Crustal Deformation of the Earth due to Inclined Dip-slip Fault

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Abstract:-The study examines a closed-form analytical solution for the plane strain problem concerning the static deformation of a homogeneous, isotropic, elastic (HIE) layer of uniform thickness lying over a rough-rigid base (RRB). This deformation is induced by a dip-slip fault at 45° with opening in the horizontal direction embedded within the elastic layer. The Airy stress function approach is employed to derive the displacement and stress (DAS) fields in the integral form by applying appropriate boundary conditions at the free surface. To analytically evaluate these integrals, the denominator term is approximated by a finite sum of exponential (FSE) terms using the method of least squares. The integral expressions for the displacements are evaluated analytically and the stresses can be derived in similar manner. This study provides insights into the role of fault geometry in crustal deformation and enhances the modeling of seismic hazard in tectonically active regions.

Key-Words: - Airy-stress function, Continental crust, Dip-slip Fault at 45°, Least Square Approximation, Plane Strain Deformation, Two-dimensional.

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1 Introduction

The deformation of the Earth's crust due to faulting is a significant concern in seismology and tectonics. Most earthquake foci occur within the Earth's crust, where seismic waves originate from the focus and propagate throughout the Earth's interior. The crustal deformation is affected not only by the faulting processes but also by the complex internal boundaries of the Earth. Understanding these deformation patterns is essential for hazard assessment and tectonic modeling. While past studies often focus on elastic or viscoelastic half-space models, the influence of fault orientation within layered media remains insufficiently unexplored. In this paper, we attempt to study crustal deformation resulting from two dimensional faulting. We examine the crustal deformation due to faulting using a simplest model, considering a HIE layer of finite width resting over a RRB induced by a 45° inclined dip-slip fault. Such fault configurations are typical in tectonically active regions. The findings may contribute to earthquake modeling, seismic hazards assessment and a deeper understanding of tectonics.

Most studies assumed the Earth's crust as an elastic layer overlying an elastic or viscoelastic half-space, focusing on the deformation due to various source dislocations. Ben-Menahem and Singh [1] obtained a solution for the static deformation field caused by an arbitrary multipolar source in a layered half-space. In contrast, Ben-Menahem and Gillon [2] demonstrated the pronounced effect of low rigidity crustal layers on the displacement field. The postseismic displacement and strains for a rectangular dip-slip fault within a three layer medium derived by Ma and Kusznir [3]. Singh and Garg [4] developed integral expressions for the Airy stress function for two-dimensional sources in an unbounded medium, while Singh et al. [5] examined the plane strain deformation of a layered half-space due to a very long dip-slip fault within the upper layer by using FSE terms. Savage [6] computed the displacement field resulting from an edge dislocation in Earth model comprising an elastic layer welded to an elastic half-space. Extending their prior study [5], Singh et al. [7] calculated the postseismic deformation in a layered half-space driven by a very long dip-slip fault. Nespoli et al. [8] explored the crack growth model in an elastic medium consisting of two welded half-spaces with various rigidities. Rani and Rani [9] derived the analytical expressions for the displacements due to dip-slip faulting for a HIE layer situated on a RRB. Additionally, several researchers [10-18] have investigated the deformation in Earth models comprising a layer of finite thickness over a half-space, focusing on strike-slip faulting.

In the present paper, the deformation for a two-dimensional plane strain model for a HIE layer resting on a RRB resulting from a 45° inclined dip-slip fault within a layer has been studied. The integral expressions for the DAS have been obtained using Airy stress function approach by applying the suitable boundary conditions at the free surface. To evaluate the integrals analytically, the complicated expression of denominator has been approximated as a finite sum of exponential terms [22] using Least square approximation [23] in such a way that the error is minimized. Then the integrals are evaluated analytically for the DAS can be derived similarly. This study provides insights into the role of fault geometry in crustal deformation and enhances the modeling of seismic hazard in tectonically active regions. Earthquakes due to dip-slip fault at 45° are generally observed in tectonically regions and the 45° dip-slip motion in these regions plays a crucial role in the release of tectonic stress, often triggering the seismic events.

2 Problem Formulation

We consider a two-dimensional approximation wherein the displacement components (u_x, u_y, u_z) are independent of a single co-ordinate x, say $\partial/\partial x \equiv 0$. This simplification allows the decoupling of the plane strain and anti-plane strain problems. Under this assumption, the plane strain problem $(u_x = 0)$ and the anti-plane strain problem get decoupled, and can be addressed separately. Here, we discuss the plane strain case only. An elastically isotropic medium is characterized by two elastic constants, namely, shear modulus (μ) and Poisson's ratio (ν).

Let us define a Cartesian co-ordinate system (x, y, z) with z-axis vertically downwards. The model consists of a HIE layer of finite width H occupying the region $-\infty \le y \le \infty,\ 0 \le z \le H$, lying over a RRB. The layer is in contact with a RRB at $z = H$. A dip-slip fault inclined at 45° to the horizontal is embedded within this layer at depth $z = h$ and extends infinitely along y-direction, producing the deformation throughout the layer (Fig. 1).

The plane strain problem for an elastic isotropic medium can be solved in terms of the Airy stress function U such that [19]

$$\sigma_{yy} = \frac{\partial^2 U}{\partial z^2}, \sigma_{yz} = -\frac{\partial^2 U}{\partial y \partial z}, \sigma_{zz} = \frac{\partial^2 U}{\partial y^2} \tag{1}$$

satisfies

$$\nabla^2 \nabla^2 U = 0 \tag{2}$$

where $\sigma_{ij}\,(i, j = y, z)$ are the stress components. The displacements components are obtained on integrating the constitutive equations:

$$\begin{aligned} \sigma_{yy} &= \frac{2\mu}{1-2\nu}\left[(1-\nu)e_{yy} + \nu e_{zz}\right] \\ \sigma_{zz} &= \frac{2\mu}{1-2\nu}\left[\nu e_{yy} + (1-\nu)e_{zz}\right] \\ \sigma_{yz} &= 2\mu e_{yz} \end{aligned} \tag{3}$$

where, $e_{ij}\,(i, j = y, z)$ are the strain components. The Airy stress function used to express the displacement components. Here, the expressions for the displacements in terms of Airy stress function are given by

$$2\mu u_y = -\frac{\partial U}{\partial y} + \frac{1}{2\alpha}\int \nabla^2 U\, dy \tag{4}$$

$$2\mu u_z = -\frac{\partial U}{\partial z} + \frac{1}{2\alpha}\int \nabla^2 U\, dz \tag{5}$$

where

$$\alpha = \frac{1}{2(1-\nu)}, \quad \nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \tag{6}$$

The Airy stress function U_0 for a line source passing through $(0, 0, h)$ parallel to the y-axis in an unbounded, isotropic, elastic medium is of the form [4]

$$U_0 = \int_0^\infty (S_1 + S_2 \varepsilon k Z) e^{-\varepsilon k Z} \begin{pmatrix} \sin ky \\ \cos ky \end{pmatrix} \frac{dk}{k} \tag{7}$$

where,

$$Z = z - h,\ \varepsilon = \pm 1 \tag{8}$$

the upper sign is for $Z > 0$ and the lower sign for $Z < 0$. The source coefficients S_1 and S_2 are independent of k and are listed for various sources by [20], as presented in Table 1 for dip-slip faulting at 45°. We refer the notations of [21] for labelling these sources. The double couple (33) - (22) denotes the double couple of strength D_{23} where forces bisect the angles between the dipoles (22) and (33).

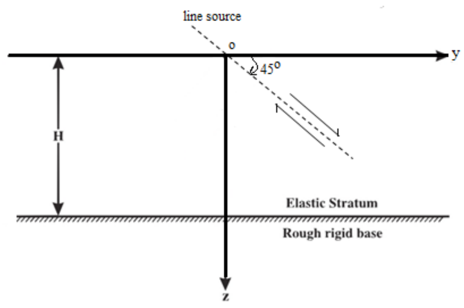

Fig.1. Geometry of a 45° inclined dip slip line dislocation in a HIE layer of finite width H overlying a RRB at $z=H$.

Table 1

Source	S_1	S_2	Solution
Double couple (33) - (22)	0	$\alpha D_{23}/\pi$	Lower
Double couple (23) + (32)	0	$\varepsilon\alpha D_{23}/\pi$	Upper

Table 1. Source coefficients for inclined dip-slip fault and vertical dip-slip fault.

Using Fourier transform to eq.(2) and solving the resulting ordinary differential equation for the elastic stratum $\left(0\le z\le H\right)$ consisting of a line dislocation situated at the location $\left(0,0,h\right)$ parallel to x-axis, the solution of Eq. (2) is obtained as

$$U=U_0+\int_0^\infty\left[\left(c_1+c_2kz\right)e^{kz}+\left(c_3+c_4kz\right)e^{-kz}\right]\begin{pmatrix}\sin ky\\ \cos ky\end{pmatrix}dk \tag{9}$$

where, $c_i\left(i=1,2,3,4\right)$ are unknowns. Using Eqs. (1), (4)-(5) and (9), we derive the integral expressions for the DAS:

$$\begin{aligned}\sigma_{yy}&=\int_0^\infty\left[S_1+S_2\left(-2+\varepsilon kZ\right)\right]e^{-\varepsilon kZ}\begin{pmatrix}\sin ky\\ \cos ky\end{pmatrix}kdk\\&+\int_0^\infty\left\{\left[c_1+c_2\left(2+kz\right)\right]e^{kz}+\left[c_3+c_4\left(-2+kz\right)\right]e^{-kz}\right\}\begin{pmatrix}\sin ky\\ \cos ky\end{pmatrix}k^2dk\end{aligned} \tag{10}$$

$$\begin{aligned}\sigma_{yz}&=\int_0^\infty\varepsilon\left[S_1-S_2\left(1-\varepsilon kZ\right)\right]e^{-\varepsilon kZ}\begin{pmatrix}\cos ky\\ -\sin ky\end{pmatrix}kdk\\&+\int_0^\infty\left\{-\left[c_1+c_2\left(1+kz\right)\right]e^{kz}+\left[c_3-c_4\left(1-kz\right)\right]e^{-kz}\right\}\begin{pmatrix}\cos ky\\ -\sin ky\end{pmatrix}k^2dk\end{aligned} \tag{11}$$

$$\begin{aligned}\sigma_{zz}&=-\int_0^\infty\left[S_1+S_2\varepsilon kZ\right]e^{-\varepsilon kZ}\begin{pmatrix}\sin ky\\ \cos ky\end{pmatrix}kdk\\&-\int_0^\infty\left\{\left[c_1+c_2kz\right]e^{kz}+\left[c_3+c_4kz\right]e^{-kz}\right\}\begin{pmatrix}\sin ky\\ \cos ky\end{pmatrix}k^2dk\end{aligned} \tag{12}$$

$$\begin{aligned}2\mu u_y&=-\int_0^\infty\left[S_1+S_2\left(-\frac{1}{\alpha}+\varepsilon kZ\right)\right]e^{-\varepsilon kZ}\begin{pmatrix}\cos ky\\ -\sin ky\end{pmatrix}dk\\&-\int_0^\infty\left\{\left[c_1+c_2\left(kz+\frac{1}{\alpha}\right)\right]e^{kz}+\left[c_3+c_4\left(kz-\frac{1}{\alpha}\right)\right]\right\}e^{-kz}\begin{pmatrix}\cos ky\\ -\sin ky\end{pmatrix}k\ dk\end{aligned} \tag{13}$$

$$\begin{aligned}2\mu u_z&=\int_0^\infty\varepsilon\left[S_1+S_2\left(\frac{1}{\alpha}-1+\varepsilon kZ\right)\right]e^{-\varepsilon kZ}\begin{pmatrix}\sin ky\\ \cos ky\end{pmatrix}dk\\&+\int_0^\infty\left\{-\left[c_1+c_2\left(kz-\frac{1}{\alpha}+1\right)\right]e^{kz}+\left[c_3+c_4\left(kz+\frac{1}{\alpha}-1\right)\right]e^{-kz}\right\}\begin{pmatrix}\sin ky\\ \cos ky\end{pmatrix}k\ dk\end{aligned} \tag{14}$$

Equations (10)-(14) give the integral form solution for a finite layer due a two dimensional faulting.

Assuming that the upper surface of the elastic layer is traction-free. The boundary conditions are

$$\sigma_{zz}=\sigma_{yz}=0 \tag{15}$$

at $z=0$. Additionally, the lower surface $z=H$ rests on a RRB, hence

$$u_y=0,\ u_z=0 \tag{16}$$

for $z=H$. Using Eqs. (15)-(16), a system of four equations in four unknowns is obtained in $c_i\left(i=1,2,3,4\right)$ and on solving by Cramer's rule, the values of unknowns are obtained as:

$$\begin{aligned}c_1&=\frac{1}{Dk}\Big[S_1^-e^{-kh}\left\{-\delta e^{-4kH}+\left(4k^2H^2+2kH+\delta^2\right)e^{-2kH}\right\}\\&+S_2^-e^{-kh}\left\{-kh\delta e^{-4kH}+\left(4k^3H^2h-2k^2H(H-h)+kh\delta^2+\frac{1-\delta^2}{2}\right)e^{-2kH}\right\}\\&+S_1^+e^{kh}\left\{\delta e^{-4kH}-\left(2kH+1\right)e^{-2kH}\right\}\\&+S_2^+e^{kh}\left\{-kh\delta e^{-4kH}+\left(2k^2H(h-H)+kh+\frac{1-\delta^2}{2}\right)e^{-2kH}\right\}\Big]\\c_2&=\frac{1}{Dk}\Big[S_1^-e^{-kh}\left(-4kHe^{-2kH}\right)+S_2^-e^{-kh}\left\{\delta e^{-4kH}-\left(4k^2Hh-2kH+1\right)e^{-2kH}\right\}\\&+2S_1^+e^{kh}\left\{-\delta e^{-4kH}+e^{-2kH}\right\}+S_2^+e^{kh}\left\{\left(2kh+1\right)\delta e^{-4kH}+\left\{2k(H-h)-1\right\}e^{-2kH}\right\}\Big]\\c_3&=\frac{1}{Dk}\Big[S_1^-e^{-kh}\left\{\left(1-2kH\right)e^{-2kH}-\delta\right\}\\&+S_2^-e^{-kh}\left\{\left(2k^2H\left(H-h\right)+kh-\frac{1-\delta^2}{2}\right)e^{-2kH}-kh\delta\right\}\\&+S_1^+e^{kh}\left\{-\delta e^{-4kH}+\left(2kH+1\right)e^{-2kH}\right\}\\&+S_2^+e^{kh}\left\{kh\delta e^{-4kH}+\left(2k^2H\left(H-h\right)-kh-\frac{1-\delta^2}{2}\right)e^{-2kH}\right\}\Big]\\c_4&=\frac{1}{Dk}\Big[2S_1^-e^{-kh}\left(e^{-2kH}-\delta\right)+S_2^-e^{-kh}\left\{\left(2k\left(h-H\right)-1\right)e^{-2kH}+\delta\left(1-2kh\right)\right\}\\&+S_1^+e^{kh}\left(4kH\right)e^{-2kH}+S_2^+e^{kh}\left\{-\delta e^{-4kH}+\left(4k^2H(H-h)-2kH+\delta^2\right)e^{-2kH}\right\}\Big]\end{aligned} \tag{17}$$

where

$$D=\delta\left[1+\left(A+Bk^2H^2\right)e^{-2kH}+e^{-4kH}\right]$$

$$A=-\left(\delta+\frac{1}{\delta}\right),\quad B=-\frac{4}{\delta},\ \delta=4\nu-3=1-\frac{2}{\alpha}. \tag{18}$$

On substituting the values of unknowns in Eqs. (10)-(14), the integral forms for the DAS are obtained.

3 Problem Solution

From Table 1, the source coefficients for a dip-slip fault inclined at 45° with opening in the y–direction are specified by double couple (33)-(22):

$$S_1 = S_1^- = S_1^+ = 0, S_2 = S_2^- = S_2^+ = \frac{\alpha D_{23}}{\pi} = \frac{\alpha bds}{\pi} \quad (19)$$

where b denotes the slip magnitude and ds represents the width of the line dislocation. By substituting these source coefficients from Eq. (19) into the integral form of DAS, a solution for the dip-slip fault at 45° in an elastic isotropic layer $\left(0 \le z \le H\right)$ is obtained. The displacements are expressed as follows:

$$\begin{aligned} u_y = \frac{\alpha bds}{2\pi} \Bigg[& \int_0^\infty \left(\varepsilon kZ - \frac{(1-\delta)}{2} \right) e^{-\varepsilon kZ} \sin ky\, dk \\ & + \frac{1}{\delta} \int_0^\infty \Bigg[\left\{ c_1' + c_2' \left(kz + \frac{1-\delta}{2} \right) \right\} e^{kz} \\ & + \left\{ c_3' + c_4' \left(kz - \frac{1-\delta}{2} \right) \right\} e^{-kz} \Bigg] \frac{\sin ky}{D} dk \Bigg] \end{aligned} \quad (20)$$

$$\begin{aligned} u_z = \frac{\alpha bds}{2\pi} \Bigg[& \int_0^\infty \left(kZ + \varepsilon \frac{(3-\delta)}{2} \right) e^{-\varepsilon kZ} \cos ky\, dk \\ & + \frac{1}{\delta} \int_0^\infty \Bigg[-\left\{ c_1' + c_2' \left(kz + \frac{1+\delta}{2} \right) \right\} e^{kz} \\ & + \left\{ c_3' + c_4' \left(kz - \frac{1+\delta}{2} \right) \right\} e^{-kz} \Bigg] \frac{\cos ky}{D} dk \Bigg] \end{aligned} \quad (21)$$

where, the coefficients $c_i' (i = 1,2,3,4)$ are given below:

$$\begin{aligned} c_1' = & e^{-kh} \Big[-kh\delta e^{-4kH} + \Big\{ 4k^3 H^2 h - 2k^2 H(H-h) + \delta^2 kh \\ & + \frac{1-\delta^2}{2} \Big\} e^{-2kH} \Big] + e^{kh} \Big[-kh\delta e^{-4kH} + \Big\{ 2k^2 H(h-H) \\ & + kh + \frac{1-\delta^2}{2} \Big\} e^{-2kH} \Big] \\ c_2' = & e^{-kh} \left\{ \delta e^{-4kH} - \left(4k^2 Hh - 2kH + 1 \right) e^{-2kH} \right\} \\ & + e^{kh} \left[(1+2kh)\delta e^{-4kH} + \left\{ 2k(H-h) - 1 \right\} e^{-2kH} \right] \\ c_3' = & e^{-kh} \left[\left\{ 2k^2 H(H-h) + kh - \frac{1-\delta^2}{2} \right\} e^{-2kH} - kh\delta \right] \\ & + e^{kh} \left[kh\delta e^{-4kH} + \left\{ 2k^2 H(H-h) - kh - \frac{1-\delta^2}{2} \right\} e^{-2kH} \right] \\ c_4' = & e^{-kh} \left[(1-2kh)\delta + \left\{ 2k\left(h-H\right) - 1 \right\} e^{-2kH} \right] \\ & + e^{kh} \left[-\delta e^{-4kH} + \left\{ 4k^2 H(H-h) - 2kH + \delta^2 \right\} e^{-2kH} \right] \end{aligned} \quad (22)$$

To analytically solve the integrals in Eqs. (20)-(21), the denominator expression $1/D$ is replaced with FSE terms to reduce error. Following [22], it is expanded as a sum of exponential terms using binomial expansion and truncated to the second order of kH [23]:

$$\frac{1}{D} \approx 1 - \left(A + Bk^2 H^2 \right) e^{-2kH} + \left[C + \beta \left(kH \right)^n \right] e^{-\gamma kH} \quad (23)$$

where, C is a constant independent of kH ; β, γ and n are determined using least square approximation. To find C, we use the asymptotic approximation taking the limit as $kH \to 0$, resulting in

$$C = \frac{A^2 + A - 1}{2 + A} = \frac{\delta^2 + 1}{1 - \delta} - \frac{1}{\delta(\delta - 1)^2} \quad (24)$$

To determine the best-fitted values of β, γ and n depending upon kH, ν in equation (23) to minimize error, we apply the least square approximation as described by Scarborough [23]. We assume n = 2 as suggested by Ben-Menahem and Gillon [2] for rapid convergence and assume $\nu = 0.23$ for continental layer and $\nu = 0.3$ for oceanic layer. The best-fitted values of β, γ are obtained as Rani and Rani [9]:

$\beta = 1.1757, \gamma = 2.6025$ for $\nu = 0.23$

and $\beta = 2.9247, \gamma = 3.2453$ for $\nu = 0.3$ (25)

From Eqs. (23)-(25), $1/D$ is approximated with best fitted values of β, γ . Using this expression of $1/D$ in Eqs. (20)-(21), the displacements can be expressed as a linear combination of integrals, which are evaluated by using the standard integrals (Gradshteyn and Ryzhik [24]).The analytical expressions for the displacements are obtained using the method of least squares such as:

$$u_y=\frac{\alpha bds}{2\pi\delta}\left[\frac{\delta(\delta-1)y}{2R^2}-\frac{2\delta y(z-h)^2}{R^4}-\frac{2\delta y(z-h\delta)(z+h)}{S^2}\right.$$
$$-\frac{4hzy\delta}{S^4}\left(\frac{4(z+h)^2}{S^2}-1\right)-\frac{\delta y(1-\delta)}{2S^2}+\left|\phi_1\right|_{n=2}+\left|\phi_2\right|_{n=4}$$
$$-A\left\{\left|\phi_1\right|_{n=4}+\left|\phi_2\right|_{n=6}+\left|\phi_3\right|_{n=2}\right\}-BH^2\left\{\left|\phi_4\right|_{n=6}+\left|\phi_5\right|_{n=4}+\left|\phi_6\right|_{n=2}\right\}$$
$$+C\left\{\left|\phi_1\right|_{n=2+\gamma}+\left|\phi_2\right|_{n=4+\gamma}+\left|\phi_3\right|_{n=\gamma}\right\}$$
$$\left.+\beta H^2\left\{\left|\phi_4\right|_{n=4+\gamma}+\left|\phi_5\right|_{n=2+\gamma}+\left|\phi_6\right|_{n=\gamma}\right\}\right] \tag{26}$$

$$u_z=\frac{\alpha bds}{2\pi\delta}\left[\frac{\delta(z-h)}{R^2}\left(\frac{2(z-h)^2}{R^2}-1\right)+\frac{\delta(3-\delta)(z-h)}{2R^2}\right.$$
$$+\frac{\delta(z+h\delta)}{S^2}\left(\frac{2(z+h)^2}{S^2}-1\right)-\frac{4hz\delta(z+h)}{S^4}\left(\frac{4(z+h)^2}{S^2}-3\right)$$
$$-\frac{\delta(1+\delta)(z+h)}{2S^2}+\left|\psi_1\right|_{n=2}+\left|\psi_2\right|_{n=4}$$
$$-A\left\{\left|\psi_1\right|_{n=4}+\left|\psi_2\right|_{n=6}+\left|\psi_3\right|_{n=2}\right\}-BH^2\left\{\left|\psi_4\right|_{n=6}+\left|\psi_5\right|_{n=4}+\left|\psi_6\right|_{n=2}\right\}$$
$$+C\left\{\left|\psi_1\right|_{n=2+\gamma}+\left|\psi_2\right|_{n=4+\gamma}+\left|\psi_3\right|_{n=\gamma}\right\}$$
$$\left.+\beta H^2\left\{\left|\psi_4\right|_{n=4+\gamma}+\left|\psi_5\right|_{n=2+\gamma}+\left|\psi_6\right|_{n=\gamma}\right\}\right] \tag{27}$$

where,
$R^2=y^2+(z-h)^2,\ S^2=y^2+(z+h)^2$
and $\phi_i,\ \psi_i\ (i=1,2,...6)$ are given in Appendix.

On taking the limit as H $\to\infty$, the displacements coincide with the corresponding results for a vertical tensile fault given by Rani et al [25], confirming the validity of the model.

4 Conclusion

The elastic residual field due to a dip-slip fault at 45º with horizontal opening buried in a layer of finite thickness overlying a RRB is obtained. The linear combination of exponential terms occurring in the integral expressions of the deformation field is approximated by a FSE terms using the method of least squares. Then the integrals have been evaluated for the displacement field analytically. The stresses can be computed similarly.

Appendix

$$\phi_1=96Hhy(H-z)\frac{p_n}{T_n^{\,6}}\left(\frac{2p_n^{\,2}}{T_n^{\,2}}-1\right)+\frac{\delta y(1-\delta)}{2T_n^{\,2}}$$
$$-4Hy(H-z-\delta h)\frac{1}{T_n^{\,4}}\left(\frac{4p_n^{\,2}}{T_n^{\,2}}-1\right)+\frac{y(\delta^3-1)}{2U_n^{\,2}}$$
$$+2y\left(h\delta^2-z+(1-\delta)H\right)\frac{p_n}{T_n^{\,4}}$$
$$+2y\left(\delta^2 z-h+(1-\delta)H\right)\frac{q_n}{U_n^{\,4}}$$
$$+96Hzy(H-h)\frac{q_n}{U_n^{\,6}}\left(\frac{2q_n^{\,2}}{U_n^{\,2}}-1\right)$$
$$+4Hy\left(\delta(H-h)-z\right)\frac{1}{U_n^{\,4}}\left(\frac{4q_n^{\,2}}{U_n^{\,2}}-1\right)$$
$$+4y(z-H)(H-h)\left\{\frac{1}{V_n^{\,4}}\left(\frac{4r_n^{\,2}}{V_n^{\,2}}-1\right)+\frac{1}{W_n^{\,4}}\left(\frac{4s_n^{\,2}}{W_n^{\,2}}-1\right)\right\}$$
$$+2y\left(h\delta+H(1-\delta)-z\right)\left\{\frac{r_n}{V_n^{\,4}}+\frac{s_n}{W_n^{\,4}}\right\}$$
$$+\frac{\delta y(1-\delta)}{2}\left\{\frac{1}{V_n^{\,2}}+\frac{1}{W_n^{\,2}}\right\}$$

$$\phi_2=2\delta y(z-h)\left\{\frac{p_n}{T_n^{\,4}}-\frac{q_n}{U_n^{\,4}}\right\}+\frac{\delta y(1-\delta)}{2}\left\{\frac{1}{T_n^{\,2}}+\frac{1}{V_n^{\,2}}+\frac{1}{U_n^{\,2}}\right\}$$
$$+4\delta hzy\frac{1}{V_n^{\,4}}\left(\frac{4r_n^{\,2}}{V_n^{\,2}}-1\right)+2\delta y(z-\delta h)\frac{r_n}{V_n^{\,4}}$$

$$\phi_3=\frac{2\delta y(z-h\delta)s_n}{W_n^{\,4}}-\frac{4\delta hzy}{W_n^{\,4}}\left(\frac{4s_n^{\,2}}{W_n^{\,2}}-1\right)-\frac{\delta y(1-\delta)}{2W_n^{\,4}}$$

$$\phi_4=24\delta y(z-h)\left\{\frac{p_n}{T_n^{\,6}}\left(\frac{2p_n^{\,2}}{T_n^{\,2}}-1\right)+\frac{q_n}{U_n^{\,6}}\left(\frac{2q_n^{\,2}}{U_n^{\,2}}-1\right)\right\}$$
$$+\delta y(1-\delta)\left\{\frac{1}{T_n^{\,4}}\left(\frac{4p_n^{\,2}}{T_n^{\,2}}-1\right)+\frac{1}{U_n^{\,4}}\left(\frac{4q_n^{\,2}}{U_n^{\,2}}-1\right)\right\}$$
$$+\frac{48\delta hzy}{V_n^{\,6}}\left(\frac{16r_n^{\,4}}{V_n^{\,4}}-\frac{12r_n^{\,2}}{V_n^{\,2}}+1\right)+\delta y(1-\delta)\frac{1}{V_n^{\,4}}\left(\frac{4r_n^{\,2}}{V_n^{\,2}}-1\right)$$
$$+24\delta y(z-\delta h)\frac{r_n}{V_n^{\,6}}\left(\frac{2r_n^{\,2}}{V_n^{\,2}}-1\right)$$

$$\psi_2=\delta(h-z)\left\{\frac{1}{T_n^{\,2}}\left(\frac{2p_n^{\,2}}{T_n^{\,2}}-1\right)+\frac{1}{U_n^{\,2}}\left(\frac{2q_n^{\,2}}{U_n^{\,2}}-1\right)\right\}$$
$$-\frac{\delta(1+\delta)}{2}\left(\frac{p_n}{T_n^{\,2}}+\frac{r_n}{V_n^{\,2}}-\frac{q_n}{U_n^{\,2}}\right)-4\delta hz\frac{r_n}{V_n^{\,4}}\left(\frac{4r_n^{\,2}}{V_n^{\,2}}-3\right)$$
$$-\delta(z+\delta h)\frac{1}{V_n^{\,2}}\left(\frac{2r_n^{\,2}}{V_n^{\,2}}-1\right)$$

$$\psi_3 = \frac{\delta(z+h\delta)}{W_n^2}\left(\frac{2s_n^2}{W_n^2}-1\right) - \frac{4\delta hzs_n}{W_n^4}\left(\frac{4s_n^2}{W_n^2}-3\right) - \frac{\delta(1+\delta)s_n}{2W_n^4}$$

$$\phi_5 = 960Hhy(H-z)\frac{p_n}{T_n^8}\left(\frac{16p_n^4}{T_n^4}-\frac{16p_n^2}{T_n^2}+3\right)$$

$$-\frac{48Hy(H-z-\delta h)}{T_n^6}\left(\frac{16p_n^4}{T_n^4}-\frac{12p_n^2}{T_n^2}+1\right)$$

$$+24y\left(h\delta^2 - z + (1-\delta)H\right)\frac{p_n}{T_n^6}\left(\frac{2p_n^2}{T_n^2}-1\right)$$

$$+\delta y(1-\delta)\left\{\frac{1}{T_n^4}\left(\frac{4p_n^2}{T_n^2}-1\right)+\frac{1}{V_n^4}\left(\frac{4r_n^2}{V_n^2}-1\right)+\frac{1}{W_n^4}\left(\frac{4s_n^2}{W_n^2}-1\right)\right\}$$

$$+48y(z-H)(H-h)\left\{\frac{1}{V_n^6}\left(\frac{16r_n^4}{V_n^4}-\frac{12r_n^2}{V_n^2}+1\right)\right.$$

$$\left.-\frac{1}{W_n^6}\left(\frac{16s_n^4}{W_n^4}-\frac{12s_n^2}{W_n^2}+1\right)\right\}+\frac{y\left(\delta^3-1\right)}{U_n^4}\left(\frac{4q_n^2}{U_n^2}-1\right)$$

$$+24y\left(h\delta - z + H(1-\delta)\right)\left\{\frac{r_n}{V_n^6}\left(\frac{2r_n^2}{V_n^2}-1\right)+\frac{s_n}{W_n^6}\left(\frac{2s_n^2}{W_n^2}-1\right)\right\}$$

$$+960Hzy(H-h)\frac{q_n}{U_n^8}\left(\frac{16q_n^4}{U_n^4}-\frac{16q_n^2}{U_n^2}+3\right)$$

$$+\frac{48Hy\left(\delta(H-h)-z\right)}{U_n^6}\left(\frac{16q_n^4}{U_n^4}-\frac{12q_n^2}{U_n^2}+1\right)$$

$$+24y\left(\delta^2 z - h + (1-\delta)H\right)\frac{q_n}{U_n^6}\left(\frac{2q_n^2}{U_n^2}-1\right)$$

$$\psi_1 = \frac{24Hh(z-H)}{T_n^4}\left(\frac{8p_n^4}{T_n^4}-\frac{8p_n^2}{T_n^2}+1\right)$$

$$+4Hy(H-z+\delta h)\frac{p_n}{T_n^4}\left(\frac{4p_n^2}{T_n^2}-3\right)$$

$$-\frac{\left(h\delta^2 - z + (1+\delta)H\right)}{T_n^2}\left(\frac{2p_n^2}{T_n^2}-1\right)$$

$$+\frac{\delta(1+\delta)}{2}\left(\frac{p_n}{T_n^2}+\frac{s_n}{W_n^2}+\frac{r_n}{V_n^2}\right)-\frac{\left(1+\delta^3\right)q_n}{2U_n^2}$$

$$+4(z-H)(h-H)\left\{\frac{r_n}{V_n^4}\left(\frac{4r_n^2}{V_n^2}-3\right)-\frac{s_n}{W_n^4}\left(\frac{4s_n^2}{W_n^2}-3\right)\right\}$$

$$-\left(z+h\delta-H(1+\delta)\right)\left\{\frac{1}{V_n^2}\left(\frac{2r_n^2}{V_n^2}-1\right)-\frac{1}{W_n^2}\left(\frac{2s_n^2}{W_n^2}-1\right)\right\}$$

$$+\frac{24Hz(H-h)}{U_n^4}\left(\frac{8q_n^4}{U_n^4}-\frac{8q_n^2}{U_n^2}+1\right)$$

$$-4H\left(z+\delta(H-h)\right)\frac{q_n}{U_n^4}\left(\frac{4q_n^2}{U_n^2}-3\right)$$

$$+\frac{\left(\delta^2 z - h + (1+\delta)H\right)}{U_n^2}\left(\frac{2q_n^2}{U_n^2}-1\right)$$

$$\psi_6 = \frac{6\delta(z+h\delta)s_n}{W_n^4}\left(\frac{8s_n^4}{W_n^4}-\frac{8s_n^2}{W_n^2}+1\right)-\frac{48\delta hzs_n}{W_n^6}\left(\frac{16s_n^4}{W_n^4}-\frac{20s_n^2}{W_n^2}+5\right)$$

$$-\frac{\delta(1+\delta)s_n}{W_n^4}\left(\frac{4s_n^2}{W_n^2}-3\right)$$

$$\phi_6 = \frac{24\delta y(z-h\delta)s_n}{W_n^6}\left(\frac{2s_n^2}{W_n^2}-1\right)-\frac{48\delta hzy}{W_n^6}\left(\frac{16s_n^4}{W_n^4}-\frac{12s_n^2}{W_n^2}+1\right)$$

$$-\frac{\delta y(1-\delta)}{W_n^4}\left(\frac{4s_n^2}{W_n^2}-1\right)$$

$$\psi_4 = 6\delta(z-h)\left\{\frac{1}{T_n^4}\left(\frac{8p_n^4}{T_n^4}-\frac{8p_n^2}{T_n^2}+1\right)+\frac{1}{U_n^4}\left(\frac{8q_n^4}{U_n^4}-\frac{8q_n^2}{U_n^2}+1\right)\right\}$$

$$-\delta(1+\delta)\left\{\frac{p_n}{T_n^4}\left(\frac{4p_n^2}{T_n^2}-3\right)-\frac{q_n}{U_n^4}\left(\frac{4q_n^2}{U_n^2}-3\right)\right\}$$

$$-\frac{48\delta hzr_n}{V_n^6}\left(\frac{16r_n^4}{V_n^4}-\frac{20r_n^2}{V_n^2}+5\right)-\frac{\delta(1+\delta)r_n}{V_n^4}\left(\frac{4r_n^2}{V_n^2}-3\right)$$

$$-\frac{6\delta(z+\delta h)}{V_n^4}\left(\frac{8r_n^4}{V_n^4}-\frac{8r_n^2}{V_n^2}+1\right)$$

$$\psi_5 = 480Hh(z-H)\frac{1}{T_n^6}\left(\frac{32p_n^6}{T_n^6}-\frac{48p_n^4}{T_n^4}+\frac{18p_n^2}{T_n^2}-1\right)$$

$$+48H(H-z+h\delta)\frac{p_n}{T_n^6}\left(\frac{16p_n^4}{T_n^4}-\frac{20p_n^2}{T_n^2}+5\right)$$

$$-6\left(h\delta^2 - z + H(1+\delta)\right)\frac{1}{T_n^4}\left(\frac{8p_n^4}{T_n^4}-\frac{8p_n^2}{T_n^2}+1\right)$$

$$+\frac{\delta(1+\delta)}{2}\left\{\frac{p_n}{T_n^4}\left(\frac{4p_n^2}{T_n^2}-3\right)+\frac{r_n}{V_n^4}\left(\frac{4r_n^2}{V_n^2}-3\right)\right.$$

$$\left.+\frac{s_n}{W_n^4}\left(\frac{4s_n^2}{W_n^2}-3\right)\right\}$$

$$+48(h-H)(z-H)\left\{\frac{r_n}{V_n^6}\left(\frac{16r_n^4}{V_n^4}-\frac{20r_n^2}{V_n^2}+5\right)\right.$$

$$\left.-\frac{s_n}{W_n^6}\left(\frac{16s_n^4}{W_n^4}-\frac{20s_n^2}{W_n^2}+5\right)\right\}$$

$$-6\left(H(1+\delta)-h\delta-z\right)\left\{\frac{1}{V_n^4}\left(\frac{8r_n^4}{V_n^4}-\frac{8r_n^2}{V_n^2}+1\right)\right.$$

$$\left.-\frac{1}{W_n^4}\left(\frac{8s_n^4}{W_n^4}-\frac{8s_n^2}{W_n^2}+1\right)\right\}$$

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Conflict of Interest

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