] Calcagni, G., L. Modesto, and G. Nardelli, *Non-perturbative spectrum of non-local gravity.* Physics Letters B, 2019. **795**: p. 391-397.

[4] Myers, J.M. and F. Hadi Madjid *Incompleteness theorem for physics*. 2018. arXiv:1803.10589 DOI: 10.48550/arXiv.1803.10589.

[5] A. Gonzalo, J., *Hawking on "Gödel and the End of Physics"*, in *Everything Coming Out of Nothing vs. A Finite, Open and Contingent Universe*. 2012. p. 20-25.

[6] Hilberdink, T.W., *Well-behaved Beurling primes and integers.* Journal of Number Theory, 2005. **112**(2): p. 332-344.

[7] Artur, K., *The inverse Riemann zeta function.* Computational Methods in Science and Technology, 2021. **27**(2): p. 57–91.

[8] Hejhal, D.A. and A.M. Odlyzko, *Alan Turing and the Riemann Zeta Function.* 2011.

[9] Pachankis, Y.I., *White Hole Observation: Quadrant Relations in Riemann Hypothesis.* American Journal of Planetary and Space Science, 2023. **2**(3).

[10] Daly, R.A., *Black Hole Spin and Accretion Disk Magnetic Field Strength Estimates for More Than 750 Active Galactic Nuclei and Multiple Galactic Black Holes.* The Astrophysical Journal, 2019. **886**(1): p. 37.

[11] Kim, K. and F.W. Roush, *Algebra, Abstract*, in *Encyclopedia of Physical Science and Technology (Third Edition)*, R.A. Meyers, Editor. 2003, Academic Press: New York. p. 435-464.

[12] Baite, L., *Riemann zeta function evaluated at zero*. 2021, YouTube.

[13] Dvureěenskij, A., *CHAPTER 20 - Measures on Quantum Structures*, in *Handbook of Measure Theory*, E. Pap, Editor. 2002, North-Holland: Amsterdam. p. 827-868.

[14] Pták, V., *Nondiscrete mathematical induction and iterative existence proofs.* Linear Algebra and its Applications, 1976. **13**(3): p. 223-238.

Contribution of Individual Authors

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

Funding

No funding was received for conducting this study.

Use of Artificial Intelligence

Only Fig. 2A is generated by the use of AI. The author attempted to polish the manuscript with the AI, but decided to remain the original style for the complexion of human imagination in mathematics, instead of the robustness of relevant theorems.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.