The wave function equation Ψ associated with the microparticle – the equation of Schrӧdinger which is governing the entire Universe

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Abstract **:** It would be quite interesting if we could ever answer to physical theories elaborate by Bohr, Einstein, Schrӧdinger, Heisenberg that had not only common but also contradictory points which revolutionized physics. The fact that one at the same time, or otherwise, an object in one place may influence the action/behavior of another one located at a fairly large distance, without any interaction between these objects, concluding that there are such influences, which Einstein called "scary actions" provoking E. Schrӧdinger to become the father of Quantum Mechanics and to start a new scary age through the wave function equation associated with the micro particle/les or with the Schrӧdinger's eat, which was closed in a box contaminated with radioactive material, that can be alive or dead, at the same time. That's what mode professor Stephan Hawking think that it would be better to shut it. Nowadays these at range theories are current too and cause debates in the quantum description of particle state but also of the interaction between them.

Keywords: scary actions, the wave function Ψ, Schrӧdinger's eat, atomics models, quantum mechanics, the physical bases of the laser, the equation of Schrӧdinger.

1 Introduction

The Danish physicist Niels Bohr elaborated The Hydrogen atom theory, in 1913 According to it, we have to do with the simplest atomic model, the model of hydrogen atom, formed of a nucleus around which an electron is graviting. The nucleus, in its turn is formed of a proton and a neutron, so that we have $e^- = -1,6 \cdot 10^{-19} C$, and $e^- = -1,6 \cdot 10^{-19} C$, respectively ${}^{1}_{0}n$ - neutron, these being the first elementary particles discovered in physics. Bohr has formulated two postulates:

I) The stationary (bonded) states of the atom $E_1, E_2, ..., E_n$, are states in which it does not absorb or emit energy, having a life time ar infinite, in the absence of perturbations due to other interactions.

II) The atoms emit or absorb (radiation) electromagnetic waves, only when going form state $m \to n$, m to n, resulting in frequency:

$$\vartheta_{mn} = \frac{E_m - E_n}{h} \quad (1)$$

Using $\lambda = \frac{h}{p}$ (2) and the condition that the wave to be stationary, the radius of the orbit a_0, calculated

$$2\pi a_0 = n\lambda$$

[1]: $$2\pi a_0 = n\frac{h}{p} \quad (3)$$

$$a_0 \cdot p = n\frac{h}{2\pi} = n\hbar$$

The equilibrium condition of the orbit imposes: $F_c = F_e$ - (the centrifugal force is equal to the electric force) (4) or

$$\frac{m_0 v^2}{a_0} = \frac{e^2}{4\pi\varepsilon_0 a_0^2}$$

$$(m_0 v)^2 a_0^2 = \frac{m_0 e^2}{4\pi\varepsilon_0}\cdot a_0$$
$$(a_0 p)^2 = \frac{m_0 e^2}{4\pi\varepsilon_0} a_0 \quad (5)$$
$$n^2\hbar^2 = \frac{m_0 e^2}{4\pi\varepsilon_0} a_0$$

$\Rightarrow$ radius of the orbit $a_0 = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_0 e^2} = \frac{\varepsilon_0 n^2 h^2}{\pi m_0 e^2}$ (6)

So the Kinetic momentum $J = (mv)\times a_0$ (7), and the radius of the orbit a_0, $a_0 = \frac{\varepsilon_0 n^2 h^2}{\pi m_0 e^2}$ (8).

For $n=1 \Rightarrow$ the radius of the first orbit Bohr $a_1 = \frac{\varepsilon_0 h^2}{\pi m_0 e^2} \simeq 0,53\cdot 10^{-10} m = 0,53 \mathring{A}$ (9).

The total quantified energy of the electron results from the planetary model of e^- in the bound state.
In the unrelative case

$$E_{tot} = E_{pot} + E_{cin} \simeq -\frac{e^2}{4\pi\varepsilon_0 a_0} + \frac{1}{2} m_0 v^2 \quad (10).$$

The total energy of the system e^- - nucleus, is the sum of potential and kinetic energy:

$$E_{tot} \simeq -\frac{e^2}{4\pi\varepsilon_0 a_0} + \frac{e^2}{8\pi\varepsilon_0 a_0} = -\frac{e^2}{8\pi\varepsilon_0 a_0} \quad (11)$$

E_{tot} can take values in the range $[0,-\infty]$. But using the radius of the orbit Bohr quantifies a_0, $a_0 = \frac{\varepsilon_0 n^2 h^2}{\pi m_0 e^2}$ (6), results the total quantified energy E_n, with the expression:

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 a_0} = -\frac{e^2}{8\pi\varepsilon_0 \cdot \frac{\varepsilon_0 n^2 h^2}{\pi m_0 e^2}} = -\frac{m_0 e^4}{8h^2}\cdot\frac{1}{\varepsilon_0^2 n^2} = -\frac{m_0 e^4}{8h^2\varepsilon_0^2}\cdot\frac{1}{n^2}$$

(12)

$n \in \mathbb{Z}^+ = [1,+\infty)$, $n =$ the main quantum number.

E_n - takes infinite values according to the number of quantified energy levels of e^- bound.

Example: Calculate the speed e^- for Bohr s model on the first Bohr orbit.

$$\frac{1}{2} m_0 v_n^2 = \frac{m_0 e^4}{8h^2}\cdot\frac{1}{\varepsilon_0^2 n^2} \quad (13)$$

$$v_n^2 = \frac{e^4}{4h^2}\cdot\frac{1}{\varepsilon_0^2 n^2} \quad (14)$$

Putting $n = 1 \Rightarrow$

$$v_1^2 = \frac{e^4}{4h^2\varepsilon_0^2} \Rightarrow v_1 = \frac{e^2}{2h\varepsilon_0} \Rightarrow v_1 \simeq 21{,}86\cdot 10^5 m/s$$

(15)

2 Mathematical modeling – Quantum Theory a dream, a shock, a reality?

Albert Einstein has always been attentive to the quantum problem and analyzed it in mind, in thought. It bothered him because if you "notice an atom" in a certain place, the fact that you have seen it, makes him realty be only there but it wasn t there, when you are looking at it, not before [2].

What does Albert Einstein about the Quantum Theory: observing an object located in a certain place makes it only there. Erwin Schrődinger as the founder of mechanical quantum theory discovered the mathematical equation that is governing the entire Universe.

The equations:

$$(16) -\frac{h^2}{2m}\nabla^2\psi(x,t) = [E - V(x)]\psi(x,t)$$

- the stationary equation,

$$(17) -\frac{h^2}{2m}\nabla^2\psi(x,t) + V(x)\psi(x,t) = it\frac{\upsilon}{\upsilon t}\psi(x,t)$$

- the temporal equation,

are the famous equations of Erwin Schrődinger [1].

So Mechanics Quantum introduces a *function of states* called function of wave of the particle which depends on its coordinates on a reference system considered fixed and of time $\psi = \psi(x,y,z,t)$ (18)

For the unidimensional case we have: $\psi = \psi(x,t)$.

Statisticaly, the possibility to find the particle in the interval $[x, x+\Delta x]$, $\Delta P = |\psi(x,t)|^2 \cdot \Delta x$ (19).

The condition of normalization, that is the of probabilities of all possible events is 1.

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1 \quad (20)$$

$$\int_{-\infty}^{\infty} \psi(x,t)\cdot\psi^*(x,t)\,dx = 1 \quad (21)$$ the condition of normalization, where $\psi^*(x,t)$ is wave function conjugated complex.

We define $\rho(x,t) = \left|\psi(x,t)\right|^2 =$ density of probability of localization [3].

Observabile: position *x*, impulse *p(v)*, Kinetic orbital moment *L*, energy *E*. The great professor, Richard Fayman, who started a new age in physics in 1960, was asked by a student: " what is Ψ?, the wave function, and the answer was shown! Firstly close the door.

The probability that the particle Is found in the *x* – point of the space, at the moment t is

$$P(x,t) = \left|\psi(x,t)\right|^2 = \psi(x,t)\cdot\psi^*(x,t) \quad (22)$$

The physical sense of $\psi(r,t)$ is described easier with the help of probability density $\rho(r,t)$ to find the particle at some point *t* in the volume element $dV = dxdydz$, domain described by $(x, x+dx), (y, y+dy), (z, z+dz)$. Probability density is not dependent *t*, so that

$$\rho(\vec{r},t) = \frac{dP}{dV}\left|\psi(r,t)\right|^2 = \left|\psi(\vec{r})\right|^2 = \rho(r) \quad (23)$$

The physical sense $\left|\psi^2(\vec{r},t)\right|^2$ demonstrates us that Quantum Mechanics has a statistical character.

1. Quantum Mechanics indicates precisely the place, in the space where the micro particle is located.
2. Quantum Mechanics shows us the probability to find the micro particle is a certatin region of the space [4].
3. Quantum Mechanics studies the motion of atomic particles.

Quantum Mechanics studies the motion lows of molecules, atoms, ions, nuclei, micro particles e^-, p^+, n_0^1 and the interaction due to their movement [4], [5], [6].

3 Quantum problem: The famous equation of Erwin Schrődinger

The differential equation of the elastic wave/electromagnetic is

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = \frac{1}{v^2}\cdot\frac{\partial^2\psi(x,t)}{\partial t^2} \quad (24)$$

Being Broglie wave associated to the particle:

$$\psi(x,t) = e^{-i\omega t}\psi(x) \quad (25)$$

from (24) and (25) results:

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{\omega^2}{v^2}\cdot\psi(x) = 0$$

$$\lambda = v\cdot T \Rightarrow v = \frac{\lambda\omega}{2\pi}$$

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{\omega^2}{v^2}\cdot\psi(x) = \frac{\partial^2\psi(x)}{\partial x^2} + \frac{\omega^2}{\frac{\lambda^2\omega^2}{4\pi^2}}\psi(x) = \frac{\partial^2\psi(x)}{\partial x^2} + \frac{4\pi}{\lambda^2}$$

$$\lambda = \frac{h}{p} = \frac{h\cdot 2\pi}{2\pi\cdot p} = \frac{2\pi\hbar}{p}$$

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{4\pi^2}{\frac{4\pi^2\hbar^2}{p^2}}\psi(x) = 0$$

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{p^2}{\hbar^2}\psi(x) = 0$$

The low on the conservation of energy:

$$\frac{p^2}{2m} + U(x) = E$$

$$p^2 = 2m\left[E - U(x)\right]$$

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m\left[E - U(x)\right]}{\hbar^2}\psi(x) = 0 \quad (26)$$

The equation of Schrődinger one – dimensional independently determined.

3.1 The equation of Schrődinger of Quantum Mechanics

To describe that on moment *t* the electron e^- is situated in a point described by the coordinates of the tridimensional system (x, y, z) to a reference system considered fixed in the center of nucleus, that attaching to the wave function Ψ which describes the condition of the elementary particle, to the electron through the temporal wave function $\psi(x,y,z,t)$ or a temporal $\psi(x,y,z)$, that independent of t. My the teacher, Adrian Chiriac paned my way to Quantum

[7]. We begin from the differential equation of electromagnetic waves:

$$\frac{\partial^2\psi}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad (27)$$

The length of wave $\lambda = v\cdot T$ associated to the electron $\lambda=\frac{v}{\nu}$, $\omega=2\pi\nu$, pulsation $\Rightarrow$ $\nu=\frac{\omega}{2\pi}$

$$\lambda=\frac{v}{\nu}=\frac{v}{\frac{\omega}{2\pi}}=\frac{2\pi v}{\omega},\quad \lambda=\frac{h}{p}=\frac{h}{mv}$$

The dependence between Cartesion coordinates and the spherical ones of are described by relations

$$\begin{aligned} x&=r\sin\theta\cos\varphi \\ y&=r\sin\theta\sin\varphi \\ z&=r\cos\theta \\ x^2&+y^2+z^2=r^2 \end{aligned} \quad (27)$$

where r – Ray radius, θ - polar angle, φ - azimuthal angle

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=-\frac{4\pi^2}{\lambda^2}\psi \quad (28)$$

Laplacian ψ is:

$$\Delta\psi=\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2} \quad (29)$$

The relation of Broglie for the non-relativistic case of the particle:

$$\lambda=\frac{h}{p}=\frac{h}{mv} \quad (30)$$

The low on energy conservation:

$$E=E_c+E_p \Rightarrow E_c=E-E_p \quad (31)$$

$$\frac{1}{2}mv^2=E-E_p \quad (32)$$

$$\frac{p^2}{2m}=E-E_p$$

$$\lambda^2=\frac{h^2}{p^2}\Rightarrow p^2=\frac{h^2}{\lambda^2}$$

$$\frac{h^2}{2m\lambda^2}=E-E_p$$

$$\lambda^2=\frac{h^2}{2m\left(E-E_p\right)} \quad (33)$$

$$\Delta\psi+\frac{4\pi^2}{\lambda^2}\psi=0 \quad (34)$$

$$\Delta\psi+\frac{4\pi^2}{\frac{h^2}{2m\left(E-E_p\right)}}\psi=0$$

$$\Delta\psi+\frac{8\pi^2m\left(E-E_p\right)}{h^2}\psi=0 \quad (35)$$

$$\Delta\psi+\frac{8\pi^2m}{h^2}\left(-E_p\right)\psi=-\frac{8\pi^2m}{h^2}E\psi$$

$$(-\frac{h^2}{8\pi^2m}\Delta+E_p)\Psi=E\psi$$

$$H\cdot\psi=E\cdot\psi \quad (36)$$

$H=-\frac{h^2}{8\pi^2m}\Delta+E_p$ (37), is the Hamiltonian operator of the system.

The equation $H\cdot\psi=E\cdot\psi$ (36) is considered the fundamental equation of the Quatum Mechanics. The solving of this equation shows us the action of *H* operator on the wave function Ψ which is equal to $E\cdot\psi$. It is considered the example by analyzing the particle *m*, located in a pit of potential that moves in the direction of the Ox axis Fig 1 [7].

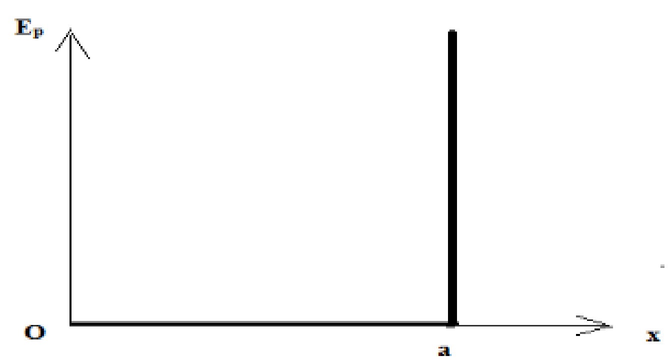

Fig.1 Analyzing of the particle *m*.

$$E_p=\begin{Bmatrix} 0 if\ 0\le x\le a \\ \infty if x\langle 0 and \\ x\rangle a \end{Bmatrix} \quad (37)$$

In the pit of potential the particles has $E_p = 0$ and

$E_c = \frac{1}{2}mv^2 = \frac{1}{2}\frac{p^2}{m} = \frac{p^2}{2m}$ (38)

$\left(-\frac{h^2}{8\pi^2 m}\Delta + E_p\right)\cdot\psi = E\cdot\psi$ (39)

Using $\hbar = \frac{h}{2\pi}$ and $E_p = 0$

$-\frac{\hbar}{2m}\Delta\psi = E\cdot\psi$ (40)

$-\frac{\hbar}{2m}\cdot\frac{\partial^2\psi}{dx^2} = E\cdot\psi$ (41)

$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}E\cdot\psi = 0$ (42)

By convection we note with α the term $\frac{2m}{\hbar^2}E = \alpha^2$ (43)

$\frac{\partial^2\psi}{\partial x^2} + \alpha\psi = 0$ (44), that is differential equation of order II whose solution is $\psi = A\sin(\alpha\cdot x+\varphi)$ (44) of the equation of the harmonic oscillator. For $x = 0 \Rightarrow \psi(0) = 0 \Rightarrow \varphi = 0$,

$x = a \Rightarrow \psi(a) = 0 \Rightarrow \sin\alpha a = 0 \Rightarrow \alpha a = n\pi$,

$\frac{2m}{\hbar^2}E = \alpha^2$,

$\alpha\cdot a = n\cdot\pi$ (45)

$\alpha = \frac{n\pi}{a}$ (46)

$\frac{(2mE)^{1/2}a}{\hbar} = n\cdot\pi$ (47)

$\frac{\sqrt{2mE}}{\hbar} = n\frac{\pi}{a}$ (48)

$E_n = \frac{\hbar^2}{2m}\cdot\frac{\pi^2}{a^2}\cdot n^2 = \frac{h^2}{8ma^2}\cdot n^2 \Rightarrow$ (49) The energy $E_n =$ is quantified, n = main quantum number, $E_n = f(n)!$ (50)

Appling the normative condition: $\int_0^a |\psi|^2 dx = 1$ (51)

$$\int_0^a \psi^2 dx = \int_0^a A^2\sin^2\alpha x dx = A^2\int_0^a \sin^2\alpha x dx = A^2\int_0^a (1-\cos^2\alpha x)d$$

$$= \frac{A^2}{2}\left(a - \frac{1}{2}\alpha\sin\alpha a\right) = \frac{A^2}{2}\cdot a$$

(52)

$$\frac{A^2}{2}a = 1$$

$$A = \sqrt{\frac{2}{a}} \quad (53)$$

$\Rightarrow \psi_n = A\sin\alpha x = \sqrt{\frac{2}{a}}\sin\frac{n\pi}{a}\cdot x$ (54) $\Rightarrow$ results that it is solved Schrődinger equation ψ_n for discrete values quantified E_n, with φ=0, $n \in Z$ [7], .

4 Conclusion

The Theory of Schrődinger has an important role in Quantum Physics, being based on two important approximations.

1.The number of particles remains constant over time - *n*, removing the absorbtion processes or issued during the phenomenon.

2. Particle velocities are small enough, so that the nonrelativistic approximation is valid being described by classical physics as a limit case. It was successfully applied to the study of atoms or molecules of chemical elements.

It was considered the simplest case of moving the free particle, e.g. electron e^- in which it was discovered and solved the equation Schrődinger. The condition of the particle is described by wave function $\psi(x,t)$ that describes the probability in space and time of the elementary particle. The equation of Schrődinger describes a new low of movement of particles. He did not agree with N. Bohr quantum values [2]. Schrődinger liked Einstein Ideas that material objects can be described by equation of the harmonic oscillator and that they have a wave behavior, in in accordance with the mathematical differential calculus. He described successfully the quantum behavior of electrons and atoms in a reasonable way, discovering the new universal equation becomes the base of many phenomena that appeared after its discovery, as the production of X rays, included emissions and laser transitions optical pumping through electrons

collisions the exercise and dextervations of atoms, and also Hamiltonian calculus associated with the system, the array elements on the main diagonal indicating the energy levels of the impurity ion in the case of solid laser etcetera.
Richard j. Davison [8] in his book – The brain and the emotional intelligence indicates Big Five model of personality (opening up to new experiences, conscientiousness, extraversion agreeability, neuroticism) resulted from the six dimensions of the emotional Style EQ (resilience, perspective, social intuition, self – consciousness, sensitivity to context, attention) combined with IQ with are IQ over 130, makes the understanding of N. Bohr model, Quantum Mechanics of Schrӧdinger, and and of its founders, e.g., Heisenberg, Pauli, De Broglie, Fermi, etc, the theory of gravitation of Newton, and the curning of light around a big star yhe cone of light. It remains to appreciate the revolutionary works of Richard Fayman, Stephan Hawking which complete creatively and intelligently the conclusions of modern physics to another ways of describing the material, on the light, solving the space and time problems, and the development of artificial intelligence IA that exceeds the human one over three years and the effects that result from this. The humanity is subject to transformation quickly on information technology, the conquest of the Universe, interpretation and development of new physical and information theories that can makes us communicating with a superior world, an extraterrestrial one. As a sign of gravitude for the equation of Erwin Schrӧdinger to whom we owe and which applies to microparticles/particles it would be better to study if it applies to macroscopic bodies, to find the condition and the behavior of these.

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