Control of Supersonic Boundary Layer and its Stability by means of Foreign Gas Injection through the Porous Wall

SERGEY A. GAPONOV, BORIS V. SMORODSKY
Khristianovich Institute of Theoretical and Applied Mechanics
Novosibirsk, 630090, RUSSIA
gaponov@itam.nsc.ru smorodsk@itam.nsc.ru http://www.itam.nsc.ru

Abstract: Properties of binary-mixture compressible boundary layers have been investigated by means of the self-similar boundary-layer equations. Mach=2 boundary-layer velocity, density, temperature and concentration profiles have been computed. It was established that action of heavy gas injection on the boundary-layer is similar to the action of wall cooling and leads to an increase of boundary-layer stability and to the delay of the laminar-turbulent transition (LTT). Influence of a foreign gas injection on skin friction and heat transfer coefficients has been investigated. It has been found that increase of foreign gas injection leads to the monotonous reduction of surface friction and heat transfer.

Linear stability of Mach=2 boundary-layer has been investigated by means of the linear stability theory (LST). It has been established that injection of a heavy gas leads to the reduction of disturbance amplification rates. A principal possibility to enlarge the transition Reynolds numbers in approximately two times has been found.

Key-Words: compressible boundary-layer, binary-mixture, laminar-turbulent transition, hydrodynamic stability.

1 Introduction

Importance to study the binary-mixture boundary-layers is determined by a number of various factors. One of the reasons was to find possibilities to control the skin friction and the wall heat flux in stationary conditions. For the first time a foreign gas injection into the two-dimensional (2D) flat-plate boundary-layer was investigated in [1-3]. It was shown there that in order to reduce the skin friction it is better to inject in the flow a gas lighter than an air. Solutions presented in the mentioned above papers have a lot of limitations. In [1] the skin friction was not calculated explicitly but its value can be deduced from the velocity profiles computed there. However the boundary-layer equations have been solved there with the boundary conditions suitable for the impermeable surface only. Also the foreign gas concentration at the wall was assumed to be large enough whereas the injection velocity was assumed to be zero that is out of logic. In [2,3] the solutions have been obtained in the assumption of the zero heat transfer. In addition in [3] it was supposed that the binary-mixture viscosity does not depend on the foreign-gas concentration and varies linearly with the temperature, while the Schmidt number was taken equal to unity.

The non-zero heat transfer has been considered for the first time in [4], where the influence of an injected gas properties on the skin friction and the wall heat flux variation has been investigated. The author has got a numerical solution to the problem using a simple model of solid spheres to describe the collisional processes, for the case when the injection magnitude varies downstream inversely proportional to the square root of the streamwise coordinate. It has been shown in this paper that the light gas injection leads to the reduction of both – the skin friction and the wall heat transfer. Moreover, the injection of a large specific heat gas reduces the heat flux significantly, but affects the skin friction only weakly. The combination of a large molecular collision diameter and high specific heat of various polyatomic gases can be much more advantageous to reduce the skin friction and the heat transfer in comparison to the injection of such a light monatomic gas as helium. Additional investigations of the laminar boundary layers of binary mixtures can be found in [5-7].

Along with the problem of a thermal protection and drag reduction there is another important problem of the laminar-turbulent transition control. In the framework of our discussion of the binary gas mixture boundary layers this problem is in fact reduced to the investigation of hydrodynamic stability of such boundary layers. Influence of a foreign gas injection on the linear stability of laminar boundary layers has been studied for the first time in [8]. It was shown there that a heavy gas injection can principally lead to a boundary layer stabilization. This possibility has been further investigated in [9], where they have performed a research on the influence of a foreign gas molecular mass on the boundary layer stability. In particular, it has been shown that injection of a light gas leads to a certain reduction of the critical Reynolds number. However the results of [9] have been obtained for Mach number $M=0$ and it was not possible to solve the problem for the Mach $M>1.3$ because of the certain limitations inherent to the asymptotic procedure used there.

The complete enough equations of the binary-mixture boundary-layer stability with respect to 2D disturbances in the parallel flow approximation have been obtained in [10]. However those equations have never been used: the author has not published the results of a numerical solution of his equations.

After a while a research on the influence of a gas nonequilibrium dissociation on the boundary-layer stability has been published in [11,12]. After that there was no further systematic investigations of the effect of a foreign gas injection on a boundary-layer properties and its stability.

Present paper fills up this gap in the knowledge of a binary-mixture boundary-layer stability. Some first results of ongoing investigations on possibilities to control the flat-plate supersonic boundary-layer and its linear stability by means of a foreign gas injection from the surface are presented here.

2 Problem Formulation

The time-dependent dynamics of a binary gas mixture is described by the system of partial differential equations [13-15]:

$$\begin{gathered}
\frac{d\rho^*}{dt}+\rho^*\operatorname{div}\mathbf{V}^*=0 \ , \\
\frac{d\mathbf{v}^*}{dt}=-\frac{2}{\rho}\operatorname{div}\mathbf{\Pi} \ , \\
\rho^*\frac{dc}{dt}=-\operatorname{div}\mathbf{j}_1^* \ , \\
\rho^*\frac{dh^*}{dt}=\frac{dP^*}{dt}-\operatorname{div}\mathbf{q}^*+2\mu^*\dot{S}^{*2} \ , \\
P^*=\frac{\rho^*RT^*}{m}=\frac{\rho^*R}{m}\frac{h^*}{C_p^*} \ ,
\end{gathered} \tag{1}$$

where $\dfrac{1}{m}=\dfrac{m_1+(m_2-m_1)c}{m_1m_2}$; the enthalpy $h^*=C_p^*T^*$, $C_p^*=\left(C_{p1}^*-C_{p2}^*\right)c+C_{p2}^*$; $C_p^*, C_{p2}^*, C_{p1}^*$ – are constant pressure specific heats of the mixture, of the primary and of the foreign gases, m_2, m_1 – molecular masses of primary and foreign gases. In Cartesian coordinates:

$$\dot{S}^{*2}=\sum_{i,j=1}^{3}\dot{S}_{ij}^{*2} \ , \ \dot{S}_{11}^*=\frac{\partial U^*}{\partial x}-\frac{1}{3}\operatorname{div}\mathbf{V}^* \ ,$$

$$\dot{S}_{22}^*=\frac{\partial V^*}{\partial y}-\frac{1}{3}\operatorname{div}\mathbf{V}^* \ , \ \dot{S}_{33}^*=\frac{\partial W^*}{\partial z}-\frac{1}{3}\operatorname{div}\mathbf{V}^* \ ,$$

$$\dot{S}_{12}^*=\dot{S}_{21}^*=\frac{1}{2}\left(\frac{\partial U^*}{\partial y}+\frac{\partial V^*}{\partial x}\right) \ ,$$

$$\dot{S}_{13}^*=\dot{S}_{31}^*=\frac{1}{2}\left(\frac{\partial U^*}{\partial z}+\frac{\partial W^*}{\partial x}\right) \ ,$$

$$\dot{S}_{23}^*=\dot{S}_{32}^*=\frac{1}{2}\left(\frac{\partial V^*}{\partial z}+\frac{\partial W^*}{\partial y}\right) \ ,$$

$$\Pi_{ij}=P^*\delta_{ij}-\mu^*S_{ij}+\frac{1}{3}\mu^*\operatorname{div}\mathbf{V}^*\delta_{ij} \ ,$$

$$\frac{d}{dt}=\frac{\partial}{\partial t}+U^*\frac{\partial}{\partial x}+V^*\frac{\partial}{\partial y}+W^*\frac{\partial}{\partial z} \ .$$

In the equations above we used the dimensional values for: $\mathbf{V}^*=\left(U^*,V^*,W^*\right)$ – velocity components in (x,y,z) – directions respectively; ρ^* – density; P^* – pressure; T^* – temperature; h^* – specific enthalpy per unit mass; μ^* – dynamic viscosity; $\mathbf{q}^*$ – total heat flux; $\mathbf{j}_1^*$ – foreign gas mass flux.

Formulas for the binary-mixture heat flux and for the foreign gas mass flux are taken from [13]:

$$\mathbf{q}^*=-\lambda^*\nabla T^*+\left(h_1^*-h_2^*\right)\mathbf{j}_1^* \ , \tag{2}$$

$$\mathbf{j}_1^*=-\rho^*D_{12}^*\left[\nabla c_1+\frac{m_2-m_1}{m}c_1\left(1-c_1\right)\nabla\left(\ln P^*\right)\right] \ . \tag{3}$$

2.1 Boundary-layer equations

2D stationary supersonic flat-plate boundary-layer of the binary gas mixture without chemical reactions is considered here. Foreign gas with molecular mass m_1 is injected into the main flow of the primary gas with a molecular mass m_2 through a permeable surface. In this case the dimensional boundary layer equations can be presented as [14, 15]:

$$\frac{\partial(U\rho)}{\partial x}+\frac{\partial(V\rho)}{\partial y}=0 \ ,$$

$$\rho\left(U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial U}{\partial y}\right) \ ,$$

$$\rho\left(U\frac{\partial h}{\partial x}+V\frac{\partial h}{\partial y}\right)=-\frac{\partial q}{\partial y}+\mu\left(\frac{\partial U}{\partial y}\right)^2 \ ,$$

$$\rho\left(U\frac{\partial c_1}{\partial x}+V\frac{\partial c_1}{\partial y}\right)=-\frac{\partial j_1}{\partial y} \ ,$$

$$P = \frac{\rho RT}{m} = \rho \tilde{R} T \ .$$

Here x is the streamwise coordinate, y – is normal to the surface,

$$q = -\lambda \frac{\partial T}{\partial y} + (h_1 - h_2) j_1 + \frac{RT}{m_2} \frac{\alpha_T j_1}{m_1/m_2 + (1 - m_1/m_2) c_1},$$

$$j_1 = -\rho D_{12} \left[\frac{\partial c_1}{\partial y} + \frac{\alpha_T c_1 (1-c_1)}{T} \frac{\partial T}{\partial y} \right],$$ U and V are the velocity components of the mixture in the x- and y- directions; ρ is the density; h – enthalpy per unit mass; P – pressure; T – temperature; q – heat flux of the mixture, j – mass flux of the foreign gas in the y-direction; c – concentration of the foreign gas; R – universal gas constant; μ, λ, D_{12} – coefficients of dynamic viscosity, heat conductivity and binary diffusion; α_T – thermal diffusion coefficient.

Boundary layer equations are solved with the following boundary conditions at the wall:

$$U = 0, \quad V = V_w, \quad \left(a_1 T + a_2 \frac{dT}{dy} \right) = 0,$$
$$V_w (1 - c_w) = D_{12} \left[-\frac{\partial c}{\partial y} + \frac{\alpha_T c_1 (1-c_1)}{T} \frac{\partial T}{\partial y} \right]. \qquad (4)$$

The last equality is obtained from the condition that the primary gas mass flux $j_2 + (\rho V)(1 - c_1)$ to the permeable wall is zero, [7]. Conditions at the boundary layer outer edge are:

$$U = U_e \ , \quad T = T_e \ , \quad c_1 = 0 \ \text{ at } (y \to \infty) \ . \qquad (5)$$

Boundary layer equations in self-similar variables and in the absence of thermal diffusion can further be written as follows:

$$\frac{d}{d\bar{y}} \left(\bar{\mu} \frac{d\bar{U}}{d\bar{y}} \right) + F \frac{d\bar{U}}{d\bar{y}} = 0 \ ,$$

$$\frac{d\bar{q}}{d\bar{y}} = F \frac{d\bar{h}}{d\bar{y}} + (\gamma - 1) \mathrm{M}_e^2 \bar{\mu} \left(\frac{d\bar{U}}{d\bar{y}} \right)^2 , \frac{d\bar{j}_1}{d\bar{y}} = F \frac{dc_1}{d\bar{y}}, \qquad (6)$$

$$\bar{q} = -\bar{\lambda} \frac{d\bar{T}}{d\bar{y}} + (\bar{h}_1 - \bar{h}_2) \bar{j}_1, \ \bar{j}_1 = -\bar{\rho} \bar{D}_{12} \frac{dc_1}{d\bar{y}}.$$

Here $\bar{y} = y \Big/ \sqrt{x\mu_e / U_e \rho_e}$, $\bar{q}(\bar{y}) = \dfrac{\sqrt{x\mu_e / U_e \rho_e}}{\mu_e h_e} q$,

$$\bar{j} = j \frac{\sqrt{x\mu_e / U_e \rho_e}}{\mu_e}, \ \bar{U} = \frac{U}{U_e} = \frac{2}{\bar{\rho}} \frac{dF}{d\bar{y}}, \ \bar{h} = \frac{h}{C_{p2} T_e},$$

$$\bar{T} = \frac{T}{T_e}, \ \bar{\mu} = \frac{\mu}{\mu_e}, \ \bar{\lambda} = \frac{\lambda}{\mu_e C_{p2}}, \ \bar{D}_{12} = \frac{\rho_e D_{12}}{\mu_e}.$$

Boundary conditions take the following shape with new variables at the permeable wall

$$F = -f_w \ , \quad \bar{U} = 0 \ , \quad \left(b_1 \bar{T} + b_2 \frac{d\bar{T}}{d\bar{y}} \right) = 0 \ ,$$

$$f_w (1 - c_{1w}) = \bar{\rho}_w \bar{D}_{12} \left[-\frac{\partial c_1}{\partial \bar{y}} + \frac{\alpha_T c_1 (1-c_1)}{\bar{T}} \frac{\partial \bar{T}}{\partial \bar{y}} \right],$$

$$f_w = \bar{\rho}_w \bar{V}_w \, \mathrm{Re} \ ,$$

while at the boundary layer outer edge:

$$(\bar{U}, \bar{T}) = 1 \ , \quad c_1 = 0 \ \text{ at } (\bar{y} \to \infty) \ .$$

To calculate the viscosity of a binary mixture the relations [16] have been used, which can be written non-dimensionally:

$$\bar{\mu} = \frac{X_1 \bar{\mu}_1}{X_1 + G_{12} X_2} + \frac{X_2 \bar{\mu}_2}{X_2 + G_{21} X_1} \ ,$$

$$X_1 = \frac{c_1 m}{m_1} \ , \quad X_2 = \frac{(1 - c_1) m}{m_2} \ ,$$

$$(\bar{\mu}, \bar{\mu}_1, \bar{\mu}_2) = (\mu, \mu_1, \mu_2) / \mu_{2e} \ ,$$

$$G_{12} = \frac{\left[1 + (\mu_1/\mu_2)^{1/2} (m_2/m_1)^{1/4} \right]^2}{8^{1/2} \left[1 + (m_1/m_2) \right]^{1/2}} \ ,$$

$$G_{21} = \frac{[1 + (\mu_2/\mu_1)^{1/2} (m_1/m_2)^{1/4}]^2}{8^{1/2} \left[1 + (m_2/m_1) \right]^{1/2}} \ .$$

The formula for the thermal conductivity coefficient of binary mixtures is looking similar, non-dimensionally:

$$\bar{\lambda} = \frac{X_1 \bar{\lambda}_1'}{X_1 + 1.065 \tilde{G}_{12} X_2} + \frac{X_2 \bar{\lambda}_2'}{X_2 + 1.065 \tilde{G}_{21} X_1} \ ,$$

$$\bar{\lambda}_i' = \bar{\lambda}_i \mathrm{Eu} \ , \quad \mathrm{Eu} = 0.115 + 0.354 C_{pi} / R_i \ ,$$

$$(\bar{\lambda}, \bar{\lambda}_1', \bar{\lambda}_1') = (\lambda, \lambda_1', \lambda_2') / \mu_e c_{pe} \ , \ \lambda_i = 15 R \mu_i / m_i \ ,$$

$$\tilde{G}_{12} = \frac{[1 + (\lambda_1/\lambda_2)^{1/2} (m_1/m_2)^{1/4}]^2}{8^{1/2} [1 + (m_1/m_2)]^{1/2}} \ ,$$

$$\tilde{G}_{21} = \frac{\left[1 + (\lambda_2/\lambda_1)^{1/2} (m_2/m_1)^{1/4} \right]^2}{8^{1/2} \left[1 + (m_2/m_1) \right]^{1/2}} \ .$$

Viscosity and thermal conductivity coefficients of a monoatomic gas according to the kinetic theory can be expressed as:

$$\mu_i^* = 2.6693\cdot 10^{-6}\frac{\sqrt{m_i T^*}}{d_i^2\Omega^{(2,2)^*}}\ ,\ \lambda_i^* = \frac{15}{4}\frac{R}{m_i}\mu_i^*\ .$$

The dimensionless coefficient of binary diffusion is

$$\bar{D}_{12} = \frac{\rho_e}{\mu_e}D_{12} = 262.8\cdot 10^{-5}\frac{\left[T^3\left(m_1+m_2\right)/2m_1m_2\right]^{1/2}}{P\sigma_{12}^2\Omega^{(1,1)*}}\frac{\rho_e}{\mu_e},$$

where D_{12} is in cm^2/s, $\sigma_{12} = \left(d_1+d_2\right)/2$ – in angstroms, P – in bars, T – in $^\circ K$. Collision integrals $\Omega^{(1,1)*}$ and $\Omega^{(2,2)*}$ have been calculated using Leonard-Jones potential, according to [13].

2.2 Stability equations

To deduce equations of linear stability we use the relation:

$\nabla T^* = \dfrac{\nabla h^*}{C_p} - \dfrac{h^*{}_1 - h^*{}_2}{C_p}\nabla c$, where C_p is the specific heat of a frozen mixture. Neglecting the Dufour effect we can also write:

$$\mathrm{div}\,\mathbf{q}^* = -\mathrm{div}\left[\lambda^*\left(\frac{\nabla h^*}{C_p^*} - \frac{h_1^*-h_2^*}{C_p^*}\nabla c_1\right)\right] + \mathrm{div}\left[\left(h_1^*-h_2^*\right)\mathbf{j}_1^*\right]$$

.

In the case of linear disturbances the dimensionless flow parameters can be written as $\bar{g}^* = \bar{g} + g'$, where $\bar{g}$ are solutions of stationary boundary-layer equations. Neglecting foreign gas thermal diffusion, for a parallel flow approximation and under simplifications used in [17, 18] the linearization of the system (1-3) with respect to perturbations and using the variable $d\tau = U_e dt/\delta$ leads to the equations:

$$\begin{gathered}
\bar{\rho}\left(\frac{du'}{d\tau}\right) = -\frac{1}{\gamma_e \mathrm{M}_e^2}\frac{\partial p'}{\partial X} + \frac{\bar{\mu}}{\mathrm{Re}}\frac{\partial^2 u'}{\partial y^2}\ ,\\
\bar{\rho}\left(\frac{\partial v'}{\partial \tau}\right) = -\frac{1}{\gamma_e \mathrm{M}_e^2}\frac{\partial p'}{\partial y}\ ,\\
\bar{\rho}\left(\frac{dw'}{d\tau}\right) = -\frac{1}{\gamma_e \mathrm{M}_e^2}\frac{\partial p'}{\partial Z} + \frac{\bar{\mu}}{\mathrm{Re}}\frac{\partial^2 w'}{\partial y^2}\ ,\\
\frac{d\rho'}{d\tau} + \bar{\rho}\left(\frac{\partial u'}{\partial X} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial Z}\right) = 0\ ,\\
\bar{\rho}\left(\frac{dh'}{d\tau}\right) = \frac{\gamma_e - 1}{\gamma_e}\left(\frac{dp'}{d\tau}\right) + \bar{\mu}\frac{1}{\mathrm{RePr}}\frac{\partial^2 h'}{\partial y^2} + \\
+\bar{\mu}\frac{1}{\mathrm{Re}}\left(\bar{h}_1 - \bar{h}_2\right)\left(\frac{1}{\mathrm{Sm}} - \frac{1}{\mathrm{Pr}}\right)\frac{\partial^2 c_1'}{\partial y^2}\ ,\\
\left(\frac{dc_1'}{d\tau}\right) = \frac{\bar{\mu}}{\mathrm{Sm\,Re}}\frac{\partial^2 c_1'}{\partial y^2}\ ,\\
\frac{p'}{\bar{P}} = \frac{\rho'}{\bar{\rho}} + \frac{h'}{\bar{h}} + \left[\frac{\left(R_1-R_2\right)}{\bar{R}} - \frac{\left(C_{p1}-C_{p2}\right)}{C_p}\right]c'\ ,
\end{gathered}\tag{7}$$

where $\dfrac{d}{dt} = \dfrac{\partial}{\partial\tau} + \bar{U}\dfrac{\partial}{\partial X}$.

We seek the solution to (7) as a harmonic in space and time perturbation: $q' = \tilde{q}\left(\bar{y}\right)\exp\left[i\alpha\left(X - C\tau\right) + i\beta Z\right]$. Then we come to the system of ODEs:

$$\begin{gathered}
i\alpha(\bar{U}-C)\tilde{\rho} + \frac{d\bar{\rho}}{dy}\tilde{v} + \bar{\rho}\left(i(\alpha\tilde{u} + \beta\tilde{w}) + \frac{d\tilde{v}}{dy}\right) = 0\ ,\\
\bar{\rho}\left(i\alpha(\bar{U}-C)\tilde{u} + \frac{d\bar{U}}{dy}\tilde{v}\right) = -\frac{i\alpha\tilde{p}}{\gamma_e\mathrm{M}_e^2} + \frac{\bar{\mu}}{\mathrm{Re}}\frac{d^2\tilde{u}}{dy^2}\ ,\\
\bar{\rho}i\alpha\left(\bar{U}-C\right)\tilde{v} = -\frac{1}{\gamma_e\mathrm{M}_e^2}\frac{d\tilde{p}}{dy}\ ,\\
\bar{\rho}i\alpha(\bar{U}-C)\tilde{w} = -\frac{i\beta\tilde{p}}{\gamma_e\mathrm{M}_e^2} + \frac{\bar{\mu}}{\mathrm{Re}}\frac{d^2\tilde{w}}{dy^2}\ ,\\
i\alpha\left(\bar{U}-C\right)\tilde{c}_1 + \frac{d\bar{c}_1}{dy}\tilde{v} = \frac{\bar{\mu}}{\mathrm{Re\,Sm}}\frac{d^2\tilde{c}_1}{dy^2}\ ,\\
\bar{\rho}\left(i\alpha\left(\bar{U}-C\right)\tilde{h} + \frac{d\bar{h}}{dy}\tilde{v}\right) = \frac{\gamma_e-1}{\gamma_e}i\alpha\left(\bar{U}-C\right)\tilde{p} + \\
+\frac{\bar{\mu}}{\mathrm{Re\,Pr}}\frac{d^2\tilde{h}}{dy^2} + \frac{\bar{\mu}}{\mathrm{Re}}\left(\bar{h}_1-\bar{h}_2\right)\left(\frac{1}{\mathrm{Sm}} - \frac{1}{\mathrm{Pr}}\right)\frac{d^2\tilde{c}_1}{dy^2}.
\end{gathered}\tag{8}$$

Here $\alpha C = \omega = \omega^*\delta/U_e$, $\omega = 2\pi f\delta/U_e = F\,\mathrm{Re}$, $C = \omega/\alpha = F\,\mathrm{Re}/\alpha$, $F = 2\pi f\mu_e/\rho_e U_e^2$ – is the reduced frequency, while f is the dimensional frequency in Hertz. System (8) is solved with the following homogeneous boundary conditions:

$$\begin{gathered}
\left(u,\ w,\ h,\ f_w c_1 - \bar{\rho}_w\bar{D}_{12}\frac{dc_1}{dY}\right) = 0\ \text{ at }\ \left(Y=0\right),\\
\left(u, w, h, c_1\right)\to 0\ \text{ at }\ \left(Y\to\infty\right).
\end{gathered}\tag{9}$$

The numerical integration of the eigenvalue problem (8-9) have been performed by means of method of orthonormalizations [15].

3 Results

In this paper we present results of the boundary-layer computations performed for the flow of air over the flat-plate at Mach numbers M=2 and 0.7. Various foreign gases (such as tetrachloromethane CCl_4, Xenon *Xe*, Helium *He*) have been injected through a permeable surface of the plate. Calculations have also been performed to reveal particularly influence on the boundary layer properties of a ratio of molecular mass of a foreign gas to the main gas at identical specific heat. The described below results are presented in the dimensionless form: all physical quantities are divided to their values at the boundary layer outer edge, while the normal coordinate is referenced to the Blasius length scale $\delta = \sqrt{x\mu_e / U_e \rho_e}$.

Fig.1 shows computed distribution of concentration of tetrachloromethane with $m_1 = 154$, that is five times heavier than air, across the boundary layer, for various values of the injection factor $-f_w$. One can see that growth of the foreign gas injection leads to the concentration c_1 increase at the wall. It is worth to note here that binary-mixture boundary-layer velocity and temperature profiles are only weakly dependent on the injection factor variation at least in the range $0 \le -f_w \le 0.2$.

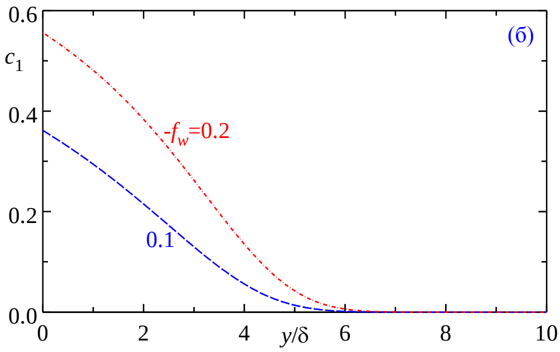

Fig.1: Distribution of the mass concentration of CCl_4 across the boundary layer for two values of $-f_w$.

Influence of an injection and a molecular mass of foreign gases on local drag is presented at Fig.2. The skin friction coefficient decreases with increasing injection independently of the molecular mass of the injected gas. Since molecular masses of *He*, *Air* and *Xe* are 4, 29 and 131 correspondingly, we see that the skin friction decreases with decreasing molecular mass of a foreign gas.

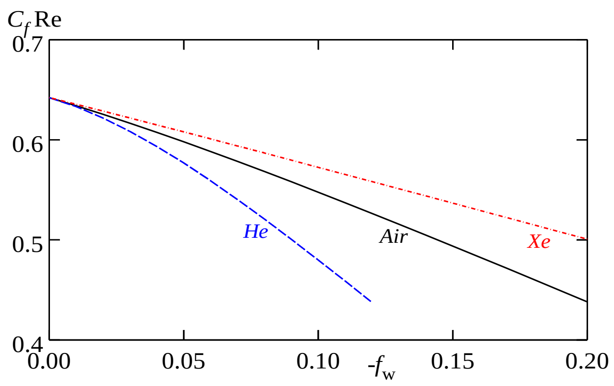

Fig.2: Normalized skin friction coefficient C_f Re versus injection factor for the adiabatic wall; $\mathrm{M} = 2$.

In Fig.3 the dependence of the local skin friction on the wall temperature is shown. The local friction drag decreases monotonously with increasing T_w for all foreign gases. As before the friction drag increases with increasing foreign gas molecular mass.

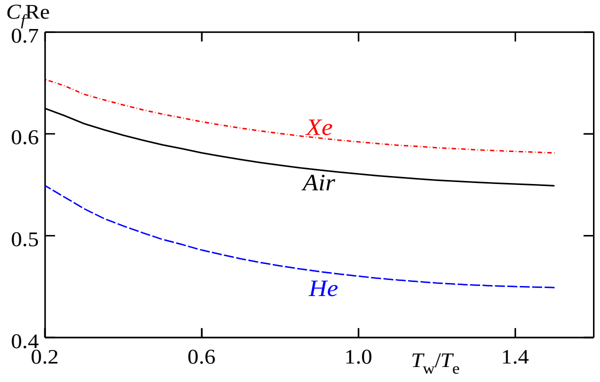

Fig.3: Normalized skin friction coefficient C_f Re versus wall temperature T_w/T_e; $\mathrm{M} = 2$, $-f_w = 0.1$.

The heat transfer data are not so uncomplicated as friction data. Fig.4 shows influence of the wall temperature on the ratio of Stanton number with injection factor $-f_w = 0.1$ to its value with no injection. One can see that only at $T_w/T_e > 0.36$ the results for helium are in conformity with the skin friction results described above. That is the decrease of the foreign gas molecular mass leads to a reduction of the wall heat transfer. However in the range $T_w/T_e < 0.3$ injection of helium gives higher heat flux in comparison with injection of air and xenon. So, decrease of the wall heat flux is achieved by the injection of *He* only for not very cold wall, while at lower values of T_w the opposite effect is observed.

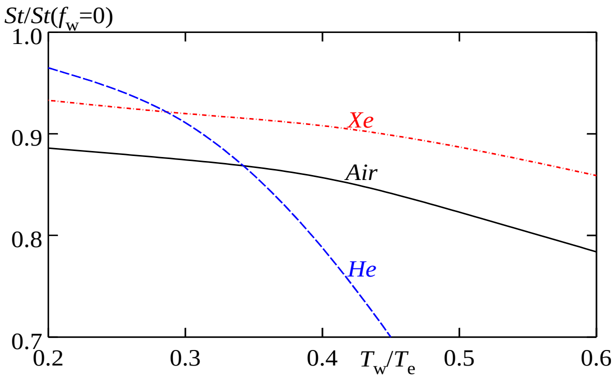

Fig.4: Influence of the wall temperature on Stanton number at M=0.7.

Stability of compressible boundary layers of binary gas mixtures depends on lots of parameters. The values of individual gases which are of importance for calculation of binary mixture viscosity, thermal conductivity and diffusion coefficients on the basis of kinetic theory in the framework of the Leonard-Jones potential are listed in table 1.

Table 1

	m_1	d_1, A	ε/k_1, °K	C_p
H_2	2	2.915	38	14320
He	4	2.576	10	5190
Ne	20	2.789	36	1030
Air	29	3.617	97	1004
Kr	84	3.498	225	248
Xe	131	4.055	229	160
SF_6	146	4.268	233	860
CCl_4	154	5.881	327	866

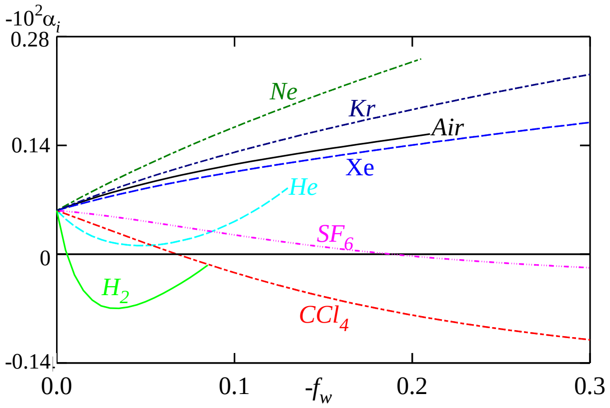

Fig.5: Spatial amplification rate $-\alpha_i$ versus injection factor $-f_w$ for variety of foreign gases; $\mathrm{Re}=720$, $F{\cdot}10^6=35$.

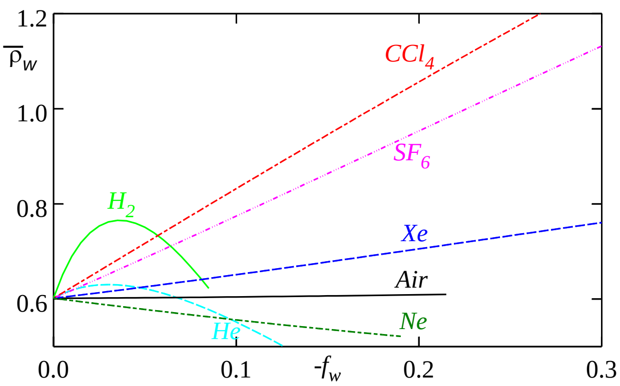

Fig.6: Density of the mixture at the wall $\bar{\rho}_w$ versus injection factor $-f_w$.

Fig.5 shows computed dependencies of the spatial amplification rate $-\alpha_i$ on the injection factor $-f_w$ for a number of foreign gases. The primary gas was always air. These functions are generally monotonous excepting dependences for hydrogen and helium. It is important to note that injection of such heavy gases as sulfur hexafluoride (SF_6) or tetrachloromethane (CCl_4) leads to a decrease of the spatial amplification rates that is to the boundary layer stabilization. This result is in agreement with the known earlier stabilizing influence of heavy gas injection. From table 1 we see that CCl_4 and SF_6 is heavier than air in 5.3 and 5.0 times. Greater efficiency in the boundary-layer stabilization by the injection of CCl_4 in comparison with SF_6 is explained by higher molecular mass of the former gas.

Destabilization of the boundary layer by the injection of heavy noble gases (Kr, Xe) is explained by their low specific heat in comparison with that of for air (table 1). The importance of an injected foreign gas specific heat becomes obvious in comparing amplification rate change at injection of krypton and air. Injection of heavy krypton gives faster the destabilization of the boundary layer than injection of air. This is because specific heat of krypton is four times smaller than that of for air.

Influence of the foreign gas molecular mass on the boundary layer stability is easier to understand by analyzing the density of a gas mixture near the wall. Fig.6 demonstrates mixture density at the wall $\bar{\rho}_w$ as a function of the injection factor $-f_w$ for the same gases like at Fig.5. Here we see a complete correlation of results presented in Figs.5 and 6. Increase of $\bar{\rho}_w$ due to the injection of a heavy foreign gas causes reduction of perturbation growth rates and vice versa.

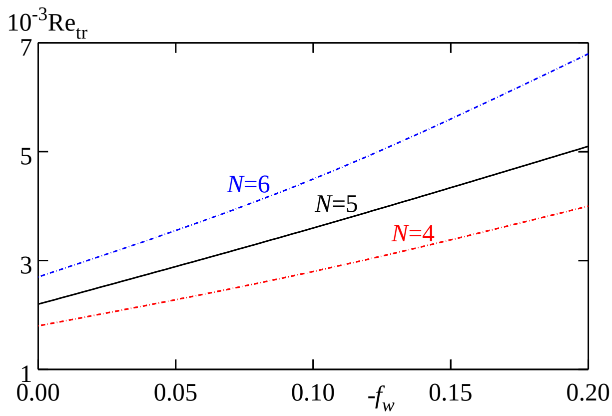

Fig.7: Transition Reynolds number Re_{tr} versus injection factor $-f_w$ for CCl_4, as calculated by the e^N-method; $\mathrm{M}=2$.

Linear stability theory gives a possibility to estimate the position of the laminar- turbulent transition by means of the well known e^N-method. According to this method the transition happens at the streamwise location where the disturbance amplification factor reaches a certain threshold e^N, where the factor $N=-\int_{\mathrm{Re}_0}^{\mathrm{Re}_{tr}} 2\,\mathrm{Im}(\alpha)d\,\mathrm{Re}$ has a certain value. Since amplitude and spectrum of external disturbances vary in different wind tunnels and in the flight, the transition appears at different values of *N*. Therefore, in this paper the estimate of the transition Reynolds number Re_{tr} has been performed for different values of the *N*-factor. The results for the injection of CCl_4 are presented at Fig.7. One can see that variation of the injection factor from zero to the value $-f_w = 0.2$ causes more than a double enlargement of the transition Reynolds number Re_{tr} independently of a value of the *N*-factor. This corresponds to a fourfold enlargement of the streamwise extent of a laminar region on a model.

4 Conclusions

System of equations of the compressible binary gas mixture boundary-layer in the approximation of a local self-similarity at foreign gas injection from a permeable model surface and its stability is developed.

Parametric calculations of the flat-plate boundary-layer profiles have been performed. It has been found that heavy gas injection influences the boundary-layer density profile similar to the influence of surface cooling. Both of them facilitate boundary-layer stabilization and noticeable laminar-turbulent transition delay.

It is obtained that increase of a foreign gas injection leads to a reduction of the skin friction and the wall heat flux. It is established that injection of a heavier gas leads to the reduction of perturbation growth rates. A possibility to enlarge in four times the length of a laminar region in Mach=2 boundary layer by means of a heavy gas injection is found.

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