Outage Probability Comparison of MRC, EGC and SC Receivers over Short Term Fading Channels

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Abstract: - In this paper, wireless communication systems with maximal ratio combining (MRC) receiver, equal gain combining (EGC) receiver and selection combining (SC) receiver operating over short term fading channels are considered. In analyzed channels, received signal experiences Rayleigh multipath fading. Probability density functions (PDF), cumulative distribution functions (CDF) and level crossing rate (LCR) of output signals are efficiently evaluated and obtained expressions are in closed form. Results for the outage probability are graphically shown to present the influence of diversity techniques on system performance. It is shown that MRC has the best performance and SC technique gives lower performance than MRC and EGC diversity techniques. .

Key-Words: - equal gain combining (EGC), maximal ratio combining (MRC), selection combining (SC), outage probability, short term fading

1 Introduction

Small scale multipath fading degrade outage probability and channel capacity of wireless communication systems. There are more diversity techniques reducing short term fading effects on the outage probability, bit error probability and channel capacity. When Gaussian noise power is equal in each branch of MRC receiver, squared output signal can be calculated as sum of squared signals from its inputs. Signal envelope at the output of the EGC receiver can be evaluated as a sum of signal envelopes at its inputs. The SC receiver selects and outputs branch with the highest signal envelope to provide service to user. Rayleigh distribution can be used to describe small scale signal envelope variation in linear, non line-of-sight multipath fading environments where signal propagates under one cluster [1].

There are many papers in open technical literature considering the outage probability, bit error probability and channel capacity of wireless communication radio systems in the presence of small scale fading. In these papers, diversity techniques are used to mitigate short term fading effects on system performance.

In [2], wireless communication system with SC receiver operating over Weibull multipath fading channel in the presence of cochannel interference subjected to Weibull multipath fading is considered. In this paper, probability density function, cumulative distribution function and moments of output signalare calculated as expressions in closed forms. In [3], wireless communication system with dual SC receiver operating over correlated Nakagami-*m* multipath fading channel is analyzed. Probability density function and cumulative distribution function of SC receiver output signal envelope are calculated. Also, the outage probability and the bit error probability for different modulation formats of proposed wireless radio system are evaluated.

Wireless communication systems with maximal ratio combining diversity receiver in the presence of small scale correlated Nakagami-*m* multipath fading channel is considered in [4]. Outage probability and symbol error probability of observed system are calculated.

The general, closed-form expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the ratio of the products of two independent α-μ variables are presented in [5]. In [6], the authors present novel closed-form expressions for the PDF and CDF of the ratio of random variable and product of two random variables for the cases where random variables are Rayleigh, Weibull, Nakagami-*m* and α-μ distributed.

The distributions of ratios of random variables are of interest in many areas of sciences. In [7], the authors present the joint probability density function and PDF of maximum of ratios *R/r* for the cases where *R* and *r* are Rayleigh, Rician, Nakagami-*m* and Weibull distributed random variables. Random variables are correlated.

The first order performance measures of wireless communication system are the outage probability and average symbol error probability. Outage probability is defined as probability that signal envelope falls bellow predetermined threshold and can be calculated from cumulative probability [1].

The second order performance measures of wireless communication system are level crossing rate and average fade duration [8]. The level crossing rate is defined as the number of crossings on determined level random process in positive or negative direction, and it can be calculated as average value of the first derivative of random process. The average fade duration is defined as average time that signal envelope falls below determined threshold and can be evaluated as the ratio of the outage probability and level crossing rate.

In this paper, wireless communication system working over Rayleigh multipath fading channel will be studied. Probability density function and cumulative distribution function of signal envelope at output of MRC receiver, EGC receiver and SC receiver will be determined. For these cases, derived expressions for the outage probability rapidly converge. The obtained results can be used in performance analysis and designing of wireless communication radio system with receivers mitigating Rayleigh fading effects on system performance.

2 The SC Receiver in Rayleigh Channel

Output signal from SC receiver is equal to the maximum of signals at its inputs:

$$x = \max\left(x_1, x_2\right) \tag{1}$$

where output signal is denoted by x and input signals by x_1 and x_2. Probability density functions of x_1 and x_2 are:

$$p_{x_1}\left(x_1\right) = \frac{2x_1}{\Omega} e^{-\frac{x_1^2}{\Omega}},\ x_1 \ge 0 \tag{2}$$

$$p_{x_2}\left(x_2\right) = \frac{2x_2}{\Omega} e^{-\frac{x_2^2}{\Omega}},\ x_2 \ge 0 \tag{3}$$

Cumulative distribution functions of x_1 and x_2 are:

$$F_{x_1}\left(x_1\right) = 1 - e^{-\frac{x_1^2}{\Omega}},\ x_1 \ge 0 \tag{4}$$

$$F_{x_2}\left(x_2\right) = 1 - e^{-\frac{x_2^2}{\Omega}},\ x_2 \ge 0 \tag{5}$$

Probability density function of output signal x is:

$$\begin{aligned} p_x\left(x\right) &= p_{x_1}\left(x\right) \cdot F_{x_2}\left(x\right) + p_{x_2}\left(x\right) F_{x_1}\left(x\right) = \\ &= 2 p_{x_1}\left(x\right) \cdot F_{x_2}\left(x\right) = \\ &= \frac{4x}{\Omega} e^{-\frac{x^2}{\Omega}} \left(1 - e^{-\frac{x^2}{\Omega}}\right). \end{aligned} \tag{6}$$

Cumulative distribution function of x is:

$$F_x\left(x\right) = F_{x_1}\left(x\right) F_{x_2}\left(x\right) = \left(1 - e^{-\frac{x^2}{\Omega}}\right)^2 \tag{7}$$

The joint probability density function of SC receiver output signal and it's the first derivative is:

$$p_{x\dot{x}}\left(x\dot{x}\right) = 2 p_{x_1\dot{x}_1}\left(x\dot{x}\right) \cdot F_{x_2}\left(x\right) \tag{8}$$

The level crossing rate of random process x is:

$$\begin{aligned} N_x &= \int_0^\infty d\dot{x}\ \dot{x}\, p_{x\dot{x}}\left(x\dot{x}\right) = \\ &= 2F_{x_2}\left(x\right) \int_0^\infty d\dot{x}\ \dot{x}\, p_{x_1\dot{x}_1}\left(x\dot{x}\right) = 2N_{x_1} F_{x_2}\left(x\right) \end{aligned} \tag{9}$$

where N_{x_1} is level crossing rate of Rayleigh random process:

$$\begin{aligned} N_{x_1} &= \int_0^\infty d\dot{x}_1\ \dot{x}_1\, p_{x_1\dot{x}_1}\left(x\dot{x}\right) = \\ &= \int_0^\infty d\dot{x}_1 \frac{2x_1}{\Omega} e^{-\frac{x_1^2}{\Omega}} \frac{1}{\sqrt{2\pi}\beta} \dot{x}_1 e^{-\frac{\dot{x}_1^2}{2\beta^2}} \end{aligned} \tag{10}$$

where

$$\beta^2 = \pi^2 f_m^2 \Omega, \tag{11}$$

and f_m is maximal Dopler frequency.

The level crossing rate of Rayleigh random process is:

$$N_{x_1} = \frac{2x_1}{\Omega} e^{-\frac{x_1^2}{\Omega}} \cdot \pi f_m \Omega^{1/2} = \frac{2x_1}{\Omega^{1/2}} e^{-\frac{x_1^2}{\Omega}} \cdot \pi f_m \,, \quad (12)$$

and level crossing rate of SC receiver output signal is:

$$N_x = 2N_{x_1} F_{x_2}(x) =$$

$$= \frac{2x\pi f_m}{\Omega^{1/2}} e^{-\frac{x^2}{\Omega}} \cdot \left(1 - e^{-\frac{x^2}{\Omega}}\right) \quad (13)$$

3 The EGC Receiver in Rayleigh Channel

Signal envelopes at inputs of EGC receiver are denoted with x_1 and x_2, and output signal from EGC receiver is denoted with x. Output signal envelope can be calculated as sum of signal envelopes from its inputs:

$$x = x_1 + x_2 \,,\; x_1 = x - x_2 \,. \quad (14)$$

Probability density function of x is:

$$p_x(x/x_2) = \left|\frac{dx_2}{dx}\right| p_{x_1}(x - x_2) \quad (15)$$

$$\frac{dx_2}{dx} = 1 .$$

After integration of conditional probability density function, the expression (15) becomes:

$$p_x(x) = \int_0^\infty dx_2 \, p_{x_1}(x - x_2)\, p_{x_2}(x_2) \quad (16)$$

Random variables x_1 and x_2 follow Rayleigh distribution (2), (3):

$$p_{x_i}(x_i) = \frac{2x_i}{\Omega} e^{-\frac{x_i^2}{\Omega}} \,,\; x_i \ge 0 \,,\; i = 1,2 \,. \quad (17)$$

Probability density function of x is now:

$$p_x(x) = \frac{x}{2\sigma^2} e^{-\frac{x^2}{2\sigma^2}} + \frac{\sqrt{\pi}}{4}\frac{x^2}{\sigma^3} erf\left(\frac{x}{2\sigma^2}\right) e^{-\frac{x^2}{4\sigma^2}} -$$

$$-\frac{\sqrt{\pi}}{2\sigma} erf\left(\frac{x}{\sigma}\right) e^{-\frac{x^2}{4\sigma^2}} \quad (18)$$

where $2\sigma^2 = \Omega$.

Cumulative distribution function of x is:

$$F_x(x) = \int_0^x dt \, p_x(t) =$$

$$= \int_0^x dt \left(\frac{t}{2\sigma^2} e^{-\frac{t^2}{2\sigma^2}} + \frac{\sqrt{\pi}}{4}\frac{t^2}{\sigma^3} erf\left(\frac{t}{2\sigma^2}\right) e^{-\frac{t^2}{4\sigma^2}} - \right.$$

$$\left. -\frac{\sqrt{\pi}}{2\sigma} erf\left(\frac{t}{\sigma}\right) e^{-\frac{t^2}{4\sigma^2}} \right) \quad (19)$$

4 The MRC Receiver in Rayleigh Channel

Signal envelopes at the inputs of MRC receiver are x_1 and x_2, and at output of MRC receiver is x. When Gaussian noise at branches of MRC receiver has the same power, squared output signal is equal to the sum of squared signal envelopes at its inputs. Square of x is:

$$x^2 = x_1^2 + x_2^2 \quad (20)$$

Random variables x_1 and x_2 follow Rayleigh distribution (17):

$$p_{x_i}(x_i) = \frac{2x_i}{\Omega} e^{-\frac{x_i^2}{\Omega}} \,,\; x_i \ge 0 \,,\; i = 1,2$$

Random variable x^2 can be written as a sum of squared Gaussian random variables. Therefore, x^2 has χ^2 distribution:

$$p_x(x) = \frac{2}{\Gamma(2)}\left(\frac{2}{2\Omega}\right)^2 x^3 e^{-\frac{2}{2\Omega}x^2} = \frac{2x^3}{\Omega^2} e^{-\frac{1}{\Omega}x^2} \quad (21)$$

Cumulative distribution function of output signal envelope is:

$$F_x(x) = \int_0^x dt \, p_x(t) =$$

$$= \frac{2}{\Omega^2}\int_0^x dt \, t^3 e^{-\frac{1}{\Omega}t^2} = \frac{2}{\Omega^2}\cdot\frac{1}{2}\Omega^2 \,\gamma\left(2, x^2\right) \quad (22)$$

The joint probability density function of x and $\dot{x}$ is:

$$p_{x\dot{x}}(x\dot{x}) = p_x(x) \cdot p_{\dot{x}}(\dot{x}) =$$

$$= \frac{2x^3}{\Omega^2} e^{-\frac{1}{\Omega}x^2} \cdot \frac{1}{\sqrt{2\pi}\beta} e^{-\frac{1}{2\beta^2}\dot{x}^2} \quad (23)$$

where β^2 is given by (11):

$$\beta^2 = \pi^2 f_m^2 \Omega \,.$$

The level crossing rate of MRC receiver output signal is:

$$N_x = \int_0^\infty d\dot{x}\, \dot{x}\, p_{x\dot{x}}\left(x\dot{x}\right) =$$

$$= \frac{2x^3}{\Omega^2} e^{-\frac{1}{\Omega}x^2} \int_0^\infty d\dot{x}\, \dot{x} \frac{1}{\sqrt{2\pi}\beta} e^{-\frac{1}{2\beta^2}\dot{x}^2} =$$

$$= \frac{2x^3}{\Omega^2} e^{-\frac{1}{\Omega}x^2} \frac{\pi f_m}{\sqrt{2\pi}} \cdot \Omega^{1/2} = \frac{2x^3}{\Omega^{3/2}} e^{-\frac{1}{\Omega}x^2} \frac{\pi f_m}{\sqrt{2\pi}} \qquad (24)$$

5 Numerical Results

In Fig. 1, the histogram for dual SC combiner with balanced inputs is shown. The abscissa of the histogram presents the amplitude value of the random process; the ordinate is the number of samples in the interval of abscissa. The outage probability for SC combiner with two inputs is given in Fig. 2.

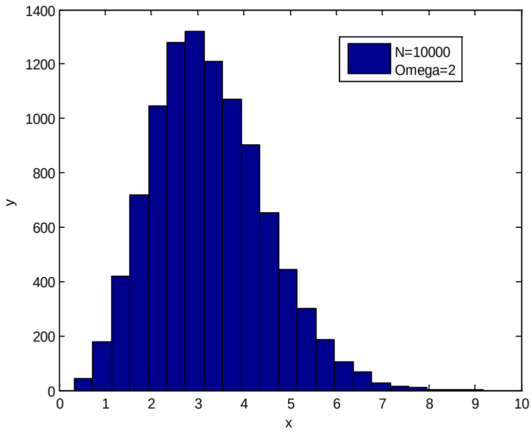

Fig.1. Histogram for dual SC combiner with balanced inputs and N=10000, Ω=2

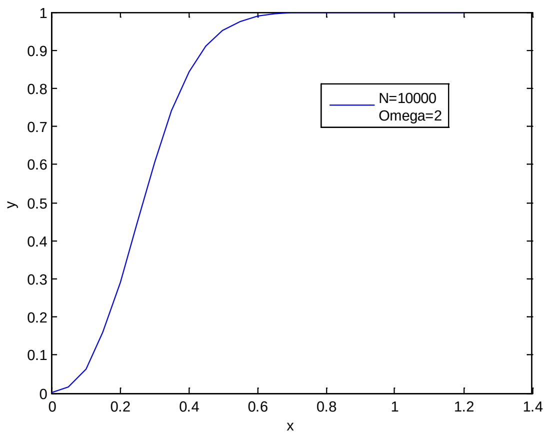

Fig.2. Outage probability for SC combiner and N=10000, Ω=2

The histogram for dual EGC combiner is presented in Fig. 3. The outage probability for this EGC combiner is given in Fig. 4.

The histogram for dual MRC combiner is shown in Fig. 5.

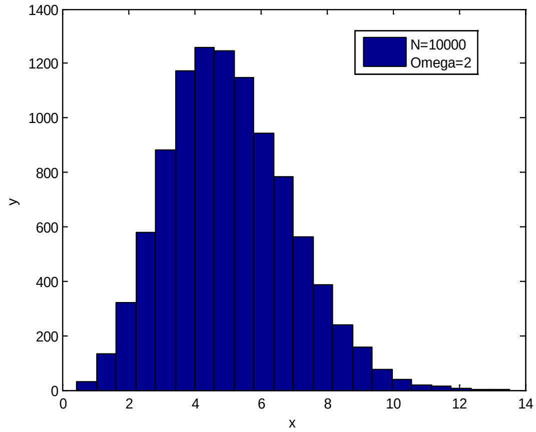

Fig. 3. Histogram for dual EGC combiner and N=10000, Ω=2

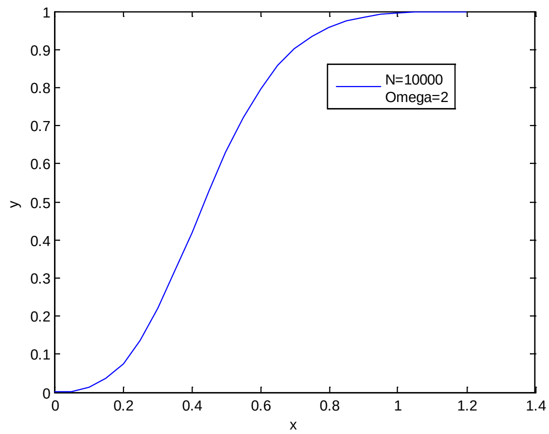

Fig.4. Outage probability for dual EGC combiner and N=10000, Ω=2

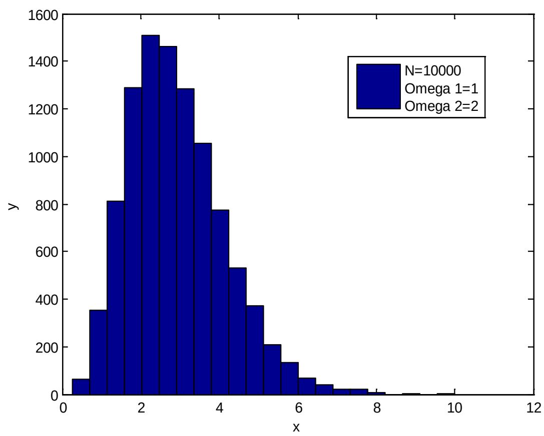

Fig.5.Histogram for dual MRC combiner and N=10000, Ω_1=1, Ω_2=2

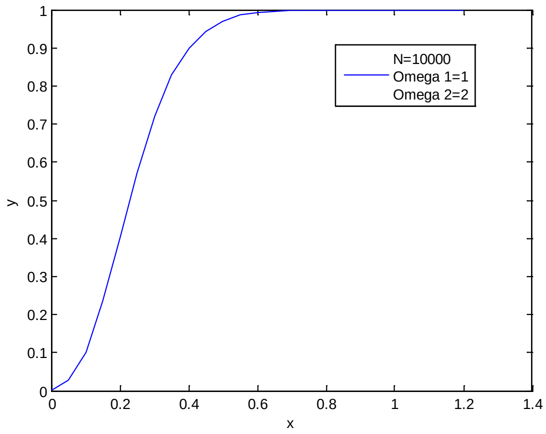

Fig. 6. Outage probability for dual MRC combiner and N=10000, Ω_1=1, Ω_2=2

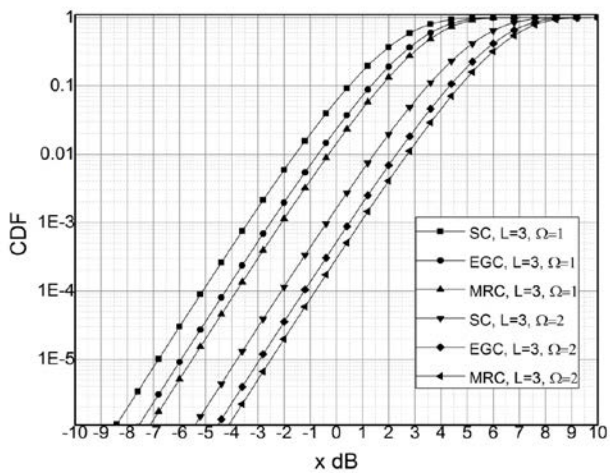

Fig. 7. Comparative analysis of outage probabilities for SC, EGC and MRC combiners

The outage probability for dual MRC combiner with two inputs is ploted in Fig. 6. The comparative analysis of the outage probabilities for MRC, EGC and SC receivers for SC, EGC and MRC combiners with three branches is given in Fig. 7. It is possible to see from this figure that theoretical knowledge and findings are validated.

Namely, the MRC diversity receiver has the best performance. The SC diversity receiver gives the highest values of the outage probability, i.e. has the worst performance. MRC receiver has the highest implementation complexity and SC reception has the lowest implementation complexity with the lowest cost of making. This investigation gives opportunity to designers of wireless communication systems to choose the best solution for fading conditions in appropriate channel.

7 Conclusion

In this paper, wireless communication system operating over Rayleigh multipath fading channel is considered. Themaximal ratio combining diversity receiver, equal gain combining diversity receiver and selection combining diversity receiver are used to reduce Rayleigh short term fading effects on the outage probability and bit error probability.Probability density function, cumulative distribution function and average level crossing rate of receiver output signal are calculated. Cumulative distribution function can be used for evaluation of the outage probability and level crossing rate can be used for calculation of the average fade duration of proposed wireless communication system. The MRC diversity techniques give the best results. SC receiver gives higher values of the outage probability. MRC reception has the highest implementation complexity and SC reception has the lowest implementation complexity with the lowest cost of production.

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