The effects of supports on optimum design of RC continuous beams

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Abstract: -The optimum design of reinforced concrete members is found according to required security measures which are defined with respect to internal forces in the critical section resulting from loading conditions. For that reason, the length of spans and number of supports are effective on the optimum cost of the RC continuous beams. In this study, the optimum cross section dimensions and detailed optimum reinforcement design were investigated for different number of supports and spans. The optimum design was performed according to the design constraints given in ACI-318 (Building Code Requirements for Structural Concrete). All live load distribution patterns were considered and the analyses of RC beam were done by using Clapeyron's three moment equations. A random search technique (RST) was employed in order to minimize the material cost of the continuous beams with different number of spans.

Key-Words: RC continuous beam, random search technique, optimization, Clapeyron's three moment equations, ACI-318, live load distribution patterns.

1 Introduction

Several uncertain variables and influences are an obstacle to obtain optimum design formulations for reinforced concrete (RC) members. The considerations of the uncertainties are standardized in several design codes. Also, the design variables of RC members (applicable dimensions for cross sectional dimensions and fixed reinforcement sizes in local market) must be discrete variables. Secondly, concrete and steel reinforcement bars are extremely different in price and these prices are not fixed in different areas. By using numerical optimization methods, a balance between the costs of these materials can be found and a practical design for construction can be provided by considering required safety conditions in design codes.

Several approaches for the optimization of RC members has been proposed. A well-known metaheuristic method called genetic algorithm was employed in the optimum design of RC beams [1-4], columns [5] and frames [6-7]. Genetic algorithm was used with the simulated annealing algorithm in the development of a methodology to find the optimum design of continuous RC beams by Leps and Sejnoha [8]. Harmony search is one of the other metaheuristic methods used in optimization of RC members such as continuous beams [9], T-shaped RC beams [10] and RC columns [11-12]. Also, several metaheuristic methods have been employed for optimum design of RC retaining walls by considering geotechnical and structural constraints [13-15]. Nigdeli and Bekdaş proposed a random search methodology for the detailed optimum design of RC continuous beams [16].

In this paper, a numerical investigation is presented for simply supported continuous RC beams. A random search methodology developed by Bekdas and Nigdeli [16] was employed. As the numerical examples, a beam was optimized by dividing to different number of spans in order to investigate the effect of supports for the optimization objective defined as the minimization of the material cost. The optimum designs were done according to the constraints given in ACI318 (Building Code Requirements for Structural Concrete) and Clapeyron's three moment equations [17-18] were employed in the analyses to find most critical stress resultants in the critical sections of continuous beams under the most unfavorable solution between all live-load distribution patterns.

2 Clapeyron's three moment equations for continuous beams

A continuous beam with n number of simple supports is given in Fig. 1a. The released structure of statically indetermined continuous beam is obtained by releasing the moments at the simple supports as seen in Fig. 1b.

Continuity condition of a support named with k can be written as;

$$\cdots\cdots\cdots + 0 + \delta_{k,k-1}X_{k-1} + \delta_{k,k}X_k + \delta_{k,k+1}X_{k+1} + 0 + \cdots\cdots\cdots + \delta_{k,0} = 0. \quad (1)$$

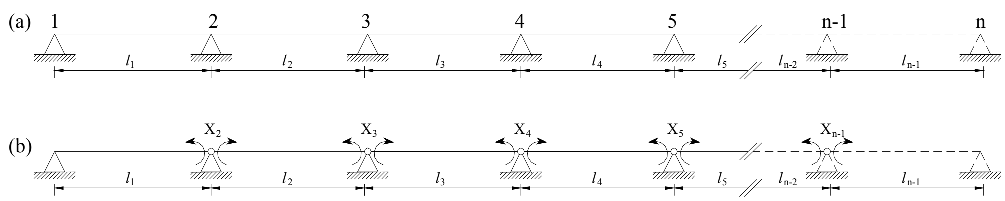

Fig. 1 A continuous beam (a) and the released structure (b)

In Eq. (1), X_{k-1}, X_k and X_{k+1} represent unknown moments at the supports named with k-1, k and k+1, respectively. $\delta_{k,k-1}$ is the virtual work obtained by X_k=1 unit loading resulted from internal forces of X_{k-1}=1 unit loading of released structure. The diagram of moments under unit loadings (X_{k-1}=1, X_k =1 and X_{k+1}=1) and the moment (M_0) under external loads can be seen in Fig. 2 for the released system.

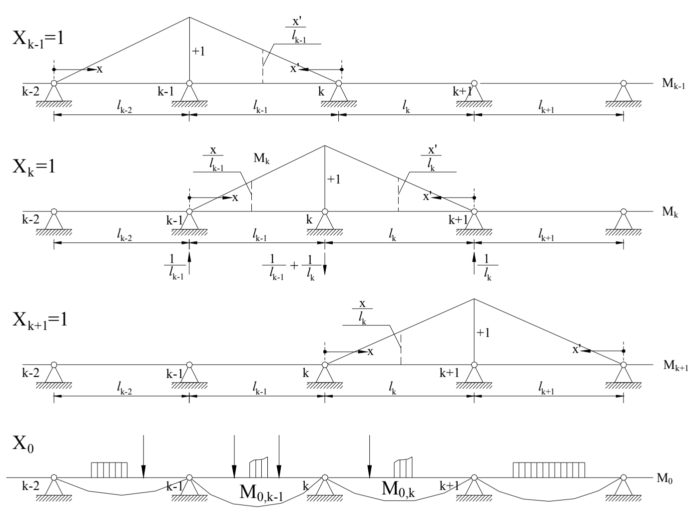

Fig. 2 The moment diagram of released structure under unit loads and external loads

The virtual works;

$$\delta_{k,k-1} = \int_{x=0}^{l_{k-1}} \frac{xx\prime}{l_{k-1}^2} \frac{dx}{EI_{k-1}}\,, \quad (2)$$

$$\delta_{k,k} = \int_{x=0}^{l_{k-1}} \frac{x^2}{l_{k-1}^2} \frac{dx}{EI_{k-1}} + \int_{x=0}^{l_k} \frac{x\prime^2}{l_k^2} \frac{dx}{EI_k}\,, \quad (3)$$

$$\delta_{k,k+1} = \int_{x=0}^{l_k} \frac{xx\prime}{l_k^2} \frac{dx}{EI_k}, \tag{4}$$

$$\delta_{k,0} = \int_{x=0}^{l_{k-1}} M_{0,k-1} \frac{x}{l_{k-1}} \frac{dx}{EI_{k-1}} + \int_{x=0}^{l_{k-1}} M_{0,k} \frac{x\prime}{l_k} \frac{dx}{EI_k} \tag{5}$$

can be written according to the moment diagrams given in Fig. 2. In Eqs. (2-5), modulus of elasticity and moment of inertia of cross-section are shown with E and I_k, respectively. By using Eqs. (2-5) in continuity condition (Eq. (1)), Clapeyron's three moment equations can be obtained as

$$X_{k-1}\left\{\frac{l_{k-1}}{I_{k-1}}\right\} + X_k\left\{2\left(\frac{l_{k-1}}{I_{k-1}} + \frac{l_k}{I_k}\right)\right\} + X_{k+1}\left\{\frac{l_k}{I_k}\right\} = -\frac{l_{k-1}}{I_{k-1}}\Re_{k-1} - \frac{l_k}{I_k}\mathcal{L}_k, \tag{6}$$

in which;

$$\left.\begin{array}{l} \Re_{k-1} = \frac{6}{l_{k-1}^2}\int M_{0,k-1} x dx \\ \mathcal{L}_k = \frac{6}{l_k^2}\int M_{0,k} x' dx \end{array}\right\}. \tag{7}$$

By determining Clapeyron's equation for each support, the unknown moments of statically indetermined continuous beam can be found. As an example, the Clapeyron's equations are obtained for a continuous beam with five supports (Fig. 3) in this section.

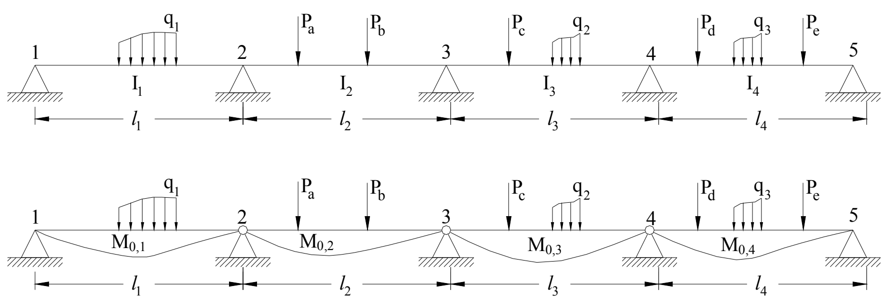

Fig. 3 A continuous beam with five supports and the solution of the released structure under external loading

Clapeyron's three moment equations for the supports 2, 3 and 4 can be respectively written as;

$$M_1(\ldots\ldots) + M_2\left[2\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right)\right] + M_3\left[\frac{l_2}{I_2}\right] = -\frac{l_1}{I_1}\Re_1 - \frac{l_2}{I_2}\mathcal{L}_2, \tag{8}$$

$$M_2\left[\frac{l_2}{I_2}\right] + M_3\left[2\left(\frac{l_2}{I_2} + \frac{l_3}{I_3}\right)\right] + M_4\left[\frac{l_3}{I_3}\right] = -\frac{l_2}{I_2}\Re_2 - \frac{l_3}{I_3}\mathcal{L}_3, \tag{9}$$

$$M_3\left[\frac{l_3}{I_3}\right] + M_4\left[2\left(\frac{l_3}{I_3} + \frac{l_4}{I_4}\right)\right] + M_5(\ldots) = -\frac{l_3}{I_3}\Re_3 - \frac{l_4}{I_4}\mathcal{L}_4. \tag{10}$$

Since the moments at exterior supports (M_1 and M_5) are equal to zero, Clapeyron's equations can be written in matrix form as given in Eq. (11). By solving this matrix, the non-zero support moments (M_2, M_3 and M_4) can be obtained.

$$\begin{bmatrix} 2\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) & \left[\frac{l_2}{I_2}\right] & 0 \\ \left[\frac{l_2}{I_2}\right] & \left[2\left(\frac{l_2}{I_2} + \frac{l_3}{I_3}\right)\right] & \left[\frac{l_3}{I_3}\right] \\ 0 & \left[\frac{l_3}{I_3}\right] & \left[2\left(\frac{l_3}{I_3} + \frac{l_4}{I_4}\right)\right] \end{bmatrix} \begin{bmatrix} \mathrm{M}_2 \\ \mathrm{M}_3 \\ \mathrm{M}_4 \end{bmatrix} = \begin{bmatrix} -\frac{l_1}{I_1}\Re_1 - \frac{l_2}{I_2}\mathcal{L}_2 \\ -\frac{l_2}{I_2}\Re_2 - \frac{l_3}{I_3}\mathcal{L}_3 \\ -\frac{l_3}{I_3}\Re_3 - \frac{l_4}{I_4}\mathcal{L}_4 \end{bmatrix} \tag{11}$$

3 Random Search Technique

The methodology considers all possible live-load patterns and the approach can be explained in six steps.

i. First, the properties of the member are defined. These properties are the coordinates of the supports, design variable ranges and loading conditions.

ii. After the problem is defined, the design variables of a continuous beam such as breadth and height of the RC cross-sections are randomly assigned. All optimum design dimensions were assigned as the multiples of 10 mm in the numerical example.

iii. Since the cross-sections dimensions are defined, internal forces including flexural moments and shear forces can be analyses for the critical sections of the beam. Clapeyron's three moment equations are employed in the analyses. The most unfavorable solution according to the live loads is found for all spans and supports.

iv. After the internal forces are analyzed, the required reinforcements (longitudinal reinforcements and stirrups) are calculated according to ACI-318. For flexural reinforcements, the positioning of the steel bars is checked and the depth of the beam is depending on it. Since reinforcement bars are found in fixed sizes in local markets, the reinforcing bars are randomly chosen from a user defined template. If the maximum reinforcement area is less than the singly reinforcement beam, doubly reinforcement beams are taken into consideration during the optimization process. For that reason, the reinforcement bars can be designed in two lines if it is needed. Also, minimum reinforcement conditions are checked. In the first design, the depth of the beam is calculated by considering clear cover, minimum stirrup diameter and minimum longitudinal reinforcement. After reinforcement bars are defined, the exact value of the depth is calculated and the required reinforcement area is recalculated. The randomizing of the steel reinforcement bars (the process in this step) is repeated until the following conditions are met.

a. The total area of the reinforcement bars of the tensile section must be less than the maximum reinforcement area defined in ACI-318. If the beam is doubly reinforcement, the area of the reinforcement bars is reduced by considering the stress of the reinforcement bars of compression section.

b. The total area of the reinforcement bars must be less than 5% more of the required reinforcement area.

c. If a doubly reinforced design is needed, the total area of the reinforcement of the compression section must be less than the bars at the tensile section.

v. In the fifth step, the total material cost is calculated.

vi. In the last step, the iteration number is checked. If the iteration number is satisfied, the optimum results are outputted. Otherwise, the range of the cross-section dimensions is modified according to the best existing solution and the process is repeated from the second step. The upper or the lower bounds of the range are updated with the best cross-section dimensions of the span with the smallest values of cross sections. This modification is done with 50% probability and the initial ranges are also used with 50% probability. By this operation, the local optimization problem is prevented.

4 Numerical Examples

In this section, four cases of a 15 meter length continuous beam are presented. In these cases, the beam is divided into 2, 3, 4 and 5 spans. The cost of concrete and steel was taken as 40\$/m^3 and 400\$/ton, respectively and the ratio of concrete and steel costs is 1/10. The spans of the beam are loaded with trapezoid loads as seen in Fig. 4. This figure is for Case 1. In that case, the beam is divided into two spans. For Cases 2, 3 and 4, the number of spans are 3, 4 and 5, respectively. All beams are loaded with 15 kN/m dead load (D) and maximum 5 kN/m live load (L). The length defined with a is 1.5 m for all cases since it is defined according to length of the structure in the other direction. The other length defined as b is found according to the number of spans. For example, b is 4.5, 2, 0.75 and 0 for the cases 1, 2, 3 and 4, respectively. In that situation, the beam is loaded with triangular loads in Case 4.

In the optimization, breadth (b_w) and height (h) of the beams were searched for the ranges 200 mm - 600 mm and 300 mm - 600 mm, respectively. The diameter ranges of longitudinal reinforcements and stirrups are between 10 mm - 36 mm and 8 mm - 14 mm, respectively. The clear cover of reinforcement and the biggest aggregate diameter are taken as 30 mm and 16 mm, respectively. The all cases of live loads were considered in the optimization since dead live always applies, but live loads may not always exist. The optimum results are presented in Tables 1-4.

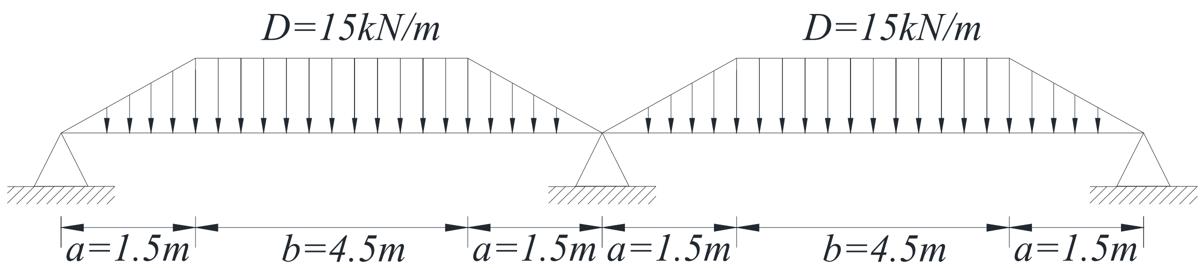

Figure 4 Two-span symmetric continuous beam (Case 1)

TABLE I. THE OPTIMUM RESULTS OF TWO-SPAN CONTINUOUS BEAM (CASE 1)

	First span	Middle support	Second span
Cross-section (B/H) (mm)	220/540	-	220/540
First line of tensile section	2Φ20+1Φ10	2Φ22+1Φ24	2Φ20+1Φ10
Stirrup steel diameter /distance (mm)	Φ8/240	-	Φ8/240
Optimum Cost ($)		125.3783	

TABLE II. THE OPTIMUM RESULTS OF THREE-SPAN CONTINUOUS BEAM (CASE 2)

	First span	Second support	Second span	Third support	Third span
Cross-section (b_w/h) (mm)	200/330	-	200/330	-	200/330
First line of tensile section	2Φ18+ 1Φ10	1Φ24+ 1Φ18	1Φ12+ 2Φ10	1Φ24+ 1Φ18	2Φ18+ 1Φ10
Stirrup steel diameter /distance (mm)	Φ8/140	-	Φ8/140	-	Φ8/140
Optimum Cost ($)			77.7782		

TABLE III. THE OPTIMUM RESULTS OF FOUR-SPAN CONTINUOUS BEAM (CASE 3)

	First span	Second support	Second span	Third support	Third span	Fourth support	Fourth span
Cross-section (b_w/h) (mm)	200/300	-	200/300	-	200/300	-	200/300
First line of tensile section	2Φ12+ 1Φ10	2Φ14+ 1Φ10	1Φ12+ 1Φ10	1Φ16+ 1Φ10	1Φ12+ 1Φ10	2Φ14+ 1Φ10	2Φ12+ 1Φ10
Stirrup steel diameter /distance (mm)	Φ8/120	-	Φ8/120	-	Φ8/120	-	Φ8/120
Optimum Cost ($)				59.7365			

TABLE IV. THE OPTIMUM RESULTS OF FIVE-SPAN CONTINUOUS BEAM (CASE 4)

	First span	Second support	Second span	Third support	Third span	Fourth support	Fourth span	Fifth support	Fifth span
Cross-section (b_w/h) (mm)	210/340	-	210/340	-	210/340	-	210/340	-	210/340
First line of tensile section	1Φ12+ 1Φ10	1Φ12+ 1Φ10	1Φ12+ 1Φ10	1Φ12+ 1Φ10	1Φ12+ 1Φ10	1Φ12+ 1Φ10	1Φ12+ 1Φ10	1Φ12+ 1Φ10	1Φ12+ 1Φ10
Stirrup steel diameter /distance (mm)	Φ8/140	-	Φ8/140	-	Φ8/140	-	Φ8/140	-	Φ8/140
Optimum Cost ($)					61.5276				

5 Conclusions

According to the optimum of continuous beam with two spans (Case 1), the optimum cost is 125.38 $. The optimum design is symmetric and the height of the beam is significantly high. Also, the diameter sizes are quite big. For an economic solution, singly reinforced design is chosen and the cross-sectional area of the beam is enlarged. Since the height is more effective than the breadth of the beam in mean of moment capacity, the breadth is near to the lower limit of the range. For that reason and adherence, the number of bars are low while the sizes are big.

In the second case, the beam is divided into three spans. The design is also symmetrical. Since the resultant moments and shear forces are reduced, a significant reduction of the optimum cost is obtained comparing to Case 1 (38% reduction).

If the number of supports are increased to five (four span), the positive effect on the optimum cost is also observed. In that case (Case 3), significant reduction of the steel reinforcements are clearly seen. The last case is not effective on reduction of the optimum cost.

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