Some questions of computational models in continuum mechanics

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Abstract: The aim of the work is to study some issues of establishing the generality of experimental and numerical methods, the possibilities of describing a continuous medium using discrete representations of computational mathematics, the influence of the choice of basis functions on calculation errors. The previously proposed model of continuum mechanics is discussed, including the general action of forces and distributed moments of forces, as well as the role of boundary conditions in Hamiltonian mechanics, especially in frequently used computational solutions to problems in mechanics. The Runge-Kutta difference scheme for a system of partial differential equations is analyzed. Examples are given.

Keywords: angular momentum, conservation laws, no symmetrical stress tensor, open systems, Runge-Kutta method, open system

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1. Introduction

The purpose of the work is to establish the generality of experimental and numerical methods, the adequacy of the description of the continuum using discrete representations of computational mathematics, and the study of the choice of basis functions for calculation errors. A possible reason for the fact that the impossibility of description using regular theories is practically recognized lies in the imperfection of mathematical models and calculation algorithms. The experiment deals with material objects, i.e. the results are averaged data over the elementary volumes, which can be presented in the form of an integral form. Computational mathematics always deals with dimensional quantities. Differential equations are related to the field representation of physical quantities and are obtained from integral equations under the additional condition that functions are smooth, i.e., the space of piecewise continuous functions becomes the space of smooth functions. Therefore, mathematically, we are dealing with another space, i.e., instead of discrete space (for example, piecewise-continuous), we work in a narrower-continuous one. These spaces have different representations when expanded in a Fourier series. The first can be represented by a Fourier series only in the region of smoothness (in difference schemes, between nodes with stitching at the nodes), in the second case, in the entire region. This means that an exact adequate representation of functions using difference schemes is impossible even for schemes of a high order of accuracy. You can only count on the optimal solution to the problem. Currently, there are two areas of research: one is related to the development and use of computational programs without proper detailed analysis, the other involves the use of the apparatus of mathematical physics without checking the convergence of the obtained asymptotic formulas and their practical verification by numbers. However, a mixed approach is needed. It is impossible to start solving a problem without an analytical study of the problem, at least on the simplest reference problems. Physics dictates the solution method and the choice of numerical scheme. This should be done especially carefully when calculating on parallel computers. Verification of the results of numerical studies by analytical methods and comparison with experiment make it possible to guarantee the reliability of the results.

Most problems are non-linear. This does not allow proving theorems and writing the theory in a general form. That is why the role of mathematical analysis and the construction of reference problems of the required type, containing the main features of the problems being solved, increases. Analysis is very important in constructing a physical picture of a phenomenon based on the results obtained by previous researchers by numerical or asymptotic methods. Errors in solving more "rough" problems can be an order of magnitude larger or of the same order as the desired values The question of building a model is very important. Closed systems and representations of fields are currently being investigated. The theory is based on Hamilton's formalism and Liouville's formula for a closed system. Mathematical properties are studied without boundary conditions [1-3] The phenomenological representation was also used: normal flows, volume and surface effects were evaluated, all forces were considered additive. Dimensions rushed to zero. It was at this stage that the phenomenological consideration of the observed changes in the modules of quantities for elementary volumes established the symmetry of the tensor for equilibrium conditions. Since mathematicians worked with one-dimensional and two-dimensional systems, everyone was satisfied with the idea of the symmetry of the tensor: at relatively low speeds, the results of determining quantities important for practice coincided with the experiment. Separate phenomena did not fit into the theory, in particular, turbulence, description of processes in nanostructures, and others. Experimental results have appeared that speak about the influence of not only the physical quantities themselves, but also their gradients
[4,5]. In mechanics, it is customary to consider the Lagrange function for non-interacting and collectively interacting particles for closed and open systems in the same way, which is doubtful, especially for metallic and ionic bonds. It is believed that the reliability of classical mechanics has been verified by experiment and kinetic theory. To rely on the kinetic theory, which gives the same results as continuum mechanics, do not allow the hypotheses underlying the kinetic theory. The theory is statistical and is suitable for a large number of particles. The distribution function depends on time through the time dependence of macro parameters. Hilbert's hypothesis is fulfilled, the consequence of which is the symmetry of the stress tensor. In all works, including the works of N.N. Bogolyubov [3], this hypothesis underlies the theory. The hypothesis assumes that the values of macroparameters can be calculated from the zeroth approximation for the equilibrium distribution function. Therefore, according to the hypothesis, the values for the equilibrium distribution function with the macroparameters found from the Euler equations and the Navier-Stokes equations coincide. As is known, the differences are significant in areas with large slopes. Any equilibrium distribution function ensures the symmetry of the stress tensor and the absence of the influence of the angular momentum. In the classical approach, the law of conservation of angular momentum is constructed, but not applied. Theoretically, this is due to the incomplete taking of the integral by parts when obtaining conservation laws and ignoring the out-integral term present in the Ostrogradsky-Gauss theorem [6]. The symmetry of the stress tensor leads to a violation of the “continuity” of the medium. Mathematically, this circumstance follows from the choice of the conditions for the balance of forces as the conditions for the equilibrium of an elementary volume.

The choice of joint conditions for the balance of forces and moments of forces leads to new formulations of the equations. Consequently, under the condition of the balance of forces, we arrive at one or another classical formulation of continuum mechanics.The transition to differential equations is carried out in two ways: using the main lemma (the theory of elasticity) or taking the integral by parts. When taking the integral by parts, as already noted, in addition to the well-known classical integrals of mechanics, an external integral term (the Ostrogradsky-Gauss theorem) should have appeared, but in mechanics it was ignored. The possibility of using the Ostrogradsky-Gauss theorem when constructing the stress tensor only in the normal direction has not been substantiated. There is no reason to choose a pressure equal to one third of the normal pressures on the sides of an elementary volume; Pascal's law for the equilibrium case is transferred to the non-equilibrium one.

Physical models differ from mathematical ones by replacing a point with finite objects, that is, open non-stationary systems that are close to reality are investigated. The openness of the system and the exchange of physical quantities changes the equations. The dimension of elementary volumes brings computational and physical models closer to real objects.

In addition to continuum models, stochastic methods are often used in mechanics and physics. Historically, methods for describing physical processes using distribution functions (Boltzmann's equations, the Leontovich kinetic equation, the Langevin method, the Fokker-Planck equation, the Wigner functions, etc.) arose first. Methods of molecular dynamics and Byrd's method appeared later, with the growth of the possibilities of technology, although Newton's equation was always used. Then the question arises about the computational method (analytical methods can rarely be used) based on the integral representation of the problem or on the differential formulation. The most well-known methods with an integral statement belong to the group of variational methods. More often-finite element method. The choice of basis function is important here. Approximation within the selected elementary volume is different for different methods. For systems of equations, transformations are sometimes made in order to obtain one equation from the system, albeit of a high order. The transition requires additional differentiation of the equations. Thus, once again the domain of definition of the problem changes.

Difference schemes can be constructed using variational principles or using direct approximation of differential equations. In the first case, as a rule, simple basis functions that are not smooth are used. In the variational formulation of the problem, the main attention is paid to the preliminary planned approximation of the function on a given subdomain (on a selected element); in the second case, for difference schemes, an approximation on a subdomain (selected grid cell) is built after solving the problem. From the integral formulation of the problem obtained in the experiment, one can obtain various differential equations. The equations are identical and transform into each other with a continuous distribution of quantities, which is what we have in field theory, but when passing to a discrete description, they differ from each other and do not transform into each other by any transformations, except for special cases. Sometimes it is possible to bring the results closer by applying the Lagrangian formulation of the problem and constructing conservative difference schemes. It is impossible to build a completely conservative difference scheme in the Cartesian coordinate system, but one can approach such a scheme [7]. For example, in the theory of elasticity or aeromechanics. In this case, additional requirements for the boundary conditions arise. It turns out a wide matrix even with the simplest approximation. They are currently trying to apply the Rune-Kutta method to solve a system of partial differential equations. An analysis of the error at the "claimed accuracy" of a fourth-order solution yields only second-order accuracy. It should be borne in mind that the most common difference scheme, the flow method, despite the integral formulation, does not correspond to the original formulation of the problem, since it proceeds from equations with a symmetric tensor.

2. A little about the models

The role of the Ostrogradsky-Gauss theorem was discussed earlier in [6]. Here we will focus on the phenomenological derivation of the momentum conservation law. In the phenomenological definition of the action of forces, the condition of equilibrium of an immobile elementary volume (without liquid) is used. The static distribution of forces is used (Fig. 1) and the equation of projections on the axes of Cartesian coordinates [8]

$$p_{nx} = n_x p_{xx} + n_y p_{yx} + n_z p_{zx} ,$$
$$p_{ny} = n_x p_{xy} + n_y p_{yy} + n_z p_{zy} ,$$
$$p_{nz} = n_x p_{xz} + n_y p_{yz} + n_z p_{zz} . \quad (1)$$

Based on this, it is concluded that

$$\boldsymbol{p_n} = \boldsymbol{n} \cdot P .$$

$$P = \begin{pmatrix} p_{xx} & p_{yx} & p_{zx} \\ p_{xy} & p_{yy} & p_{zy} \\ p_{xz} & p_{yx} & p_{zz} \end{pmatrix} \quad (2)$$

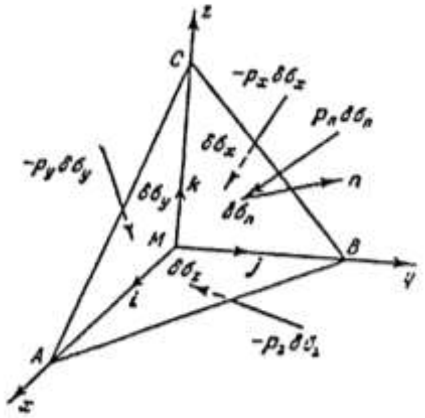

Fig. 1. Elementary volume used in the construction of a theory.

On the basis of this tensor for an equilibrium system, the equations of motion are written in integral form and the Ostrogradsky-Gauss theorem is used without an external integral term. A conclusion is also made about the symmetry of the stress tensor and the predominance of surface effects compared to volume ones. All continuum mechanics is based on these conclusions. The kinetic theory is based on a closed volume. Accordingly, flows through the boundary of the elementary volume are not taken into account. The Liouville equation, from which the rest of the equations are derived, is written for a closed system.

In [9-15], an algorithm for constructing a solution to a problem with an no symmetric tensor is given if the solution to the problem with a symmetric tensor is known. We give examples of solutions.

3. Influence of the moment in the Couette flow

Consider a flow between two parallel flat walls, one of which rests, and the other moves in its plane with a constant speed U. $\frac{dP}{dx}$ is a given function [8]. Further, u is the speed, y is the normal coordinate, h is the distance between the planes.

Let us trace the change of the solution taking into account the influence of the moment in comparison with the classical one.

When considering the fluid flow near an infinite plate, for our case we obtain

$$\frac{d}{dy}\left(\mu\frac{du}{dy}\right) + \frac{d}{dy}\left(\mu y\frac{d^2u}{dy^2}\right) = 0. \quad (3)$$

$$u = 0,\ \mu\frac{du}{dy} = \tau_w ,\ y = 0,$$

$$u = U_\infty, \quad y \to \text{h}$$

Integration gives

$$\mu\frac{du}{dy} + \mu y\frac{d^2u}{dy^2} = \text{const.} = \tau_w.$$

Due to the boundary conditions the constant is equal to τ_w and, as in a turbulent layer, the constant must be specified. Then

$$\text{i}u = C \cdot lny + \frac{\tau_w}{\mu} \cdot y + const. \quad (4)$$

A possible variant of satisfying the boundary conditions on the wall is that at $y = \frac{\nu}{\nu_*}$, where, $\nu = \frac{\mu}{\rho}$, $\nu_* = (\frac{\tau_w}{\rho_w})^{1/2}$, large, but the zero velocities at the two boundaries of the wall layer do not allow flow in the opposite direction. The thickness of the "resting" liquid at Reynolds numbers is 10-3 cm. Indeed, $C \cdot lny = 0$. A further decrease in velocity occurs before it vanishes, the derivative becomes very large for air at one atmosphere, when the plate moves at a speed of 300 km / h at a distance of 0.2 m with a total plate length of 2 m for m^2/ s.

It should be noted that such a profile will always be present inside the boundary layer and corresponds to the inertial interval (A. N. Kolmogorov). It should be noted that there is no asymptotic transition from a solution for a semi-infinite plate to a solution for an infinite plate. For a semi-infinite plate, friction at infinity tends to zero and at τ_w= 0 one can obtain the Prandtl-Karman mixing length. The classical version corresponds to the linear profile. Hence it can be concluded that the viscosity should work in two areas: directly near the surface and at the outer boundary. In the stationary case, the amount of dissipated energy near the external boundary will be equal to the amount of this energy near the wall. The behavior of the velocity u will depend mainly on the vertical component of the velocity and the perturbations of the longitudinal velocity at the outer boundary.

For our case, the classical equation and solution have the form

$$\frac{d}{dy}\left(\mu\frac{du}{dy}\right) = \frac{dP}{dx}, \quad \text{for } \mu = cons \qquad u = \frac{y}{h}U - \frac{h^2}{2\mu}\frac{dP}{dx}\frac{y}{h}\left(1-\frac{y}{h}\right). \qquad (5)$$

Equation with moment

$$\frac{d}{dy}\left(\mu\frac{du}{dy}\right) + \frac{d}{dy}\left(\mu y\frac{d^2u}{dy^2}\right) = \frac{dP}{dx}, \qquad (6)$$

$$u = \frac{dP}{\mu dx}\, lny + C\,\frac{y^2}{2} + Dy + B.$$

Near the surface, you can use the representation of ln y in a row.

$$lny = 2\,[\frac{x-1}{x+1} + \frac{(x-1)^3}{(x+1)^3} + \cdots \qquad (7)$$

Near the wall, where the velocities are small or zero, the solution becomes classical (no torque). An important factor is the identity of the formulas for laminar and turbulent motions. Let us consider a variant of movement at a low given speed of movement of the lower plate.

.$h_s = \varepsilon\cos kx$ is small

Friction must also be specified on this boundary. It can be assumed that since the velocity near the wall is small, there is an excess of boundary conditions, then to determine the constants it will be necessary to solve a system of equations and one of the solutions will be the value of friction without taking into account the moment.

4. Influence of the asymmetric stress tensor in the problem of steady oscillations of viscoelastic rectangular plates

The purpose of this part of the work is to illustrate a method for solving a three-dimensional problem with an no symmetric stress tensor [9,10]. Some provisions of the theory of elasticity are no longer valid for an no symmetric stress tensor. For example, for two opposite sides of an elementary volume, we obtain our own direction of principal stresses

$$tg\,2\theta_1 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}, \quad tg\,2\theta_2 = \frac{2\tau_{yx}}{\sigma_x - \sigma_y} \qquad (8)$$

and since $\boldsymbol{\tau_{xy}} \neq \boldsymbol{\tau_{yx}}$, we get different results. Thus, at each point there is a main direction of stresses. The common principal axis can only be determined in terms of integrals.

The problem of vibrational bending of viscoelastic rectangular plates and round cylindrical shells was solved in [16]. The basic formulas and equations are given, which are based on Kirchhoff's hypotheses for plates, deformations are considered small and obey the linear law of viscoelasticity. Here, the influence of the no symmetric stress tensor on the acting forces is traced. In the work

$$\sigma_x = \frac{1}{1-\nu^2}\int_{-\infty}^{t} K(t-\tau)[\,\varepsilon_x\,(\tau) + \nu\,\varepsilon_y\,(\tau)]\,d\tau, \quad (x \Leftrightarrow y), \qquad (9)$$

$$\tau_{xy} = \frac{1}{2\,(1+\nu)}\int_{-\infty}^{t} K\,(t-\tau)\gamma_{xy}(\tau)\,d\tau,$$

Solutions of problems on steady-state oscillations under the action of a load of the form

$$q(x,y,t) = \sum_{k=1}^{2} q_k\,(x,y)\;\cos\,[(k-1)\frac{\pi}{2} - \omega t\,] \qquad (10)$$

for a rectangular plate of finite dimensions, in which two opposite sides are hinged, and the other two are arbitrary. We have chosen a problem to illustrate the method for determining the no symmetry of the stress tensor. Even when using the Kirchhoff hypothesis without taking into account the deformation of the sections, the effect of the no symmetry of the stress tensor is significant. Consider a plate of small thickness $h = \text{const}$, in the Cartesian coordinate system, the Oxy plane of which is aligned with the middle plane of the plate. The plate is deformed by transverse load distributed over the face $z = -h/2$

$$q(x,y,t) = L\,[q_k\,(x,y)],$$

$$L\,(\varphi_k) = \sum_{-\infty}^{2} \varphi_k \cos\,\left[(k-1)\frac{\pi}{2} - \omega t\,\right]. \quad (11)$$

For projections of the displacement vector, according to the Kirchhoff hypothesis, we have

$$w = w\,(x,y,t),\;\; u = -z\frac{\partial w}{\partial x},\quad v = \frac{\partial w}{\partial y}.$$

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2},\;\; \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2},\;\; \gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y}.$$

It is assumed that the plate material obeys the linear viscoelastic law

$$\sigma_x = \frac{1}{1-\tilde{\nu}^2}\int_{-\infty}^{t} K\,(t-\tau)[\,\varepsilon_x(\tau) + \tilde{\nu}\varepsilon_y\,(\tau)d\tau,\;\; (x \Leftrightarrow y\,),$$

$$\tau_{xy} = \frac{1}{2\,(1+\tilde{\nu})}\int_{-\infty}^{t} K\,(t-\tau)\gamma_{xy}(\tau\,)d\tau, \quad (12)$$

where $\tilde{\nu}$ is Poisson's ratio and material properties do not increase with temperature. Deflection $w = w(x,y,t)$ is sought in the form $w\,(x,y,t) = L\,[w_k\,(x,y)]$,

$$\left\{\varepsilon_x\,, \varepsilon_y\,, \gamma_{xy}\,, \sigma_x\,, \sigma_y\;\,, \tau_{xy}\,\}\right) = L\left\{\varepsilon_x^{(k)}\,, {\varepsilon_y}^{(k)}\,, {\gamma_{xy}}^{(k)}\,, {\sigma_x}^{(k)}\,, {\sigma_y}^{(k)}\;\,, {\tau_{xy}}^{(k)}\right\},$$

$${\varepsilon_x}^{(k)}\;\,(x,y,t) = -z\frac{\partial^2 w_k}{\partial x^2},\;\; {\gamma_{xy}}^{(k)}\,(x,y,t) = -2z\frac{\partial^2 w_k}{\partial x \partial y}. \quad (13)$$

$${\sigma_x}^{(k)} = \frac{1}{1-\tilde{\nu}^2}\,L_E\,\left(\frac{\partial^2 w_i}{\partial x^2}(\tau) + \tilde{\nu}\,\frac{\partial^2 w_i}{\partial y^2}\right), \qquad (x \Leftrightarrow y\,),$$

$${\tau_{xy}}^{(л)} = \frac{1}{1+\tilde{\nu}}\,L_E\,\left(\frac{\partial^2 w_i}{\partial x \partial y}\right). \quad (14)$$

$$L_E\;\,(\,f_i\;) = \sum_{i=1}^{2}(-1)^i\,E_{i+k-1}f_i\,,$$

$$E_1 + \,iE_2\int_0^{\infty} K\,(s)e^{i\omega s}ds, \quad E_3 = -E_1\,.$$

Enter bending and torque moments

$$M_x(x,y,t) = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_x\,dz\;\,,$$

$$M_y(x,y,t) = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_y\,dz\;\,,$$

$$H_{xy}\,(x,y,t) = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\tau_{xy}\;dz\,, \quad (15)$$

$$Q_x\;(x,y,t) = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\tau_{xz}\;dz\;,\quad Q_y\;(x,y,t) = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\tau_{yz}\;dz\,,$$

$$\left\{M_x\,,\;M_y\,, H_{xy},\;Q_x\,, Q_y\,, \widetilde{Q_y}\,\right\} = L\left\{{M_x}^{(k)},\;{M_y}^{(k)}, {H_{xy}}^{(k)},\;{Q_x}^{(k)}, {Q_y}^{(k)}, \widetilde{Q_y}^{(k)}\right\}.$$

$\widetilde{Q_y}$ reduced shear force, determined by the relation

$$\widetilde{\mathrm{Q_y}} = \mathrm{Q_y} + \frac{\partial \mathrm{H_{xy}}}{\partial \mathrm{x}},$$

The quantities

$${M_x}^{(k)},\;\;{M_y}^{(k)}, {H_{xy}}^{(k)},\;\;{Q_x}^{(k)}, {Q_y}^{(k)}, \widetilde{Q_y}^{(k)}$$

are functions x, y. For ${M_x}^{(k)},\;{M_y}^{(k)}, {H_{xy}}^{(k)}$ we have

$${M_x}^{(k)} = L_D\;\left(\left(\frac{\partial^2 w_i}{\partial x^2} + \tilde{\nu}\,\frac{\partial^2 w_i}{\partial y^2}\right),\quad (x \Leftrightarrow y\,),\right.$$

$${H_{xy}}^{(k)} = \;(1-\widetilde{\nu})\,L_D\;\left(\frac{\partial^2 w_k}{\partial x \partial y}\right), \quad (16)$$

$$L_D\;\,(\,f_i\,) = \sum_{i=1}^{2}(-1)^i\;D_{i+k-1}f_i, \qquad D_k = \frac{E_k\,h^3}{12\,(1-\tilde{\nu}^2)},\;\; D_3 \;=\; -D_1\;.$$

Equations of motion of a plate element for bending vibrations

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \rho h\frac{\partial^2 w}{\partial t^2} = q\,(x,y,t), \quad (17)$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial H_{yx}}{\partial y} = Q_x, \quad \frac{\partial H_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} = Q_y,$$

ρ −plate material density.

$$\frac{\partial {Q_x}^{(л)}}{\partial x} + \frac{\partial {Q_y}^{(л)}}{\partial y} + \rho h\,\omega^2\,w_k = q_k\;\,(x,y),$$

$$\frac{\partial {M_x}^{(k)}}{\partial x} + \frac{\partial {H_{yx}}^{(k)}}{\partial y} = {Q_x}^{(k)},$$

$$\frac{\partial {H_{xy}}^{(k)}}{\partial x} + \frac{\partial {M_y}^{(k)}}{\partial y} = {Q_y}^{(k)}.$$

In our case, we add the equation for the moment

$$\left\{\varepsilon_x\,, \varepsilon_y\,, \gamma_{xy}\,, \sigma_x\,, \sigma_y\;\,, \tau_{xy}\,\}\right) = L\left\{\varepsilon_x^{(k)}\,, {\varepsilon_y}^{(k)}\,, {\gamma_{xy}}^{(k)}\,, {\sigma_x}^{(k)}\,, {\sigma_y}^{(k)}\;\,, {\tau_{yx}}^{(k)}\right\},$$

$$x\left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z}\right) - y\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z}\right) + \sigma_{yx} - \sigma_{xy} = 0$$

We write the equation in standard form

$\frac{\partial \sigma_{yx}}{\partial y} + \sigma_{yx}/y = \frac{x}{y}(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z}) + (\frac{\partial \sigma_{xx}}{\partial x} + + \frac{\partial \sigma_{zx}}{\partial z}) + \sigma_{xy}/y$ (18)

$\Delta_{xy} = \sigma_{yx} - \sigma_{xy}$

$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \Delta_{xy}}{\partial x} + \frac{\Delta_{xy}}{y} = \frac{x}{y}(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z}) + (\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zx}}{\partial z})$

$\Delta_{xy} = e^{-F}\,(C + \int_{\xi}^{x}(-\frac{\partial \sigma_{xy}}{\partial x} + \frac{x}{y}(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z}) + (\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zx}}{\partial z}))e^{F}dx)$

$F = \int_{\xi}^{x}\frac{1}{y}dx = \frac{1}{y}\,(x-y).$

Finally we get

$\sigma_{yx} = \sigma_{xy} - x(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z}) + y(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z}).$

We use the results of the solution for the symmetric tensor; if we take σ_{xy} as a basis, then the first item disappears.

$\sigma_{yx} = \sigma_{xy} - y(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z}),$ (19)

We need to write next equation for other coordinates. If there is a force, the parenthesis is replaced by that force. An iterative procedure is performed.

5. About some difference schemes

An important role, in addition to choosing a model, is the choice of a solution method. When solving problems of aeromechanics, the control volume method is often used at present by using for approcsimation on time Runge-Kutta schemes with various modifications [17,18]. The finite volume method is an integral method. If it comes from the initial experimental setting, then it grasps all the components involved in the change in value physical values in the volume. However, now the method is being built on the basis of existing differential equations. In addition, the method uses the values of functions on time layers without passing to intermediate points in time. Intermediate values are replaced by the introduction of additional coefficients. We give two versions of schemes that, after excluding intermediate calculations, do not lead to the Taylor series. Schemes are three-layer on time, explicit. The latter gives them an advantage over implicit ones. The latter gives them an advantage over implicit ones, but it is not to get a higher order than traditional explicitly implicit ones.

First, consider the classic version of the method

$y' = f\,(x,y), \qquad y(x_0) = y_{0.}$ (20)

$[x_0, X], \quad X > x_0.$

$x_1 = x_0 + h < X,\; h > 0.\; k_1,\; k_2,\; k_3,\; k_4$

$k_1 = h\,f(x_0, y_{0.}),$

$k_2 = hf(x_0 + \frac{h}{2}, \quad y_{0.} + \frac{k_1}{2})$

$k_3 = hf(x_0 + \frac{h}{2}, \; y_{0.} + \frac{k_2}{2})$

$k_4 = hf(x_0 + h, \quad y_{0.} + k_3)$

$y_1 = y_{0.} + \left(\frac{1}{6}\right)(k_1 + 2\,k_2 + 2\,k_3 + k_4)$

In the work, the implementation of the method is as follows

$Q_{i,j,k}^{(1)} = Q_{i,j,k}^{(0)} + \Delta t L\left(Q_{i,j,k}^{(0)}\right);$

$Q_{i,j,k}^{(2)} = \frac{3}{4}\,Q_{i,j,k}^{(0)} + \frac{1}{4}[Q_{i,j,k}^{(1)} + \Delta t\,L\,(Q_{i,j,k}^{(1)})];)$

$Q_{i,j,k}^{(2)} = \frac{1}{3}\,Q_{i,j,k}^{(0)} + \frac{2}{3}[Q_{i,j,k}^{(2)} + \Delta t\,L\,(Q_{i,j,k}^{(2)})];)$

$y^{(1)} = y^{(0)} + \Delta t\, f(x_0, y_{0.}),$ (21)

$y^{(2)} = \frac{3}{4}y^{(0)} + \frac{1}{4}[y^{(0)} + \Delta t\, f(x_0, y_{0.}) + \Delta t f(y^{(0)} + \Delta t\, f(x_0, y_{0.})\,]$

$y^{(3)} = \frac{1}{3}y^{(0)} + \frac{2}{3}\left\{\frac{3}{4}y^{(0)} + \frac{1}{4}\,[y^{(0)} + \Delta t\, f(x_0, y_{0.}) + \Delta t f\big(y^{(0)} + \Delta t\, f(x_0, y_{0.})\,\big] + \Delta t f\,\{\frac{3}{4}y^{(0)} + \frac{1}{4}[y^{(0)} + \Delta t\, f(x_0, y_{0.}) + \Delta t f\big(y^{(0)} + \Delta t\, f(x_0, y_{0.})\,]\}\right\}$

However, after expanding into a series and performing the summation, we have

$y^{(1)} = y^{(0)} + \Delta t\,\frac{dy^{(0)}}{dt} + \cdots$ (22)

The following term is equal to the coefficient 1/2 times $f(x_0, y_{0.})$ Therefore, we do not get the Taylor series.

It follows that the circuit has a second order of accuracy, but its stability increases.

Consider the simplest version of the linear approximation. Equation of a line passing through two points

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}.$$

By definition of the derivative on the right and on the left

$$\tilde{y} = y_1 + \varepsilon_1 \frac{y_2 - y_1}{x_2 - x_1}$$

$$\bar{\tilde{y}} = y_1 - \varepsilon_2 \frac{y_1 - y_0}{x_1 - x_0}$$

$$\frac{\left(\varepsilon_1 \frac{y_2 - y_1}{x_2 - x_1} - \varepsilon_2 \frac{y_1 - y_0}{x_1 - x_0}\right)}{\varepsilon_1 + \varepsilon_2} = \frac{dy}{dx}$$

For $\varepsilon_1 = \varepsilon_2$ we have Dirichlet conditions: a function satisfies the Dirichlet conditions in the interval (-π, π) if it is either continuous in this interval or has a finite number of discontinuities of the first kind, and if, in addition, the interval (-π, π) can be divided into tinite number such intervals, in each of which f(x) changes monotonically. In general case, the conditions are not met.

6. Correspondence between variational and difference formulations of problems in the theory of elasticity

In the theory of elasticity, when passing to a differential formulation of the problem, the equations are transformed in order to simplify them, acting on the resulting equations with the operators rot, div, which require differentiation of functions. For example, when obtaining a biharmonic equation.

Namely, the main task of the theory of elasticity (written in integral form, is the minimization of energy (the notation is standard : $\mathbf{V}$ –velocity, $f(\boldsymbol{V})$—function, ε_{ij} tensor of stress [19]

$$J(\boldsymbol{V}) = \frac{1}{2} \int_{\Omega} \{\lambda (div\, \boldsymbol{V})^2 + 2\mu \sum_{i,j}^{3} \left(\varepsilon_{ij}(\boldsymbol{V})\right)^2 \} dx - \left(\int_{\Omega} \boldsymbol{f} \cdot \boldsymbol{V}\, dx + \int_{\Gamma_i} g \cdot V\, d\gamma\right) = \frac{1}{2}\, a(\boldsymbol{V},\boldsymbol{V}) - f(\boldsymbol{V}) \quad (23)$$

In space

$$V \in (H^1(\Omega))^3; \quad \boldsymbol{V} = 0 \text{ on } \Gamma_0 .$$

allowable displacements, and the corresponding boundary value problem has the form

$$-\mu\Delta\, \boldsymbol{u} - (\boldsymbol{\lambda}+\boldsymbol{\mu})\text{grad div}\, \boldsymbol{u} = \boldsymbol{f} \text{ в } \Omega, \text{ на } \Gamma_0 .$$

$$\sum_{j=1}^{3} \sigma_{ij}(\boldsymbol{u})\boldsymbol{V_j} = g_i \quad \text{на} \quad \Gamma_i, \quad 1 \leq i \leq 3.$$

as already noted, in the case of application of surface forces, the div operation is applied to the equation. The result is a biharmonic equation

$$\Delta\Delta\mathbf{u} = 0 \quad (24)$$

At the same time, we narrow the class of solutions, requiring greater smoothness of the functions. This is especially important in the numerical solution of the problem. The use of smoother functions in the general case leads to a more filled matrix with the same requirements for the accuracy of the solution. The biharmonic equation often underlies a new variational formulation of the problem. After that, the domains of definition of the original problem and the new one are not required to coincide. We think we are solving the original problem. Therefore, in our opinion, it is better to use, if a differential statement is necessary, the representation of the problem in the form of a system of differential equations.

7. Conclusion

The paper presents studies of some issues of establishing the generality of experimental and numerical methods, the possibility of describing a continuous medium using discrete representations of computational mathematics, the influence of the choice of basis functions on calculation errors. The previously proposed model of continuum mechanics is discussed, including the combined action of forces and distributed moments of forces, the role of boundary conditions in Hamiltonian mechanics, and features of the most commonly used computational methods for solving problems in mechanics. The following cases are chosen: the Couette problem and consideration of the influence of the asymmetric stress tensor in the problem of establishing existing viscoelastic rectangular plates.

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