Outage Probability of Wireless Relay Communication System with Three Sections in the Presence of Nakagami-m Short Term Fading

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Abstract: - The wireless relay communication system with three sections operating over Nakagami-m multipath fading channel is the topic of this work. The outage probability of proposed relay system is calculated for two cases. In the first case, the outage probability is evaluated when it is defined as probability that signal envelope falls below the specified threshold at any section. In the second case, the outage probability is calculated when it is defined as probability that output signal envelope is lower than predetermined threshold. For the first case, the outage probability is calculated by using cumulative distribution function of minimum of three Nakagami-m random variables. For the second case, the outage probability is evaluated by using the cumulative distribution function of product of three Nakagami-m random variables. Numerical expressions for the outage probability of wireless relay communication system are presented graphically and the influence of Nakagami-m parameter from each section on the outage probability is estimated.

Key-Words: - Nakagami-m short term fading; relay communication system; outage probability

1 Introduction

Refractions, reflections, deffractions and scatterings cause multipath propagation resulting in degradation of the outage performance of wireless relay communication radio system [1], [2].

Certain numbers of channel models exist to describe the statistics of the amplitude and phase of multipath fading signals. Nakagami-m distribution has some advantages versus the other models, such as that this is a generalized distribution which can model different fading environments. It has greater flexibility and accuracy in matching some experimental data than the Rayleigh, lognormal or Rice distributions, and also, Rayleigh and onesided Gaussian distribution are special cases of Nakagami-m model. So the Nakagami-m channel model is of more general applicability in practical fading channels [3] - [8].

Nakagami-m fading modelling in the frequency domain is investigated in [6]. For frequency-Nakagami-m fading selective channels, the magnitudes of the channel frequency responses to be Nakagami-m distributed random variables with fading and mean power parameters as explicit functions of the fading and mean power parameters of the channel impulse responses are shown. Based on this model, the bit error rate (BER) performance of an orthogonal frequency-division multiplexing system with receive diversity over correlated Nakagami-m fading channels is analytically evaluated.

BER of band-limited binary phase-shift keying in a fading and cochannel interference (CCI) environment is derived for the case of perfect coherent detection in [7]. The assumed fading-andinterference model is general and of interest for microcellular systems. The model allows both desired signal and interfering signals to experience arbitrary amounts of fading severity.

In this paper, considered wireless relay system has three sections. In sections, Nakagami-m channel is present. This channel can be denoted as Nakagami- Nakagami- Nakagami channel. It has three parameters which are denoted with m_1 , m_2 and m3. Also, Nakagami- Nakagami- Nakagami relay channel is general channel and several channels can be derived from this channel. For $m_l=1$, Nakagami-Nakagami- Nakagami channel becomes Rayleigh-Nakagami- Nakagami channel; for $m_1=1$ and $m_2=1$, Nakagami- Nakagami channel becomes Rayleigh - Rayleigh - Nakagami channel, and for $m_1=1$, $m_2=1$ and $m_3=1$, Nakagami- Nakagami-Nakagami channel becomes Rayleigh- Rayleigh -Rayleigh channel. For $m_1=0.5$, Nakagami-Nakagami- Nakagami channel becomes One sided Gaussian- Nakagami- Nakagami channel; for $m_1 = 0.5$ and $m_2 = 0.5$, Nakagami-Nakagami-Nakagami channel becomes One sided Gaussian-One sided Gaussian-Nakagami channel, and for $m_1=1/2$, $m_2=1/2$ and $m_3=1/2$, Nakagami- Nakagami-Nakagami channel becomes One sided Gaussian-One sided Gaussian- One sided Gaussian relay channel. Also, for m_1 goes to infinity, Nakagami-Nakagami- Nakagami relay channel becomes no fading- Nakagami- Nakagami channel, for m_1 goes to infinity and m_2 goes to infinity, Nakagami-Nakagami- Nakagami relay channel becomes no fading- no fading - Nakagami relay channel, and for m_1 goes to infinity, m_2 goes to infinity and m_3 goes to infinity, Nakagami- Nakagami- Nakagami relay channel becomes no fading- no fading - no fading channel.

The wireless relay system with two sections in the presence of κ - μ and η - μ multipath fading is processed in [9]. The wireless relay communication mobile radio system with two sections, subjected to κ - μ short term fading is considered in [10]. The outage probability is derived and parameters influence is analyzed.

In this work, the outage probability of wireless communication relay radio mobile system with three sections operating over Nakagami multipath fading channel is considered. For relay system, the outage probability can be defined at two manners. In the first case, the outage probability is defined as probability that signal envelope at any section falls below the specified threshold. For this definition, the outage probability can be calculated by using cumulative distribution function of minimum of three Nakagami random variables.

At the second manner, the outage probability is defined as probability that signal envelope at output

of relay mobile radio system is lower than specified threshold. Signal envelope at the output of relay system with three sections can be written as product of three Nakagami random variables. Therefore, the outage probability by the second definition can be evaluated by using cumulative distribution function of product of three Nakagami random variables.

In this paper, probability density function and cumulative distribution function of minimum of three Nakagami random variables and product of three Nakagami random variables are calculated. The joint probability density function of minimum of three Nakagami random variables can be calculated and used for calculation the level crossing rate of minimum of three Nakagami random processes. By our cognition, the results obtained in this paper for the outage probability of wireless relay system in Nakagami-Nakagami channel is not reported in open technical literature.

2 Statistics of Minimum of Three Nakagami Random Variables

Random variables x_1 , x_2 and x_3 follow Nakagami-*m* distribution:

$$p_{x_{1}}(x_{1}) = \frac{2}{\Gamma(m_{1})} \left(\frac{m_{1}}{\Omega_{1}}\right)^{m_{1}} x_{1}^{2m_{1}-1} e^{-\frac{m_{1}}{\Omega_{1}}x_{1}^{2}}, x_{1} \ge 0, \quad (1)$$

$$p_{x_2}(x_2) = \frac{2}{\Gamma(m_2)} \left(\frac{m_2}{\Omega_2}\right)^{m_2} x_2^{2m_2 - 1} e^{-\frac{m_2}{\Omega_2} x_2^2}, x_2 \ge 0, \quad (2)$$

$$p_{x_3}(x_3) = \frac{2}{\Gamma(m_3)} \left(\frac{m_3}{\Omega_3}\right)^{m_1} x_3^{2m_3 - 1} e^{-\frac{m_3}{\Omega_3}x_3^2}, x_3 \ge 0, \quad (3)$$

where $\Gamma(.)$ is the (complete) gamma function [11].

Cumulative distribution functions (CDF) of x_1 , x_2 and x_3 are:

$$F_{x_{1}}(x_{1}) = \frac{1}{\Gamma(m_{1})} \gamma\left(m_{1}, \frac{m_{1}}{\Omega_{1}}x_{1}^{2}\right), x_{1} \ge 0, \qquad (4)$$

$$F_{x_2}(x_2) = \frac{1}{\Gamma(m_2)} \gamma\left(m_2, \frac{m_2}{\Omega_2} x_2^2\right), \ x_2 \ge 0, \qquad (5)$$

$$F_{x_3}(x_3) = \frac{1}{\Gamma(m_3)} \gamma \left(m_3, \frac{m_3}{\Omega_3} x_3^2 \right), \ x_3 \ge 0, \qquad (6)$$

Minimum of x_1 , x_2 and x_3 is:

$$x = \min(x_1, x_2, x_3)$$
(7)

Probability density function (PDF) of *x* is:

$$p_{x}(x) = p_{x_{1}}(x)F_{x_{2}}(x)F_{x_{3}}(x) +$$

$$+p_{x_{2}}(x)F_{x_{1}}(x)F_{x_{3}}(x) + p_{x_{3}}(x)F_{x_{1}}(x)F_{x_{2}}(x) =$$

$$= \frac{2}{\Gamma(m_{1})} \left(\frac{m_{1}}{\Omega_{1}}\right)^{m_{1}} x_{1}^{2m_{1}-1}e^{-\frac{m_{1}}{\Omega_{1}}x^{2}} \frac{1}{\Gamma(m_{2})}\gamma\left(m_{2},\frac{m_{2}}{\Omega_{2}}x^{2}\right) \cdot$$

$$\cdot \frac{1}{\Gamma(m_{3})}\gamma\left(m_{3},\frac{m_{3}}{\Omega_{3}}x^{2}\right) + \frac{2}{\Gamma(m_{2})} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} x^{2m_{2}-1}e^{-\frac{m_{2}}{\Omega_{2}}x^{2}} \cdot$$

$$\cdot \frac{1}{\Gamma(m_{1})}\gamma\left(m_{1},\frac{m_{1}}{\Omega_{1}}x^{2}\right) \cdot \frac{1}{\Gamma(m_{3})}\gamma\left(m_{3},\frac{m_{3}}{\Omega_{3}}x^{2}\right) +$$

$$+ \frac{2}{\Gamma(m_{3})} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{1}} x^{2m_{3}-1}e^{-\frac{m_{3}}{\Omega_{3}}x^{2}} \cdot \frac{1}{\Gamma(m_{1})}\gamma\left(m_{1},\frac{m_{1}}{\Omega_{1}}x^{2}\right) \cdot$$

$$\cdot \frac{1}{\Gamma(m_{2})}\gamma\left(m_{2},\frac{m_{2}}{\Omega_{2}}x^{2}\right), x \ge 0$$

$$(8)$$

Cumulative distribution function of minimum of three Nakagami-*m* random variables is:

$$F_{x}\left(x\right) = \int_{0}^{x} dt \ p_{x}\left(t\right) =$$

$$F_{x}\left(x\right) = \left(1 - \left(1 - F_{x_{1}}\left(x\right)\right) \cdot \left(1 - F_{x_{2}}\left(x\right)\right) \cdot \left(1 - F_{x_{3}}\left(x\right)\right)\right) =$$

$$= 1 - \left(1 - \frac{1}{\Gamma\left(m_{1}\right)}\gamma\left(m_{1}, \frac{m_{1}}{\Omega_{1}}x^{2}\right)\right) \cdot \left(1 - \frac{1}{\Gamma\left(m_{2}\right)}\gamma\left(m_{2}, \frac{m_{2}}{\Omega_{2}}x^{2}\right)\right) \cdot \left(1 - \frac{1}{\Gamma\left(m_{3}\right)}\gamma\left(m_{3}, \frac{m_{3}}{\Omega_{3}}x^{2}\right)\right), x \ge 0$$

$$\left(1 - \frac{1}{\Gamma\left(m_{3}\right)}\gamma\left(m_{3}, \frac{m_{3}}{\Omega_{3}}x^{2}\right)\right), x \ge 0$$

$$(9)$$

In previous expressions, parameter m_1 is severity parameter of Nakagami-m fading in the first section, m_2 is severity parameter of Nakagami-m fading in the second section and m_2 is the severity parameter of Nakagami-m fading in the third section. The Ω_1 is signal envelope average power in the first section, Ω_2 is signal envelope average power in the second section and Ω_3 is signal envelope average power in the third section.



Fig.1. PDF of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 2$.



Fig. 2. The outage probability of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 2$.



Fig. 3. PDF of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$.



Fig. 4. The outage probability of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$.

Probability density functions of *x* are shown in Figs. 1. and 3 versus of minimum of three Nakagami-m random variables. Severity parameters of Nakagami-m fading are $m_1 = m_2 = m_3 = 2$ in Fig. 1. and $m_1 = m_2 = m_3 = 3$ in Fig. 2. Signal envelope average powers are $\Omega_1 = \Omega_2 = \Omega_3 = 1$ in both figures.

In Figs. 2 and 4, the outage probability in terms of minimum of three Nakagami-m random variables are shown for several values of severity Nakagami parameters and several values of signal envelopes average powers in sections. The outage probability decreases when severity Nakagami parameter m_1 in the first section increases, severity Nakagami parameter m_2 in the second section increases, and severity Nakagami parameter m_3 in the third section increases. The influence of severity Nakagami parameter in the first section on the outage probability is the highest for higher values of severity Nakagami parameters in the second section and in the third section. The outage performance is better as signal envelope average power in the first section increases, when, also, signal envelope average power in the second section increases and when signal envelope average power in the third section increases. Signal envelope average power in the first section has higher influence on the outage probability when severity Nakagami parameters in the first, in the second and in the third section have higher values.

3 Statistics of Product of Three Nakagami Random Variables

Product of three Nakagami-m random variables is:

$$x = x_1 \cdot x_2 \cdot x_3$$
, $x_1 = \frac{x}{x_2 \cdot x_3}$ (10)

Conditional probability density function of *x* is:

$$p_x(x/x_2x_3) = \left|\frac{dx_1}{dx}\right| p_{x_1}\left(\frac{x}{x_2x_3}\right)$$
(11)

where:

$$\frac{dx_1}{dx} = \frac{1}{x_2 x_3} \,. \tag{12}$$

After substituting and averaging, probability density function of *x* becomes:

$$p_{x}(x) = \int_{0}^{\infty} dx_{2} \int_{0}^{\infty} dx_{3} \frac{1}{x_{2}x_{3}} p_{x_{1}}\left(\frac{x}{x_{2}x_{3}}\right) p_{x_{2}}(x_{2}) p_{x_{3}}(x_{3}) =$$

$$= \frac{2}{\Gamma(m_{1})} \frac{2}{\Gamma(m_{2})} \frac{2}{\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{1}}\right)^{m_{1}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot$$

$$x^{2m_{1}-1} \int_{0}^{\infty} dx_{2} x_{2}^{-1-2m_{1}+1+2m_{2}-1} e^{-\frac{m_{2}}{\Omega_{2}}x_{2}^{2}} \cdot$$

$$\cdot \int_{0}^{\infty} dx_{3} x_{3}^{-1-2m_{1}+1+2m_{3}-1} \cdot e^{-\frac{m_{1}}{\Omega_{1}}\frac{x^{2}}{x_{2}^{2}x_{3}^{2}} - \frac{m_{3}}{\Omega_{3}}x_{3}^{2}} =$$

$$= \frac{8}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{1}}\right)^{m_{1}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot x^{2m_{1}-1} \cdot$$

$$\cdot \int_{0}^{\infty} dx_{2} \ x_{2}^{2m_{2}-2m_{1}-1} e^{-\frac{m_{2}}{\Omega_{2}}x_{2}^{2}} \cdot \\ \cdot \left(\frac{m_{1}x^{2}\Omega_{3}}{\Omega_{1}m_{3}x_{2}^{2}}\right)^{m_{3}-m_{1}} \cdot K_{2m_{3}-2m_{1}} \left(2\sqrt{\frac{m_{1}x^{2}m_{3}}{\Omega_{1}\Omega_{3}x_{2}^{2}}}\right) = \\ = \frac{8}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{1}}\right)^{m_{1}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \\ x^{2m_{1}-1+2m_{3}-2m_{1}} \cdot \left(\frac{m_{1}\Omega_{3}}{\Omega_{1}m_{3}}\right)^{m_{3}-m_{1}} \\ \cdot dx_{2} \ x_{2}^{2m_{2}-2m_{3}-1} \ e^{-\frac{m_{2}}{\Omega_{2}}x_{2}^{2}} \cdot K_{2m_{3}-2m_{1}} \left(2\sqrt{\frac{m_{1}x^{2}m_{3}}{\Omega_{1}\Omega_{3}x_{2}^{2}}}\right)$$
(13)

Cumulative distribution function of *x* is:

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$$F_{x}(x) = \int_{0}^{\infty} dt \ p_{x}(t) =$$

$$= \int_{0}^{x} dt \ \frac{8}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{1}}\right)^{m_{1}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{1}}\right)^{m_{1}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{1}}\right)^{m_{1}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{2}}\right)^{m_{1}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{2}}\right)^{m_{3}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{2}}\right)^{m_{3}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{3}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{2}}\right)^{m_{3}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{3}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{3}}{\Omega_{2}}\right)^{m_{3}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{3}}{\Omega_{2}}\right)^{m_{3}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{3})} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{3})\Gamma(m_{3})} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{\Gamma(m_{3})\Gamma(m_{3})}$$

The integral I_1 is:

$$\begin{split} I_1 &= \int_0^\infty dx_2 \ x_2^{2m_2 - 2m_1 - 1} \ e^{-\frac{m_2}{\Omega_2} x_2^2} \cdot \int_0^\infty dx_3 \ x_3^{2m_3 - 2m_1 - 1} \ e^{-\frac{m_3}{\Omega_3} x_3^2} \cdot \\ &\quad \cdot \frac{1}{2} \left(\frac{\Omega_1}{m_1}\right)^{m_1} \cdot x_2^{2m_1} \cdot x_3^{2m_1} \gamma \left(m_1, \frac{x^2}{x_2^2 x_3^2} \frac{m_1}{\Omega_1}\right) = \\ &= \frac{1}{m_1} \frac{x^{2m_1}}{x_2^{2m_1} x_3^{2m_1}} \left(\frac{m_1}{\Omega_1}\right)^{m_1} \cdot \sum_{j_1 = 0}^\infty \frac{1}{(m_1 + 1)(j_1)} \frac{m_1^{j_1}}{\Omega_1^{j_1}} \frac{x^{2j_1}}{x_2^{2j_1} x_3^{2j_1}} e^{-\frac{m_1 x^2}{\Omega_1 x_2^2 x_3^2}} = \end{split}$$

=

$$\int_{0}^{\infty} dx_{2} \ x_{2}^{2m_{2}-2m_{1}-1-2j_{1}} \ e^{-\frac{m_{2}}{\Omega_{2}}x_{2}^{2}} \cdot \\ \cdot \frac{1}{2} \int_{0}^{\infty} dx_{3} \ x_{3}^{2m_{3}-2m_{1}-1-2j_{1}} \ e^{-\frac{m_{1}}{\Omega_{1}}x_{2}^{2}x_{3}^{2}} \frac{m_{3}}{\Omega_{3}}x_{3}^{2}} = \\ = \frac{1}{m_{1}} x^{2m_{1}} \cdot \sum_{j_{1}=0}^{\infty} \frac{1}{(m_{1}+1)(j_{1})} \frac{m_{1}^{j_{1}}x_{2}^{2j_{1}}}{\Omega_{1}^{j_{1}}} \\ \int_{0}^{\infty} dx_{2} \ x_{2}^{2m_{2}-2m_{1}-1-2j_{1}} \ e^{-\frac{m_{2}}{\Omega_{2}}x_{2}^{2}} \cdot \\ \left(\frac{m_{1}x^{2}\Omega_{3}}{\Omega_{1}x_{2}^{2}m_{3}}\right)^{m_{3}-m_{1}-j_{1}} \cdot K_{2m_{3}-2m_{1}-2j_{1}} \left(2\sqrt{\frac{m_{1}x^{2}m_{3}}{\Omega_{1}\Omega_{3}x_{2}^{2}}}\right)$$
(15)

By substituting the last expression in the expression (14), we obtain for cumulative distribution function of *x*:

$$F_{x}(x) = \frac{8}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \left(\frac{m_{1}}{\Omega_{1}}\right)^{m_{1}} \left(\frac{m_{2}}{\Omega_{2}}\right)^{m_{2}} \left(\frac{m_{3}}{\Omega_{3}}\right)^{m_{3}} \cdot \frac{1}{1-m_{1}} x^{2m_{1}} \cdot \sum_{j_{1}=0}^{\infty} \frac{1}{(m_{1}+1)(j_{1})} \frac{m_{1}^{j_{1}} x^{2j_{1}}}{\Omega_{1}^{j_{1}}} \cdot \frac{1}{1-m_{1}} x^{2m_{2}} \cdot \frac{1}{1-m_{1}} x^{2m_{2}} \cdot \frac{1}{1-m_{1}} x^{2m_{2}-2m_{3}-1} e^{-\frac{m_{2}}{\Omega_{2}} x^{2}_{2}} \cdot \frac{1}{1-m_{1}} \cdot K_{2m_{3}-2m_{1}-2j_{1}} \left(2\sqrt{\frac{m_{1}m_{3}x^{2}}{\Omega_{1}\Omega_{2}x^{2}}}\right)$$
(16)

Probability density functions of x are shown in Figs. 5. and 7 versus of product of three Nakagamim random variables. Severity parameters of Nakagami-m fading are $m_1 = m_2 = m_3 = 2$ in Fig. 5. and $m_1 = m_2 = m_3 = 3$ in Fig. 7. Signal envelope average powers are $\Omega_1 = \Omega_2 = \Omega_3 = 1$ in both figures.



Fig. 5. PDF of product of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 2$.



Fig. 6. The outage probability of product of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 2$.



Fig. 7. PDF of product of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$.



Fig. 8. The outage probability of product of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$.

In Figs. 6 and 8, the outage probability depending of product of three Nakagami-m random variables are shown for several values of severity Nakagami parameters and several values of signal envelopes average powers in sections. The outage probability decreases when severity Nakagami parameter m_1 in the first section increases, severity Nakagami parameter m_2 in the second section

increases, and severity Nakagami parameter m_3 in the third section increases.

4 Conclusion

In this paper, wireless mobile relay radio communication system with three sections operating over Nakagami small scale fading channel is considered. Nakagami- Nakagami- Nakagami relay channel is defined. For proposed relay system, the outage probability is determined.

The outage probability can be defined at two manners. To the first case, the outage probability is defined as probability that signal envelope at any section falls below the specified threshold. For this case, the outage probability can be calculated by using cumulative distribution function of minimum of three Nakagami random variables.

To the second manner, the outage probability is defined as probability that signal envelope at the output of wireless relay communication system with three sections is lower than the predetermined threshold. Signal envelope at the output of relay system with three sections can be written as product of signal envelopes at sections. Therefore, the outage probability for the second case can be derived from cumulative distribution function of product of three Nakagami random variables.

In this work, probability density functions and cumulative distribution functions of minimum of three Nakagami random variables and product of three Nakagami random variables are evaluated. Cumulative distribution function of minimum of three Nakagami random variables is derived in the closed form. Cumulative distribution function of product of three Nakagami random variables is obtained as expression with one integral. For the both cases, the outage probability decreases when severity parameters of Nakagami fading increase at any sections.

These results are useful for designing of wireless mobile relay radio communication system with more sections in the presence of gading.

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