

Modified 2D Model for Prediction of Forest Fire Spread

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Abstract: - A modified two-dimensional two-phase mathematical model of forest fires spread is considered. The model is derived by averaging the three-dimensional flow equations over the height of the forest fuel material layer, it includes the $(k-\varepsilon)$ turbulence model with additional turbulence production and dissipation terms in the forest layer and the Eddy Break-up Model for the combustion rate in the gas phase. The model elaborated can serve to carry out numerical simulation of the forest fire front propagation for the case of non-homogeneous forest fuel materials distribution in the area. This model can be used for computation of the fire propagation in real time.

Key-Words: - Forest fire, two-dimension mathematical model, numerical simulation

1 Introduction

Mathematical modeling of forest fires behavior is one of the most complex areas of modeling due to the diversity and complexity of the physical processes occurring in the area of the fire and in the atmosphere above the fire, the effect of weather conditions, the possibility of fire spreading over a large area for a long time and other factors.

In the past 20 years, studies in this area were carried out particularly intense, and it was created many different forest fire behavior models. Conventionally, these models can be divided into two large groups [1]. The first group of models – empirical and semi-empirical models (Simple Fire Behavior Models), the second group – physically based models (Complex Fire Behavior Models), built on the conservation laws of continuum mechanics.

Empirical models are relatively simple, they do not take into account many physical processes occurring in the fire area. Physically based models are more complicated, they can take into account various physical phenomena occurring in the burned area [1-8]. The recently created three-dimensional codes [5,6] are computationally expensive and using them is not yet possible to perform simulation of

real fires spread over a large area and for a long time. However, by using three-dimensional codes the complex physical effects which take place in the burned area and in the atmosphere above the fire can be investigated. Thus, the formation of coherent structures in the atmosphere above the fire was obtained by numerical simulation [5]. Previously created two-dimensional (x,z) -models are used to for computation of forest fire spreading in a specified direction, and the models with cylindrical coordinates (r,z) are used to for computation of this effect equally in all horizontal directions. For the latter models it is impossible to set a uniform distribution of the forest fuels, and thus it is impossible to simulate of real fires spreading using them.

A two-dimensional two-phase physically based model of forest fires propagation was developed by the authors earlier [7,8]. The model is based on the averaging of three-dimensional equations over the height of the forest fuel (FF) layer. This model does not computationally expensive as compared with the three-dimensional ones, and allows to carrying out numerical simulation of forest fires propagation in real time using updated parallel supercomputers.

This model based on the fundamental physical

laws of mass, momentum and energy conservation and takes into account all physical phenomena in the burned area that are important for fire propagation. However, a simplified description of turbulent mixing in the forest layer is used in the model. Also, the heat release rate which governs the fire propagation speed, is described by kinetics formulas for chemical reactions involving numerous uncertain parameters. In the current paper, a two-dimensional model is presented that is based on the ($k-\varepsilon$) turbulence model, modified by including additional turbulence production and dissipation terms in the forest layer. The combustion rate in the gas phase is described by the Eddy Break-up Model which assumes that the combustion rate is limited by turbulent mixing rather than by reaction kinetics.

2 Mathematical Model

2.1 Medium Composition

The proposed two-phase model treats a forest as a two-phase medium consisting of air and gaseous products of pyrolysis and combustion (the air-gas or gas phase) and FF, solid products of pyrolysis and combustion of FF (solid phase). The gas phase is a medium composed by six components: combustible gas CO (mass concentration C_1), oxidizer O_2 (C_2), carbon dioxide CO_2 (C_3), water vapor H_2O (C_4), nitrogen N_2 (C_5), and dispersed soot (C_s). Our assumption is that the disperse soot particles move together with the gas phase and when the soot particles are burned, the heat exchange process is so rapid that we can characterize it only by the temperature of the gas phase. The solid phase is also a multicomponent medium composed by the FFM (volume fraction ϕ_1) and products of FFM pyrolysis: breeze coke (ϕ_2) and ashes (ϕ_3). Breeze coke and disperse soot consist of virtually 100 percent of carbon and are burned up completely.

2.2 Two-dimensional System of Equations

Consider the forest fire behavior in the single-FF-layer and suppose that all physical and chemical parameters in this layer are approximately equal to some average values for this layer. The equation set of the two-phase, two-dimensional, single-layer model obtained by integrating the original three-dimensional equations over the FF layer height has the form:

gas phase:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = Q - J_\rho, \quad (1)$$

$$\frac{\partial \rho \mathbf{V}}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) + \nabla (\phi p) = \nabla \cdot (\mu_{eff} \nabla \mathbf{V}) + \mathbf{F} + \mathbf{F}_w - \mathbf{J}_v, \quad (2)$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho \mathbf{V} E + \phi p \mathbf{V}) = \nabla \cdot (k_{eff} \nabla T) + \alpha (T_1 - T) + \sigma (\alpha_1 T_1^4 - \alpha T^4) + f_\Gamma + f_{\Gamma R} + q_1 W + q_s W_s - J, \quad (3)$$

$$\frac{\partial \rho C_i}{\partial t} + \nabla \cdot (\rho \mathbf{V} C_i) = \nabla \cdot (\rho D_{eff} \nabla C_i) + Q_i + W_i + W_i^s - J_{C_i}, \quad i = \overline{1, 5}, \quad (4)$$

$$\frac{\partial \rho C_s}{\partial t} + \nabla \cdot (\rho \mathbf{V} C_s) = Q_s - W_s - J_s, \quad (5)$$

$$\frac{\partial \rho k}{\partial t} + \nabla \cdot (\rho \mathbf{V} k) = \nabla \cdot \left(\frac{\mu_{eff}}{\sigma_k} \nabla k \right) + G + P_k - \rho \varepsilon - L_k, \quad (6)$$

$$\frac{\partial \rho \varepsilon}{\partial t} + \nabla \cdot (\rho \mathbf{V} \varepsilon) = \nabla \cdot \left(\frac{\mu_{eff}}{\sigma_\varepsilon} \nabla \varepsilon \right) + c_1 G \frac{\varepsilon}{k} + P_\varepsilon - c_2 \frac{\rho \varepsilon^2}{k} - L_\varepsilon, \quad (7)$$

$$p = \rho^0 R T \sum_{i=1}^5 \frac{C_i}{M_i}; \quad (8)$$

solid phase:

$$\rho_j^0 \frac{\partial \phi_j}{\partial t} = R_{\phi_j}, \quad j = \overline{1, 3}; \quad (9)$$

$$\frac{\partial}{\partial t} \sum_{j=1}^3 \rho_j^0 \phi_j c_{pj} T_1 = -\alpha (T_1 - T) - \sigma (\alpha_1 T_1^4 - \alpha T^4) + Q_{T_1}; \quad (10)$$

normalization and balance relations:

$$\phi + \sum_{j=1}^3 \phi_j = 1, \quad \sum_{i=1}^5 C_i + C_s = 1, \quad (11)$$

$$\sum_{i=1}^5 Q_i + Q_s = Q, \quad \sum_{j=1}^3 R_{\phi_j} = -Q.$$

The system (1)-(11) is considered in Cartesian coordinates in a rectangular area $\Omega = \{0 \leq x \leq l_1, 0 \leq y \leq l_2\}$ on a horizontal plane XY , ρ is the partial density of the gas phase, p is the total pressure of the multiphase medium, $\mathbf{V} = (u, v)$ is velocity of the gas phase, T is the gas phase temperature, E is the total energy of the gas phase $E = 0.5(u^2 + v^2) + c_v T$, c_v is the gas specific heat capacity at constant volume, ϕ is the volume

fraction of the gas phase, $\rho^0 = \rho/\phi$ is the real density of the gas phase, R is the universal gas constant, M_i are the molecular masses of the gas phase components, ϕp is the diagonal component of the stress tensor proportional to the volume fraction of the gas phase; $\mathbf{F} = -\rho c_d s \mathbf{V}|\mathbf{V}|$ is the volume force related to the exchange of momentum between the phases (the force of friction between the phases), c_d is an empirical drag coefficient for forest vegetation, s is the FF specific surface area; $\mathbf{F}_w = -\rho \xi (\mathbf{V} - \mathbf{V}_w) |\mathbf{V} - \mathbf{V}_w|$ is the wind influence on the top border of the FF layer, \mathbf{V}_w is the wind velocity; Q is the mass source describing the gas inflow due to the processes taking place in the solid phase; $J_\rho, \mathbf{J}_V, J_E, \mathbf{J}_C, J_s$ are the mass, momentum and energy fluxes at the upper and lower boundaries of the FF layer [7,8]; $\alpha(T_1 - T)$ is the interphase heat energy exchange; $\sigma(\varepsilon_1 T_1^4 - \varepsilon T^4)$ is the interphase radiant energy exchange; $\rho_j^0, j = \overline{1,3}$ are the real densities of solid phase components $\rho_j^0 = \text{const}$, $c_{pj}, j = \overline{1,3}$ are the solid phase components' heat capacities, $R_{\phi_j}, j = \overline{1,3}$ is the rate of the volume fraction ϕ_j variation as a result of chemical reactions, T_1 is the solid phase temperature, Q_{T_1} is the heat generation in the solid phase, W is the chemical reaction rate per gas fuel mass, W_s is the soot burning rate, $W_i, W_i^s, i = \overline{1,5}$ are the production or consumption rates of gas phase components in the respective reactions. As is shown by numerical experiments carried out earlier with the two-dimensional model [7,8], the terms describing the change of the momentum and energy caused by varying the phase volume fractions are small and can be excluded.

2.3 Turbulence Closure

The system of equations (1)-(5) is closed by the modified $k-\varepsilon$ turbulence model (6), (7) which contains in the right-hand sides of the equations for the turbulent kinetic energy k and its dissipation rate ε , in addition to the usual production term G , extra terms P_k, P_ε and L_k, L_ε which describe production and dissipation of turbulence due to flow interaction with trees [9,10].

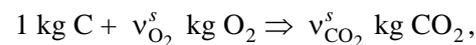
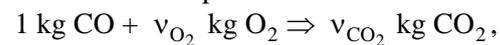
The effective viscosity μ_{eff} is the sum of the laminar μ_l and turbulent μ_t components,

$\mu_{eff} = \mu_l + \mu_t$, while the turbulent viscosity is obtained as $\mu_t = c_\mu \rho k^2 / \varepsilon$.

The total heat conductivity coefficient takes into account the heat and the turbulent heat conductivity as well as heat transfer by radiative conduction, $k_{eff} = k_l + k_t + k_r$, where $k_l = c_p \mu_l / \text{Pr}$ is the heat conductivity coefficient; $k_t = c_p \mu_t / \text{Pr}_t$ is the turbulent heat conductivity coefficient, $\text{Pr} = \text{Pr}_t = 0.7$ are the laminar and turbulent Prandtl numbers; $k_r = 16\sigma l_s T^3 / 3$ is the radiative heat conductivity, l_s is the radiation mean path, σ is the Stefan-Boltzmann constant. The effective diffusion coefficient is defined as $D_{eff} = \mu_l / \rho S c + \mu_t / \rho S c_t$, where $S c = S c_t = 0.7$ are the laminar and turbulent Schmidt numbers.

2.4 Chemical Processes in a Gas and Solid Phases

The gas phase combustion model includes two single-stage irreversible global combustion reactions for CO and dispersed soot



where $\nu_i, i = \overline{1,5}$ are the mass stoichiometric coefficients for the gas phase components in the gas phase reaction, ν_i^s are the respective values for the soot combustion. The turbulent combustion rate is described by the Eddy Breakup Model [11], according to which the combustion rate at high temperatures is determined solely by the turbulent mixing, rather than by the reaction kinetics. The mass burning rate of CO and soot are (for clarity, chemical formulas are used as subscripts):

$$W = \rho A \frac{\varepsilon}{k} \min \left(C_{\text{CO}}, \frac{C_{\text{O}_2}}{\nu_{\text{O}_2}}, \frac{B C_{\text{CO}_2}}{\nu_{\text{CO}_2}} \right),$$

$$W_s = \rho A \frac{\varepsilon}{k} \min \left(C_s, \frac{C_{\text{O}_2}}{\nu_{\text{O}_2}^s}, \frac{C_s \nu_{\text{O}_2}^s}{C_s \nu_{\text{O}_2}^s + C_{\text{CO}} \nu_{\text{O}_2}} \right),$$

where $A = 4$, $B = 0.5$. Heat release rate in the gas phase due to the CO and dispersed soot combustion is $q_1 W + q_s W_s$, where q_1, q_s are the CO and dispersed soot combustion heats respectively. The gas component production and consumption rates in Eq. (4) are expressed via the reaction rates W, W_s and the mass stoichiometric coefficients are presented as $W_i = \pm \nu_i W$, $W_i^s = \pm \nu_i^s W_s$, where the

“plus” sign is taken for reaction products, the “minus” sign for CO and oxidizer ($v_{CO}=1$ by definition of W). Similar approach was used in the earlier works [12-14].

The amount of the gases Q_i and the dispersed soot Q_s produced due to the pyrolysis process, and the rate of variation of the volume fractions of the phases R_{ϕ_j} depend on the speed of chemical reactions of the FFM pyrolysis, the heat generation in combustion of the breeze coke Q_{T_1} depends on the speed of reaction the carbon combustion, they are described by the Arrhenius law [7,8].

3 Results of the Numerical Simulation

An efficient algorithm of splitting by physical processes was used for the numerical solution of the asserted problem. The discretization in time $t_{n+1}=t_n+\Delta t$ is considered. At the time step $[t_n, t_{n+1}]$ the system (1)-(11) splits to the subsystems and each of the subsystems describes some physical process: the gas phase transfer, the turbulent viscosity, the heat conductivity, the diffusion of gases, the interphase friction, variation of the mass and energy due to chemical reactions, the generation and dissipation of the turbulent energy, the effect of the wind. On each stage the subsystems are solved using a finite-difference method on a rectangular mesh.

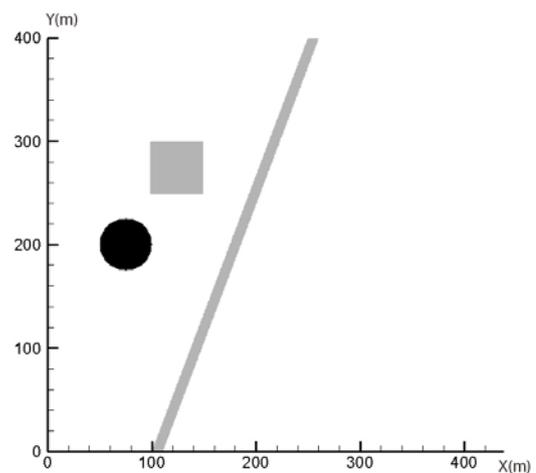
Numerical simulation of the process of fire spread is carried out on the mesh with 400×400 points.

Validation of the model was conducted. Comparison of the rate of spreading of the fire front to the data [6] gave the discrepancy in the results of calculations made no more than 15%.

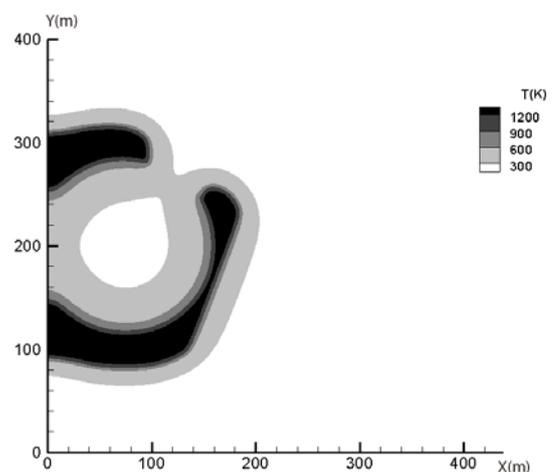
Numerical simulation of forest fire front propagation for the case of non-homogeneous FF distribution in the area was carried out.

Figure 1 shows the results of a sample numerical simulation of a forest fire spreading in a one-layer of the FF in the absence of a wind and with the assumption that the FF are distributed non-homogeneously. Figure 1a shows a schematic sketch of a rectangular glade in the forest and a road which have no vegetation, the source of the fire initially has the shape of a circle. Figure 1b shows the FFM burning out and the formation of the fire temperature front. The temperature behind the back edge of the fire front gradually decreases. The fire front propagates around the glades, but fails to cross the 10-meter-wide road (Fig. 1c).

a) $t = 0$ s



b) $t = 200$ s



c) $t = 400$ s

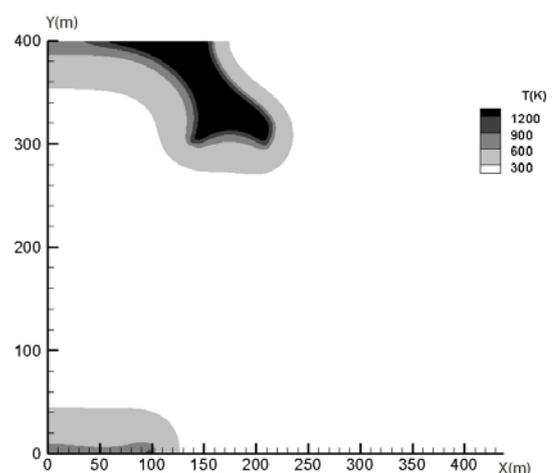


Fig. 1 The temperature front of the fire propagates around the glades, but fails to cross the 10-meter-wide road.

t = 120 s

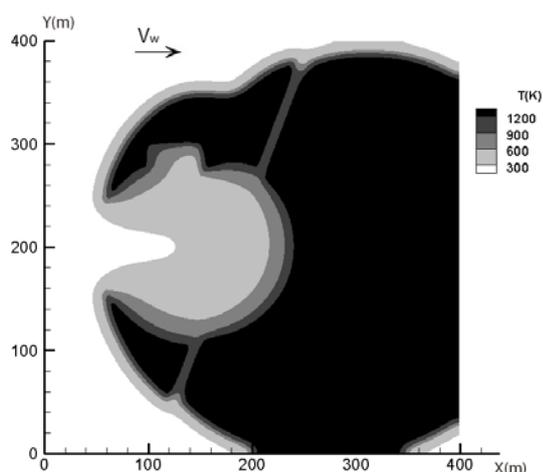


Fig. 2 The temperature front of the fire propagates around the glades and across the 10-meter-wide road.

Figure 2 shows the results of numerical simulation of the process with the same initial situation, but with the wind blowing at a speed $\mathbf{V}_w = (2, 0)$ m/s. The fire front propagates around the glades and spreads across the 10-meter-wide road.

The results of these computations show that the created model correctly describes the forest fire front propagation for the case of non-homogeneous FF distribution in the area, with such obstacles as roads, rides, rivers, glades, water bodies etc., as well as in the presence of wind, i.e., for all typical conditions in which real forest fires occur.

4 Conclusion

The model elaborated enables one to carry out numerical simulation of the fire front propagation for real forest fires.

This model can be used for the computation of the fire propagation in real time, as well as to obtain expert estimates on the progress of the emergency situation, related to the forest fires. The model can thus contribute to the decision making process concerning the fire attack and the damage assessment caused by the fire.

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