Interaction Optimization in Multibody Dynamic System

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Abstract: - The paper describes the optimization facility of interaction within the "bridge-track-car" system that concerns high-speed railway traffic on the bridges zones. The singularity of the approach to be discussed lies in the attempt of integration of the system of elements that work simultaneously and together. The model that takes into account vertical oscillation of the car body, bogies, wheels, rails, and superstructure of the bridge. Several criteria allow the estimation of various parameters of dynamic interaction and reach the optimal dynamic parameters dealing with wheel-rail contact and derailment, comfort of passengers and ballast wearing. Therefore, we can obtain the system with predetermined dynamic behavior to decrease or to increase the interaction forces in defined places. It allows decreasing stress of ballast or increasing wheel-rail contact force to prevent derailment.

Key-Words: High-speed railway, bridge dynamics, safety of motion, track, ballast wearing

1 Introduction

The research of the dynamic processes in the complicated "Bridge-track-car" system (BTCS) - consists of mechanisms and structures. It requires a big amount of information analysis dealing with behavior and trends of the BTCS changes. The trend analysis is necessary to determine the direction and resource of optimizing the system. That is why scientific methods of decision-making are needed.

System analysis could be very useful in this case. One of the tools of the system analysis is the mathematical theory of optimal control that concerned optimal control of a spaceship. This theory assumes minimizing a criterion simultaneously with the execution of constraints in the form of equations and inequations. Minimizing the criteria and performing of constraints are achieved by means of controlling functions that depend on time in the mathematical theory. Though in construction engineering the characteristics of a structure must not (and cannot) be dependent on a short period of time. It is necessary to vary the characteristics on space coordinate. Thus, it is necessary to imply stationary controlling functions.

2 Applied Theory of Optimal Control

There is a carrying system in methasystem "railroad". The aim of the carrying system is taking any load during its functioning and railing, security of the strength, stiffness, etc. of the structures. In our theory, the BTCS is one of the units of the carrying system. This unit has input and output impacts. Input impact is the boundary and starting conditions. Output impact is a state of the BTCS at the moment when the train has left the BTCS completely. In accordance with the Eigen frequency, the BTCS could be decomposed into elements: sprung mass (car body and bogies), unsprung mass (wheelsets) and track – intersystem "wheel-rail", and bridge superstructure.

It is very important to point out, that decomposition in our theory does not mean dividing the system into isolated elements as it usually happens, but dividing the system into subsystems, that continuously interact between each other while oscillations run on.

Thus, the BTCS consists of:

- discrete-continuous structure of the BTCS, containing continuous elements – track and bridge superstructure; the BTCS includes the bridge and transition zones, and discrete subsystem includes cars;

- mathematical model *Z* of the BTCS behavior (including start and boundary conditions *z* and parameter *t*);
- description of the stationary characteristics *O(x)* of the BTCS;
- constraints *F* on BTCS behavior parameters *W, W = Z(x, t, z, O)*;
- stationary controlling functions *u = u(x)* within multitude *O*, that influence the achievement of the aim of the BTCS;
- functional *D* describes the quality of the BTCS;
- quality parameters *d* of the BTSC, that have to be optimized, *d=D(u,W).*

The BTCS could be defined as “purposeful system”. The target of the BTCS is the train reaching the bound of the BTCS, while all the constraints (including limits of comfort, safety, loads, etc.) are satisfied when the train is inside the BTCS. For an optimal system, there is an additional requirement of minimizing some parameters. In this case, optimization is performed under absolute satisfaction of all the constraints.

The target of the optimal system can be achieved by means of controlling functions that determine behavior of the system because the starting and boundary conditions effect is minimized by the choice of the model parameters. The above-mentioned formulation of the problem allows using mathematical theory of optimal controlling. The difference of our applied theory from the classical one is application of stationary controlling functions, i.e. the functions that do not depend on time. The second difference is minimization of starting and boundary conditions effect while the behavior of the BTCS is determined only by the interaction of its subsystems.

The controlling functions are rigidity $E_b J_o \alpha^4(x_k)$ and mass $\alpha^2(x_k)\rho F_0$ of the bridge superstructures, rail bed stiffness and mass of rail and sleepers. The track parameters can be changed by means of implementation of different pads stiffness $\gamma(x)$ and sleeper’s spacing $\delta(x)$. All mentioned functions depend on space, but not on time. Thus, u(x): $\{\alpha(x), \gamma(x), \delta(x)\}$. For the computer program these functions are just many dimensional vectors.

The vector criterion *D* has been worked out for estimating the quality of the BTCS. D_1 is the bridge superstructure (beam) criterion. D_1 will be discussed in another paper. The D_2 criterion consists of three criteria that allow minimizing rail bed loading *(d_{21})*, energy dissipation in ballast *(d_{22})*, and volatility of rail bed loading *(d_{23})* about predetermined meaning *Q*.

$$D_2 = min \left| \begin{array}{c} \int_L \int_T \left(\gamma(x)\delta(x)U'\left(y_\text{p} - y_b\right)\right)^2 dxdt \\ \int_L \int_T \left(\text{c}_\text{p}\left(\frac{\partial y_\text{p}}{\partial t} - \frac{\partial y_b}{\partial t}\right)\right)^2 dxdt \\ \int_L \int_T \left(\gamma(x)\delta(x)U'\left(y_\text{p} - y_b\right) - Q\right)^2 dxdt \end{array} \right| \begin{array}{c} (d_{21}) \\ \vdots \\ (d_{22}) \\ \vdots \\ (d_{23}) \end{array}$$

where $U(x,t) = \gamma(x)\, \delta(x) U'(y_r - y_b)$ is rail bed loading, c_p – dissipation coefficient, L – the length of the BTCS, T – the time interval while the train is inside the BTCS, y – deflection of the rail or the beam.

The D_2 criterion includes physically different parameters and therefore application of a goal function that summarizes these parameters is wrong. Moreover, some criteria may conflict with each other, i.e. they demand opposite steps of optimization. Thus, it is necessary to find the multitude of compromise decisions. Figure 1 shows the optimization process.

3 The Carrier System Model

The model we have introduced above is sufficient enough to consider the required parameters and, on the other hand, it is not too complicated to apply the theory of the optimal controlling. Of cource, multy-body train should be considered.

The model takes into account the following parameters shown in Figure 1:

- M_k, m_{ti}, m_i, m(x), M(x) means mass of the car body, bogie, wheel, track and superstructure correspondingly. These parameters may depend on the x-coordinate if it is pointed out. For the track, it means dependence of the track mass on sleeper spacing *δ(x)*. M(x) can be varied by *α(x)* function.

J_k, j_{ti}, J(x) means the moment of inertia of the car body, bogie and superstructure correspondingly. The parameters may depend on the x-coordinate if it is pointed out. U(x) means vertical stiffness of the rail bed, including rail-sleeper fastening, ballast and embankment (if it is), that can be varied by *γ(x)* function. U(x) depends on $\delta(x)$ function as well. It is important to point out that the non-linear function U depends on the direction of vertical movement of the rail. If the vertical movement of the rail is positive (upwards) respectively the rail bed, the resistance to the movement is equal only to frictional force. This feature reflects reality and it is significant for high-speed traffic. J(x) can be varied by *α(x)* function.

- y(x,t), y(0,t), y(L,t), y_b(x,t) means vertical displacement of the rail that depends on x-coordinate and time, starting/boundary conditions and vertical displacement of the bridge superstructure.
- G_k, C_k, G_t, C_t, P_{ti}, R_i means the forces in the suspensions of the car and in the rail-wheel contact.

The *α(x), δ(x)* and *γ(x)* functions are controlling function. They are not assigned, but have to be found using by criterion of optimization. It is important to underline that the ranges of their changing are limited by the technologic possibilities.

In addition, the safety condition should be observed. The stability of a wheel motion on a rail depends on the ratio between vertical and lateral forces in wheel-rail contact.

The ratio has been determined for different cases. Obviously, the single-track superstructure itself does not cause horizontal oscillation of the car; it causes only pitching and bouncing oscillation of the car if the vertical interaction is considered. We do not take into account the wind load, but take into account the maximum lateral force that the track can bear. Therefore, we may assume that horizontal oscillation of a car is random. In the worst case, the horizontal force may be equal (but not exceed) to the force that may shunt the rail in the horizontal lateral direction. Therefore, we can estimate the minimal permissible value of the vertical force R_{min} that prevents derailment. According to our conclusion R_{min} > 23,814 N [3].

The theory of the optimal controlling supposes equality and inequality constraints bridge, rail and car oscillations. Equality constraint *F* looks like a partial differential equation of rail oscillation:

$$\left(k_b\delta(x)\rho_p F_p\right)\frac{\partial^2 y_p}{\partial t^2} + c_p\left(\frac{\partial y_p}{\partial t} - \frac{\partial y_b}{\partial t}\right) + \quad (1)$$

$$+E_p J_p\frac{\partial^4 y_p}{\partial x^4} + \gamma(x)\delta(x)U'\left(y_p - y_b\right) = P(x,t)$$

Similarly, superstructures oscillation equations were used [4]. Oscillations of the cars may be presented by ordinary differential equations [4].

The inequality constraints *F* are needed to limit minimum vertical force R_{min} in wheel-rail contact that prevents derailment and vertical acceleration of car bodies to obtain acceptable comfort:

$R_{min} > R_{lim} = 23{,}814$ N (2)

$W_{max} < W_{lim} = 0.35$ m/s2 (3)

4 Optimisation

Let us consider the optimisation of dynamic interaction of cars and track inside the bridge zone. We will use the criterion of minimal irregularity of the load on the rail bed in reference to preset meaning of the average load *Q*:

$$D = \int_L\int_T\left(\gamma(x)\delta(x)U'\left(y_p - y_b\right) - Q\right)^2 dxdt \quad (4)$$

Simultaneously we will try to maximize R_{min} while a train is passing a bridge zone *L* including transition zones for a period of time *T*, where *L* is overall length of the bridge zone including transition zones and *T* is the total time of a train motion on the bridge zone.

4.1 Wheel-rail contact forces

Figure 3 shows an example of vertical wheel-rail contact force of a single wheel of a car during its motion through the superstructure before and after optimization. Note, the possibility of wheel lift-off is common knowledge and was discussed for instance in [5]. Figure 3 demonstrates the wheel lift-off before optimization on the first iteration. Then, the wheel-rail contact force becomes more stable due to optimization and on the sixth iteration we get acceptable meaning of the force, that is more than R_{lim} = 23,814 N.

Figure 4 shows vertical forces of four wheels of the second car of a train during its motion through the superstructure at the train speed 400 km/h. We take into account the second car because oscillation of the bridge superstructure gets stabilization after the first car passing and superstructure oscillation amplitude does not grow while the train is passing the bridge superstructure. We can see dangerous decreasing of the forces behind the bridge and the wheel lift-off.

Figure 5 demonstrates the result of optimization – the wheel-rail vertical contact forces remain more than R_{lim} = 23,814 N while the train is passing and vertical force R_{min} = 39.3 kN. One of the targets of optimization has been achieved.

4.2 The main goal of optimization

The main goal of optimization is formulated in (d_{23}). Let us discuss the interrelation between (4) and demand (2).

In Figure 6 an example of evolution of *D* criterion (4) and the meaning of R_{min} during the train passing the bridge zone is shown. At the first iteration, we can see the R_{min}= 0 and demand (2) is violated.

At the second iteration, the meaning of the criterion *D* is nosedivng from 7.5 to 2.18 and simultaneously R_{min} is increasing up to 14.2 kN. Further decreasing of the *D* criterion and simultaneous increasing of R_{min} is impossible and the software is pressed to increase R_{min} because the safety demand (2) is very strong and must be satisfied in any case. At further iterations, *D* criterion is almost constant. At the ninth iteration, the demand (2) is already hold.

Nevertheless, the consequence of the optimization may be sufficient. Figure 7 shows the meaning of the rail bed load direct under the moving wheels of the second car during motion through the bridge zone before optimization and Figure 8 – after optimization.

The average meaning of the load is decreasing from 30.8 to 26.2 kN and standard deviation is decreasing from 2.79 to 1.41kN. Another important result of optimization is that the maximum load has decreased to the meaning of 30.69 kN, less than critical meaning that leads to plastic deformation in ballast.

We supposed that optimization for heavier high-speed train leads to optimal decision for lighter trains. Figure 8 demonstrates success of the optimization for the heavier perspective train and Figure 9 shows that optimal structure for the perspective train remains optimal for lighter CHR380 train. The average meaning and standard deviation for CHR380 train are lower than for the perspective train.

4.2 The optimal controlling functions

The above mentioned results were achieved with the single $\gamma(x)$ controlling function that changes the rail bed stiffness along the track. Our research showed that the required changes within allowed range of the rail bed stiffness can the obtained by grading under-sleeper pads of mass production. The other types of the controlling function in the cases mentioned are not required which makes the track structure simpler, in spite of obligatory usage of flange rails. Sometimes the application of sleeper spacing function $\delta(x)$ can be required. At last, $\alpha(x)$ controlling function is used for bridge superstructure optimization and it will be discussed in detail in another paper.

Because of integral character of the criterion (4), the optimal result can be achieved with several different controlling functions. Managing of optimization process allows obtaining technically reasonable controlling function. Figure 10 shows the unreasonable version of optimal $\gamma(x)$ function. The aim of optimization consists in getting some reasonable version of optimal controlling function (Figure 11). Slight deviations of the function result from discrete character of the optimization process. These deviations are negligible and can be presented by a line.

If the single controlling function application is deficient several controlling functions can be used simultaneously (Figure 12). Figure 11 and Figure 12 show the significance of the integral carrier system (BTCS) research instead of separate consideration of different parts of the BTCS because optimal structure of the track depends on dynamic properties of the bridge superstructure.

4 Conclusion

The developed concept of integrated carrying system including bridge superstructures, track and cars takes into account the interaction inside the system.

Controlling of dynamic interaction within “bridge-track-car” system at the design stage allows estimation and assurance of safety level. For ensuring safety and acceptable ballast wearing at the high speed, it is necessary to explore the whole system. The research showed the importance of the integral system dynamic analysis without application of multiple hypothesis. Thus, the applied theory of optimal controlling was developed. The applied theory allows design the system of predicted behavior.

Planning of motion within many-dimensional behavior and estimation spaces by means of vector-specified controlling functions may be considered as a step forward to development of artificial intelligence in a certain science. The computer program can make decisions concerning technical parameters of the BTCS on the base of analysis of great amount of information and the designer makes a final decision about the acceptability of the computer-aided design.

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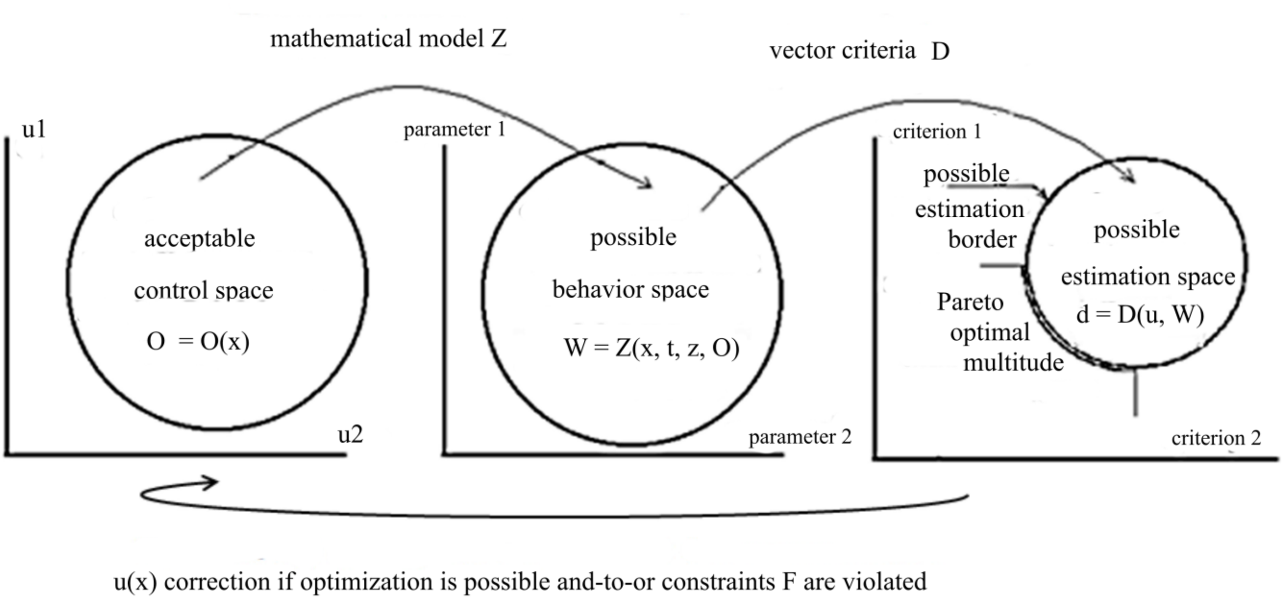

Figure 1: The scheme of solving the BTCS optimization

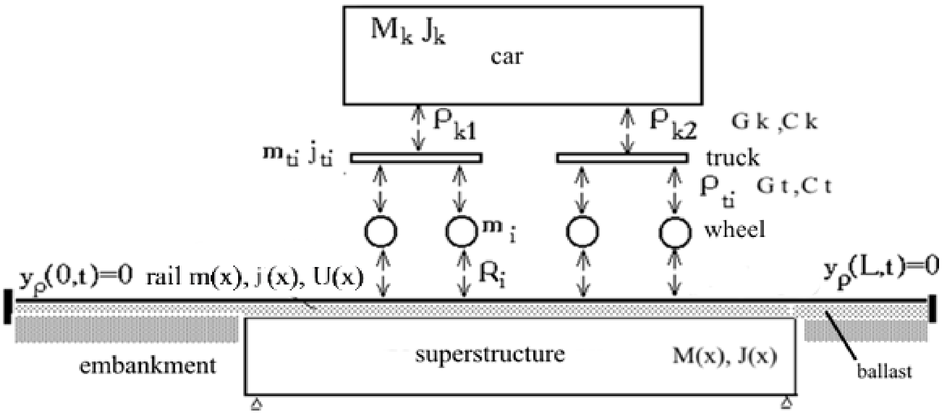

Figure 2: The model of BTCS

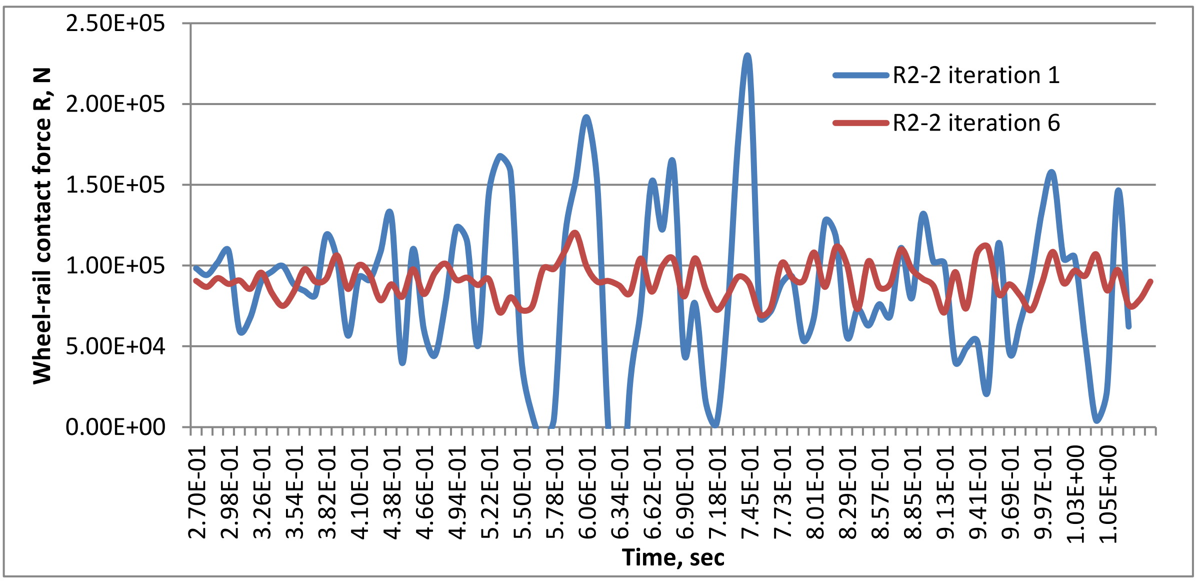

Figure 3: An example of vertical wheel-rail contact force before and after optimization.

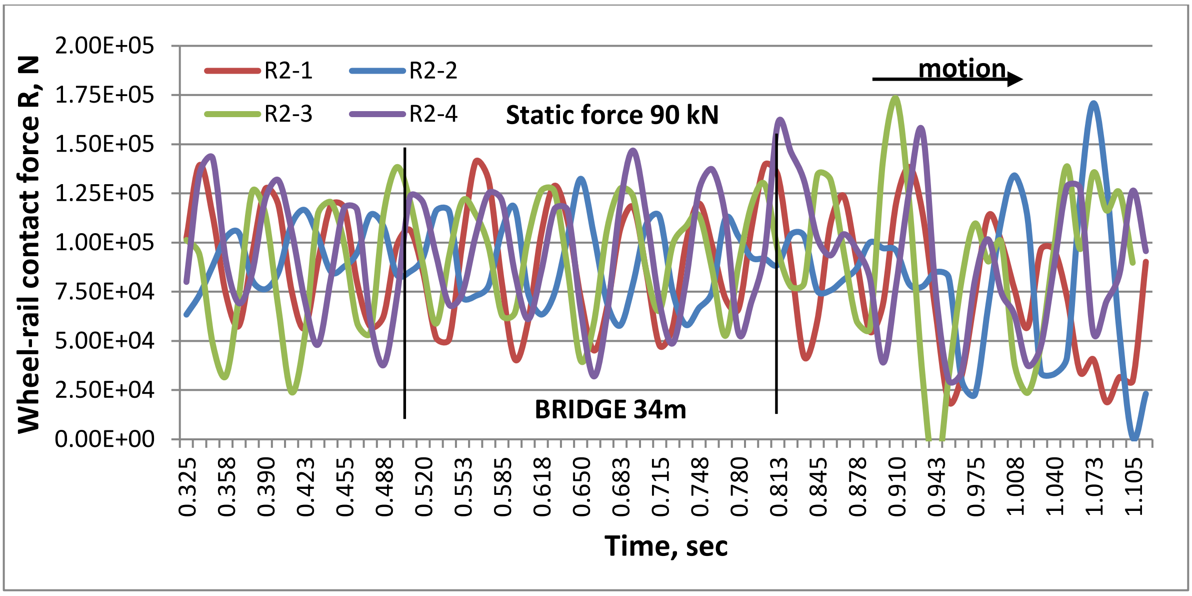

Figure 4: Vertical forces of four wheels of the car of a train during motion through the bridge before optimization

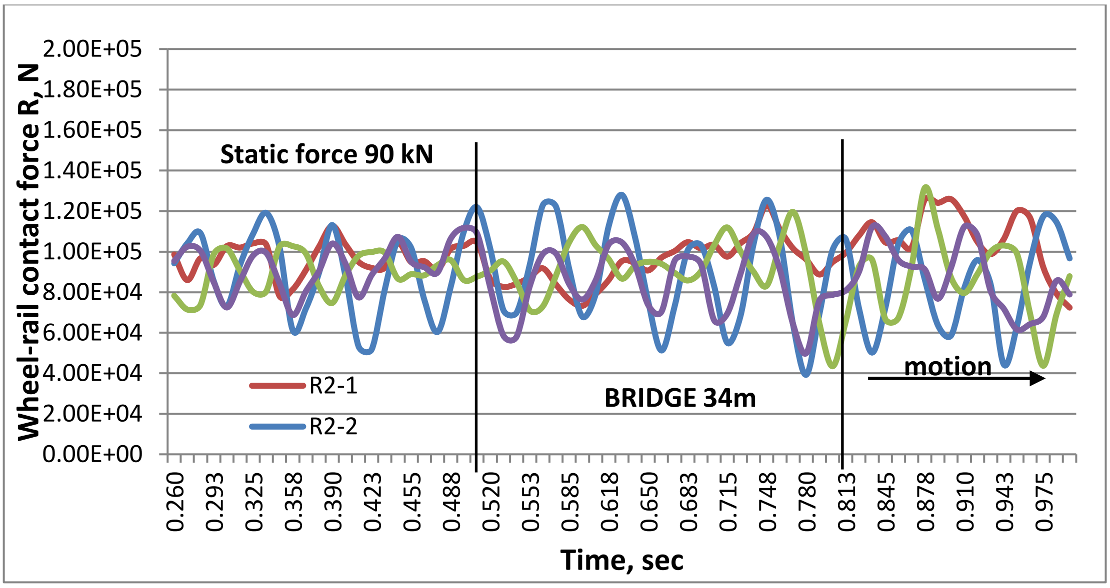

Figure 5: Vertical forces of four wheels of the car of a train during motion through the bridge after optimization

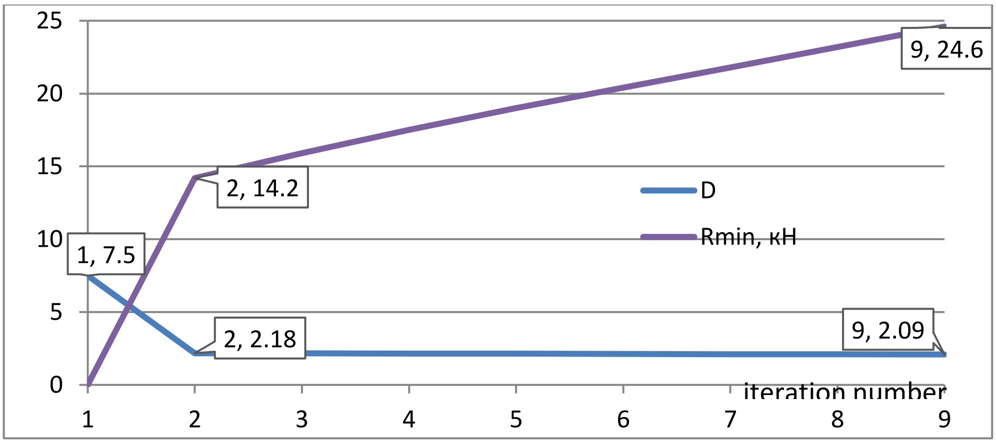

Figure 6: Vertical minimal wheel-rail contact force R_{min} and *D* criterion

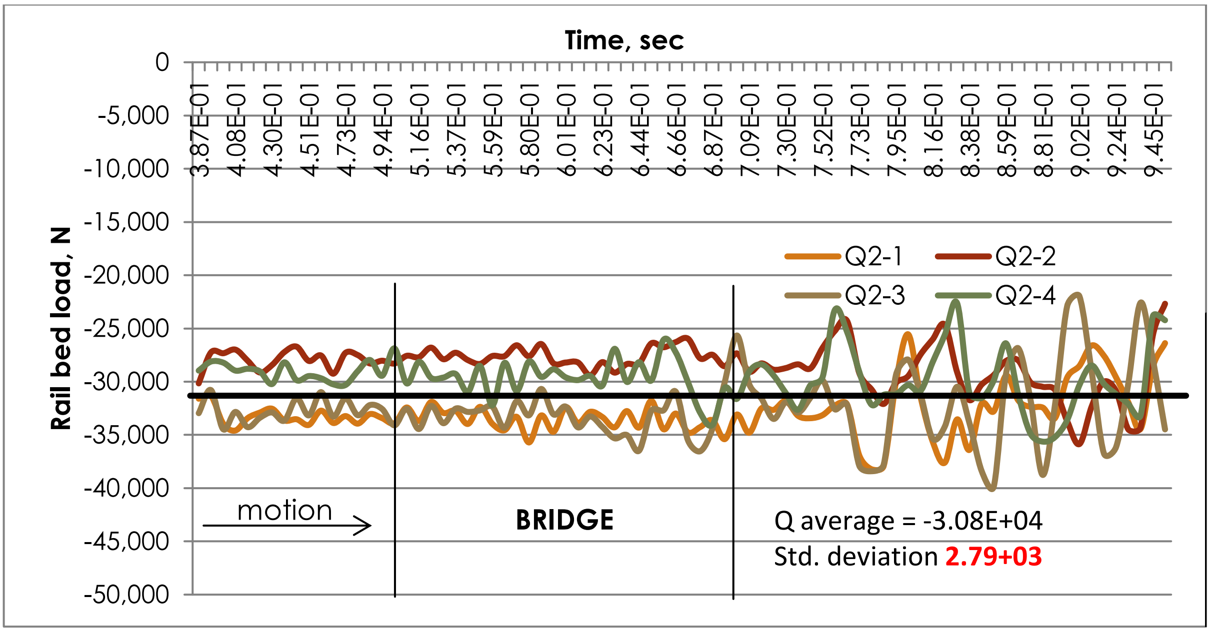

Figure 7: Rail bed load before optimization (perspective train)

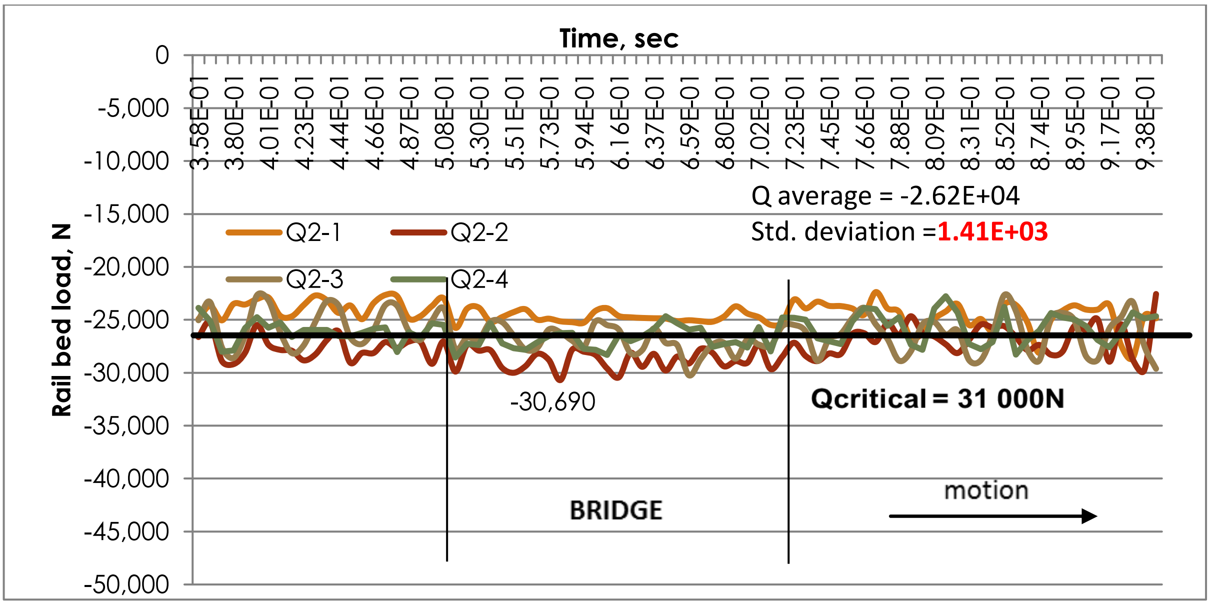

Figure 8: Rail bed load after optimization (perspective train)

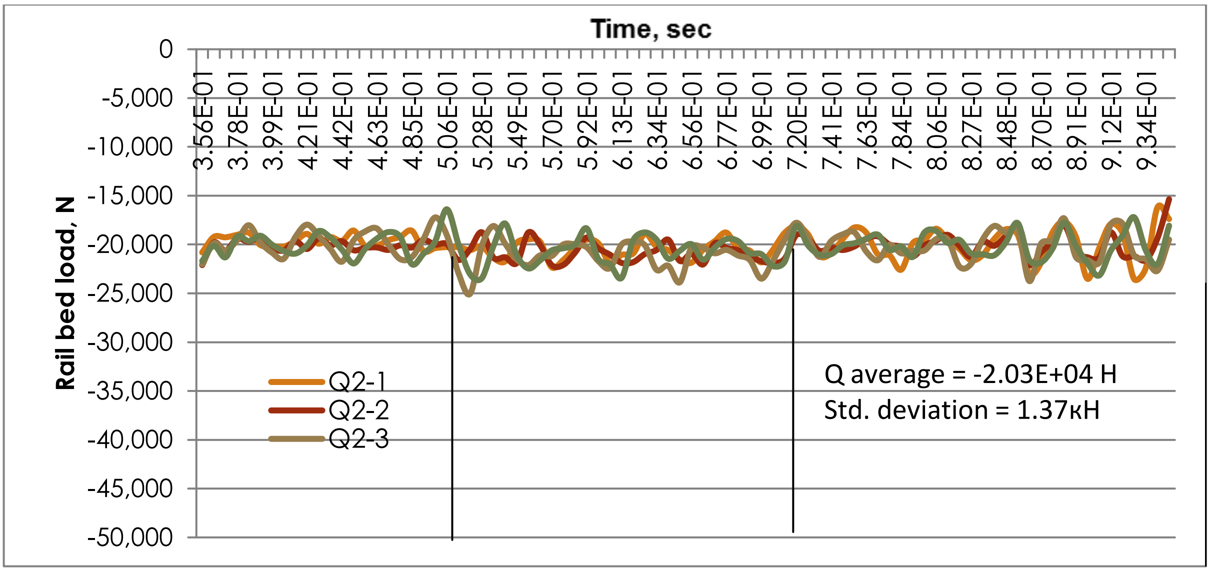

Figure 9: Rail bed load under the second car of CHR380 train after track optimization for perspective train

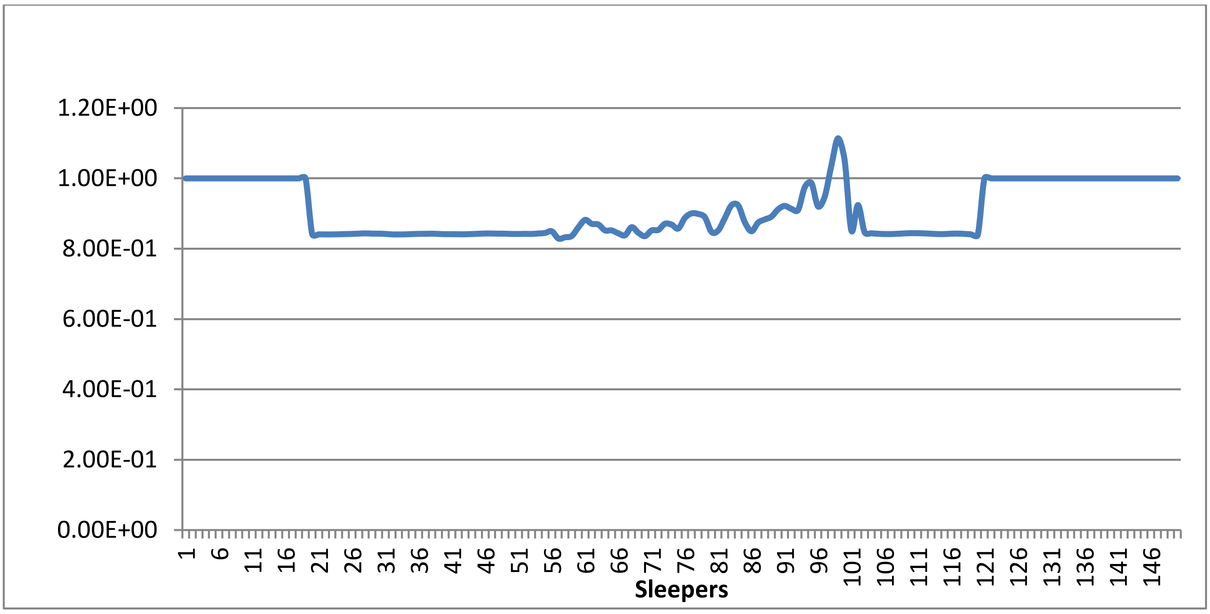

Figure 10: The unreasonable version of optimal γ(x) function

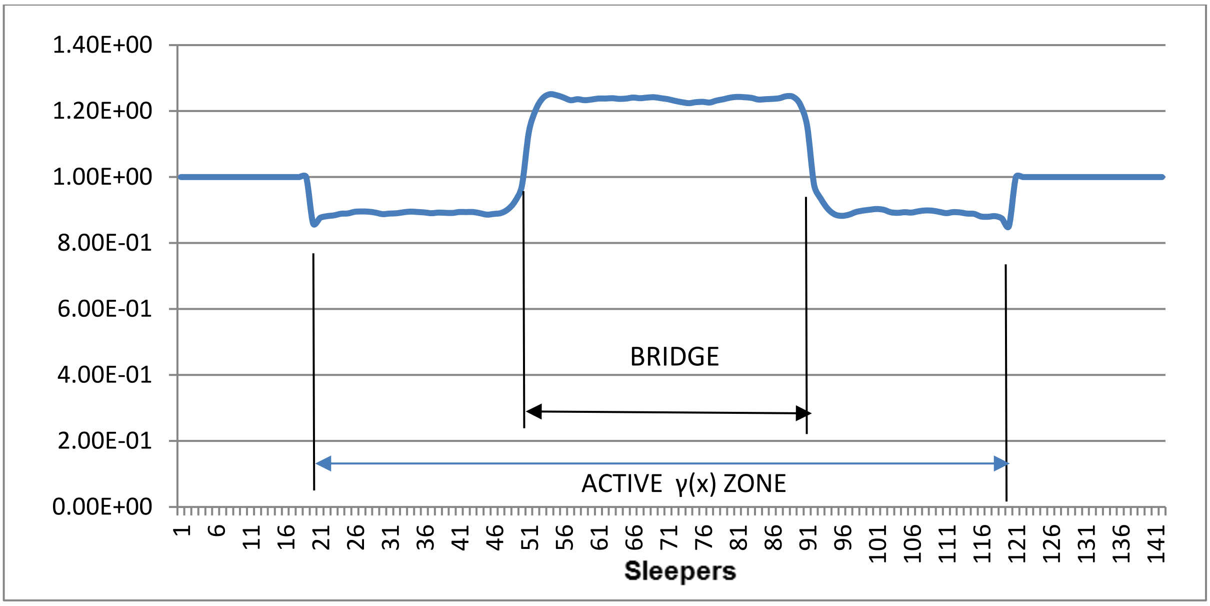

Figure 11: The reasonable version of optimal γ(x) function

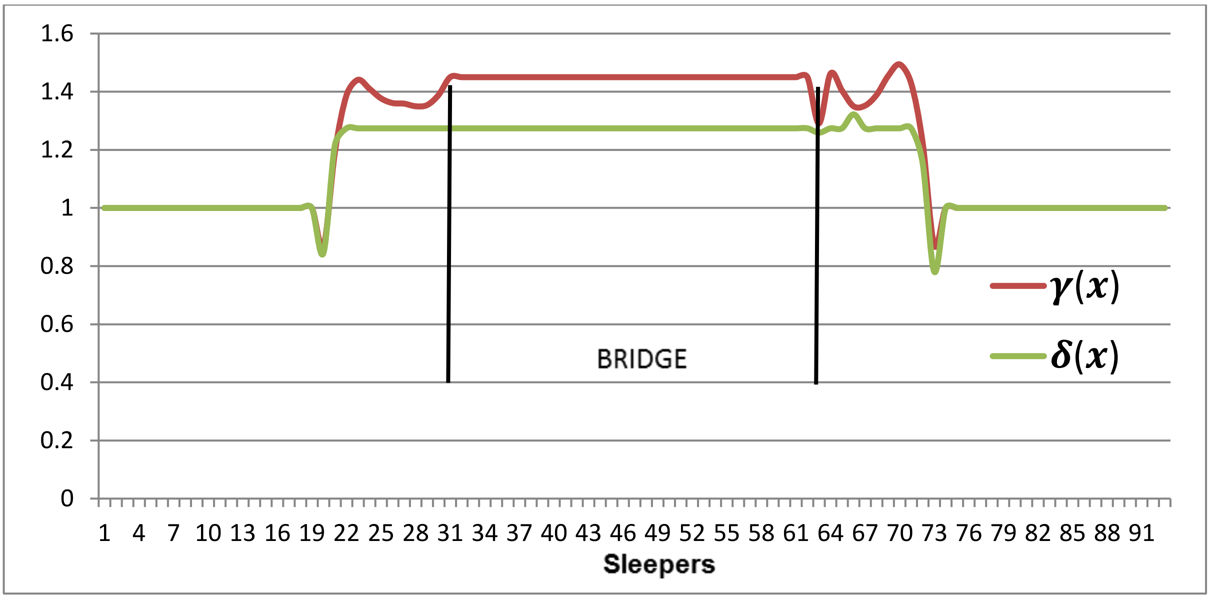

Figure 12: The optimal controlling functions in case of less rigid superstructure